Statistical Physics I
WS 2015/16, FUB

Problem sheet 5

Please hand in your solutions before the lecture on Wednesday, 18th of November.

Problem 1 - distinguishable ideal gas particles

An ideal gas is composed of N "red" atoms of mass m , N "blue" atoms of mass m, and N "green" atoms of mass m. Atoms of the same color are **indistinguishable**. Atoms of different color are **distinguishable**.

(a) Use the canonical ensemble to compute the entropy of this gas. *(3 points)*

(b) Compute the entropy of ideal gas of 3N "red" atoms of mass m. Does if differ from that of the mixture. If so, by how much. *(3 points)*

Problem 2 - anharmonic oscillator

The potential energy of a one-dimensional, anharmonic oscillator may be written as:

$$V(q) = cq^2 - gq^3 - fq^4 \tag{1}$$

where c, g, and f are positive constants; quite generally, g and f may be assumed to be very small in value.

(a) Show that the leading contribution of anharmonic terms to the heat capacity of the oscillator, assumed classical, is given by *(4 points)*

$$\frac{3}{2}k^2\Big(\frac{f}{c^2} + \frac{5}{4}\frac{g^2}{c^3}\Big)T \tag{2}$$

(b) and, to the same order, the mean value of the position coordinate q is given by *(2 points)*

$$\frac{3}{4}\frac{gkT}{c^2} \tag{3}$$

Problem 3 - Magnetization

(1) Consider a lattice of N noninteracting particles, each possessing a magnetic moment $\overrightarrow{m}_i$ of fixed magnitude m which can point in any spacial direction. (where $m_i = \pm m$.) The Hamiltonian is:

$$\hat{H} = -\sum_i \overrightarrow{m}_i.\overrightarrow{B} \tag{4}$$

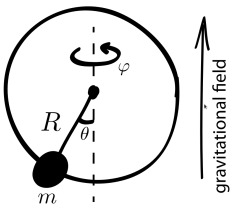
where $\overrightarrow{B}$ is the externally applied magnetic field, assumed homogeneous and in the Z direction.

(a) Calculate the canonical partition function Z of the system *(2 points)*

(b) Calculate the free energy A, internal energy U and heat capacity C. Discuss the limiting cases where $k_BT \ll mB$ and $k_BT \gg mB$. Calculate the entropy S in those cases. *(5 points)*

(c) The total magnetization in z direction is given by $M_z = \sum_i m_i^z$. Show explicitly that: *(3 points)*

$$\langle M_z \rangle = -\frac{\partial A}{\partial B_z} \tag{5}$$



(2) Consider a lattice of N classical rigid rotors. Each rotor is independent, is free to point in any spatial direction and has a moment of inertia $I = mR^2$. Its Hamiltonian is:

$$\hat{H} = \frac{1}{2I}\left(p_\theta^2 + \frac{p_\varphi^2}{\sin^2\theta}\right) \tag{6}$$

(a) Calculate the (canonical) partition function of the system of N rotors. Calculate the internal energy and the heat capacity. Study the regimes $T \to 0$ and $T \to \infty$. *(3 points)*

We now immerse the N rotors into a gravitational field with potential $V = mgz_i = -mgR\cos\theta_i$.

(b) Determine the partition function and compare it with the partition function of first part [1(a)] of this problem. Calculate the free energy, internal energy and heat capacity of the system. Discuss the limits $T \to 0$ and $T \to \infty$. *(5 points)*