VL: Mi 14-16 Uhr Prof. Dr. Kathy Lüdge
UE: Mi 16-18 Uhr (every 2nd week)
http://www.physik.fu-berlin.de/en/einrichtungen/ag/ag-luedge/teaching/

5. Homework - Nonlinear dynamics

Needs to be handed in: Wed 6.1.16(before the lecture) You can build groups of up to 3 students.

Task 9 (10 points): *Euler method for delay differential equations (DDE)*

In this task the following DDE will be solved numerically:

$$\begin{aligned}\dot{x} &= \lambda\, x + \omega\, y - K\,[x(t) - x(t-\tau)],\\ \dot{y} &= -\omega\, x + \lambda\, y - K\,[y(t) - y(t-\tau)],\end{aligned}$$

1. Determine the characteristic equation and proof that $K \geq \lambda/2$ is a necessary condition for the stabilization of the fixed point.
2. Integrate the above equation, chose $\lambda = 0.5$ and $\omega = \pi$, and plot the resulting trajectories for $K = 0,\ K = 0.2,\ K = 0.25$ and $K = 0.3.$ Interprete your results.
3. Numerically solve the characteristic equation and plot the real part of the largest eigenvalues as a function of the delay time (chose $\lambda = 0.5, \omega = \pi$, and *K=0.25*) .

Some hints for the numerics

- Use the Euler method that is given by $X_{n+1} = X_n + dt \cdot f[X_n,\, X_{n-\Delta}]$, with $\Delta = \tau/dt$,

  for a DDE of the form $\dot{X} = f[X(t),\, X(t-\tau)]$.
- You need to save the history in order to be able to evaluate the delay term. Use an arry with length delta=int(tau/dt) and initially fill it with zeros. During the iteration you will need to cyclically fill it with new values (use the modulo-operation %).
- To initialize the delay array during the times from $t = 0$ to $t = \tau$, chose $K = 0$.

Task 10 (10 points): *Optimal control*
Imagine a room at a temperature of $0°C$ at $t = 0$ that is heated and supposed to reach $20°C$ at $t = t_f$. The change in temperature is given by the following differential equation:

$$\dot{T}(t) = -aT(t) + bu(t), \quad a, b \in \mathbb{R} \tag{1}$$

where a is the loss rate of the heat and $u(t)$ the heat supplied to the room.

- Find the optimal heat supply u_*, that minimizes the following cost function:

$$\mathcal{I}[T(t), u(t)] = s/2(T(t_f) - 20)^2 + 1/2 \int_0^{t_f} u(t)^2 dt$$

$s \in \mathbb{R}$ is a parameter for the error tolerance.

- Interprete the origin of the different terms in $\mathcal{I}$.
- For a given $u_*(t)$ find the corresponding $T_*(t)$, which solves Eq. (1) .