Statistical Mechanics WS 2013/14 Sheet 4

Please hand in your solutions (in pairs) before the Monday lecture.

Problem 1 Maxwell-Boltzmann Distribution (20 points)

a) (5 points) Write down the Maxwell-Boltzmann distribution $f_{\text{MB}}(v)$ in terms of the speed v and the mass m of a molecule, the Boltzmann's constant k_B, and the absolute temperature T.

b) (5 points) The molecular weight of the air is ≈ 0.029 kg/mol. Find the mean speed of the air at $T = 300$ K. For the calculation use values of the Avogadro's constant 6.02×10^{23} mol^{-1}, and $k_B = 1.38 \times 10^{-23}$ J K^{-1}.

c) (5 points) The escape speed v_{esc}, for an object to escape from the earth's gravitation, is known as $v_{\text{esc}} \approx$ 11200 meter/sec. Find the mass m_{esc} for a gas molecule to escape from the earth, at $T = 300$ K.

d) (5 points) Calculate the probability for the Hydrogen gas (molecular weight: 0.002 kg/mol) to exist around the moon's surface, using the Maxwell-Boltzmann distribution $f_{\text{MB}}(v)$. For the calculation use $v_{\text{esc}} \approx$ 2400 meter/sec for the escape speed on the moon, and $T \approx 400$ K for the maximum temperature on the moon. You will see that the probability is not small at all. Then, why actually there is no atmosphere on the moon? Explain.

Problem 2 Magnetic Fridge (30 points)

In problem 4 on the last sheet you learned how to calculate the entropy of a spin system with spin quantum number $S = 1$ in a magnetic field B. Imagine that the spin system is connected thermally to a box with an ideal gas as shown in figure 1.

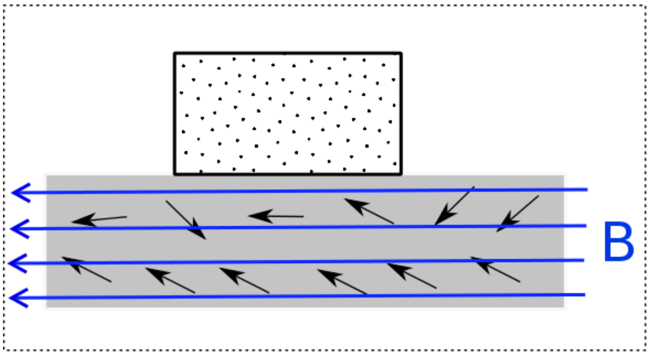

Fig. 1: Magnetic fridge.

a) (5 points) Approximate the entropy of the spin system for $x = \frac{\mu B}{k_B T} << 1$.

b) (5 points) Calculate the canonical partition function, the Helmholtz free energy and the entropy of the ideal gas at temperature T.

c) (20 points) How does the temperature of the gas change if the magnetic field is slowly reduced from $B << \frac{k_B T}{\mu}$ to $B = 0.0$? (Hint: The systems are in equilibrium and therefore at the same temperature. Since the whole system comprising the spins and the ideal gas is thermally isolated, the entropy does not change in this adiabatic process.)

Problem 3 Absorbing Surface (30 points)

An ideal gas is in chemical contact with a surface, that can absorb gas molecules. The surface has M equivalent sites, where a molecule can bind to. Only one molecule is accepted per site. Binding lowers the energy of the molecule by $\epsilon_b > 0$ as compared to the free molecule.

a) (6 points) Calculate the partition function for a single molecule that is bound to the surface.

b) (6 points) Find the canonical partition function for the surface. Do not double count equivalent configurations.

c) (6 points) Find the grand canonical partition function Ξ_{surf} and the grand potential $\Phi_G^{\text{surf}} = -\frac{1}{\beta}\log(\Xi_{\text{surf}})$ for the surface. You can use the relation $(1+x)^M = \sum_{N=0}^{M} \binom{M}{N} x^N$.

d) (6 points) Find the grand canonical partition function and the grand potential of the ideal gas. Calculate the pressure of the gas, $P = -\frac{\partial \Phi_G^{\text{gas}}}{\partial V}$.

e) (6 points) Using the fact that surface and gas are in chemical equilibrium ($\mu_{\text{gas}} = \mu_{\text{surface}}$), find the surface coverage $\theta = \frac{\langle N \rangle}{M}$ (fraction of sites on the surface that are occupied on average) as a function of the gas pressure. Plot $\theta(P)$ for a temperature and ϵ_b of your choice.