Statistical Physics I
WS 2015/16, FUB

Problem sheet 1

Please hand in your solutions before the lecture on Wednesday, 21st of October.

Problem I - Mean values and variances

The national team of Statmechania is analyzing their results of the last twenty matches to obtain some information about what they have to improve to become better. Those are the results:

match	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
goals scored	0	4	0	2	4	0	3	0	6	1	1	0	0	1	0	0	5	0	0	1
goals conceded	1	0	1	0	3	0	2	2	1	1	3	2	0	4	1	0	2	1	3	1

We see that they won 6 matches and lost 9 of them, while 5 matches were drawn, thus it seems that they have a quite weak team. However, how many goals do they score and concede on average? Assume that the amount of goals scored or conceded per match is given by the probability distribution

$$P(n) = \frac{\lambda^n}{n!}\,\mathrm{e}^{-\lambda}\,,$$

where λ is the mean value of the distribution. Calculate how often on average one should expect $n = 0, 1, ..., 7$ goals scored or conceded in 20 matches on the basis of this distribution with the mean value taken from the data. Plot both data sets in a histogram together with your results from the given distribution. Also, calculate the variances of the data sets. Describe in your own words on the basis of your plots what the variance tells you about the data. *(5 points)*
Hint: You can plot the data either with the help of a computer program of your choice or draw it by hand.

Problem II - Probability distributions

(a) Decide whether those distributions are probability distributions and comment. *(4 points)*

(1)

$$w_1(m) = \begin{cases} 1 & \text{if } m = 0 \\ 0 & \text{else} \end{cases} \qquad \text{with} \qquad m \in \mathbb{Z}$$

(2)

$$w_2(m) = \frac{1}{m!} \qquad \text{with} \qquad m \in \{\mathbb{Z}\,|m \geq 0\,\}$$

(3)

$$w_3(x) = \frac{1}{4}x^3 - 6x + 11 \qquad \text{with} \qquad x \in \{\mathbb{R}\,|2 \leq x \leq 4\,\}$$

(4)

$$w_4(x) = \frac{1}{x^2} \qquad \text{with} \qquad x \in \{\mathbb{R}\,|x \geq 1\,\}$$

(b) Consider the distribution $w_5(m) = a \cdot b^m$ with $m \in \{\mathbb{N}\,|m \geq 0\,\}$.

(1) For which values of b is $w(m)$ a probability distribution? For those cases determine the prefactor a as a function of b. *(2 points)*

(2) Calculate $\langle m \rangle$ and Δm. *(5 points)*

(c) Consider the distribution

$$w_6(x) = \begin{cases} -sx^2 + t & \text{if } -y \le x \le y \\ 0 & \text{else} \end{cases} \qquad \text{with} \qquad x \in \mathbb{R}$$

(1) Determine y such that $w_6(x)$ is continuous. *(1 point)*

(2) Compute s as a function of t such that $w_6(x)$ is a probability distribution. *(1 point)*

(3) Calculate $\langle x \rangle$ and Δx. *(3 points)*

(4) Calculate the cumulative distribution function $W_6(x)$. *(2 points)*

Problem III - Characteristic functions

(a) Compute the characteristic function $G(k)$ of the following probability densities: *(5 points)*

(1)

$$w_1(x) = \frac{a}{2}\,\mathrm{e}^{-a|x|} \qquad \text{with} \qquad x \in \mathbb{R}$$

(2)

$$w_2(x) = \begin{cases} |x| & \text{if } -1 \le x \le 1 \\ 0 & \text{else} \end{cases} \qquad \text{with} \qquad x \in \mathbb{R}$$

(b) For the first probability density, determine the first two moments. *(2 points)*