VL: Mi 14-16 Uhr Prof. Dr. Kathy Lüdge
UE: Mi 16-18 Uhr (every 2nd week)
http://www.physik.fu-berlin.de/en/einrichtungen/ag/ag-luedge/teaching/

4. Homework - Nonlinear dynamics

Needs to be handed in: Wed 9.12.(before the lecture) You can build groups of up to 3 students.

Task 7 (15 points): *Chaos control – OGY – method*

Our starting point is the Henon-Map given by the equations

$$x_{n+1} = a + p - x_n^2 + b\, y_n,$$
$$y_{n+1} = x_n,$$

with $b = 0.3$ and $a = 1.29$. We use the notation $\boldsymbol{\xi}_n := (x_n, y_n)$ for the state vector of our system. The goal is to stabilize an unstable fixed point of the Henon-map by changing the control parameter p within a small interval $-p_* < p < p_*$ (with $p_* = 0.2$) after each iteration.

1. Show that the fixed point that we want to stabilize is given by:

   $$\boldsymbol{\xi}_F := (x_F, x_F), \text{ with } x_F = \frac{1}{2}\left(b - 1 + \sqrt{(b-1)^2 + 4(a+p)}\right) \quad (x_F \approx 0.838486 \text{ for } p = 0\ )$$

   Please determine the eigenvalue λ_u for the unstable direction ($|\lambda_u| > 1$), the eigenvalue λ_s corresponding to the stable direction ($|\lambda_s| < 1$), and the respective eigenvectors $\mathbf{e}_u$ and $\mathbf{e}_s$.

2. Determine the dual vectors $\mathbf{f}_u$ and $\mathbf{f}_s$ which are defined as

   $$\mathbf{f}_u \cdot \mathbf{e}_u = 1, \qquad \mathbf{f}_u \cdot \mathbf{e}_s = 0,$$
   $$\mathbf{f}_s \cdot \mathbf{e}_s = 1, \qquad \mathbf{f}_s \cdot \mathbf{e}_u = 0.$$

3. Determine the vector g that describes how the fixed point $\boldsymbol{\xi}_F$ changes under variations of p, as well as the threshold δ after which the control sets in:

   $$\mathbf{g} := \frac{\partial}{\partial p}\boldsymbol{\xi}_F(p)\Big|_{p=0},$$
   $$\delta := p_* \left|(1 - \lambda_u^{-1})\, \mathbf{g} \cdot \mathbf{f}_u\right|.$$

4. *Numerical part:* Start the iteration with a choice of $\xi_0 \in [\,0, 1] \times [\,0, 1]$ and p =0. After you did approx. 300 iterations you need to start the control with te following rules:

   1)check whether the trajectory is close enough to the fixed point, i.e., if $(\boldsymbol{\xi}_n - \boldsymbol{\xi}_F) \cdot \mathbf{f}_u < \delta$

   2)check if the control signal lies within the allowed range $[-p_*, p_*]$. Calculate p_n from:

   $$p_n := \lambda_u(\lambda_u - 1)^{-1}((\boldsymbol{\xi}_n - \boldsymbol{\xi}_F) \cdot \mathbf{f}_u))/(\mathbf{g} \cdot \mathbf{f}_u)$$

   If both are true use the control signal $p = p_n$, if not use p =0 for the next iteration.
   Plot the time series x_n together with the value if the control signal p_n.

Task 8 (5 points): *Fourier-transformation and transfer function*

In this task two methods of control will be compared, e.g. the control via the derivative and time delayed feedback control. We start with the nonlinear system $\dot{X}(t) = f(X(t)) + u(t)$ with

the control signal *u(t)*. Please determine the transfer function $T(\omega) = \hat{u}(\omega)/\hat{X}(\omega)$ for the two cases

1) control via the derivative $u(t) = -\gamma \dot{X}(t)$

2) time-delayed feedback control $u(t) = \gamma[X(t-\tau) - X(t)]$

Note that $u\hat{}(\omega)$ and $\hat{X}(\omega)$ are the Fourier-transforms of $u(t)$ and $X(t)$. What is the behaviour of $|\,T(\omega)|$ for high frequencies? Plot $|\ T(\omega)|$ for appropriate values of γ and τ for both cases.