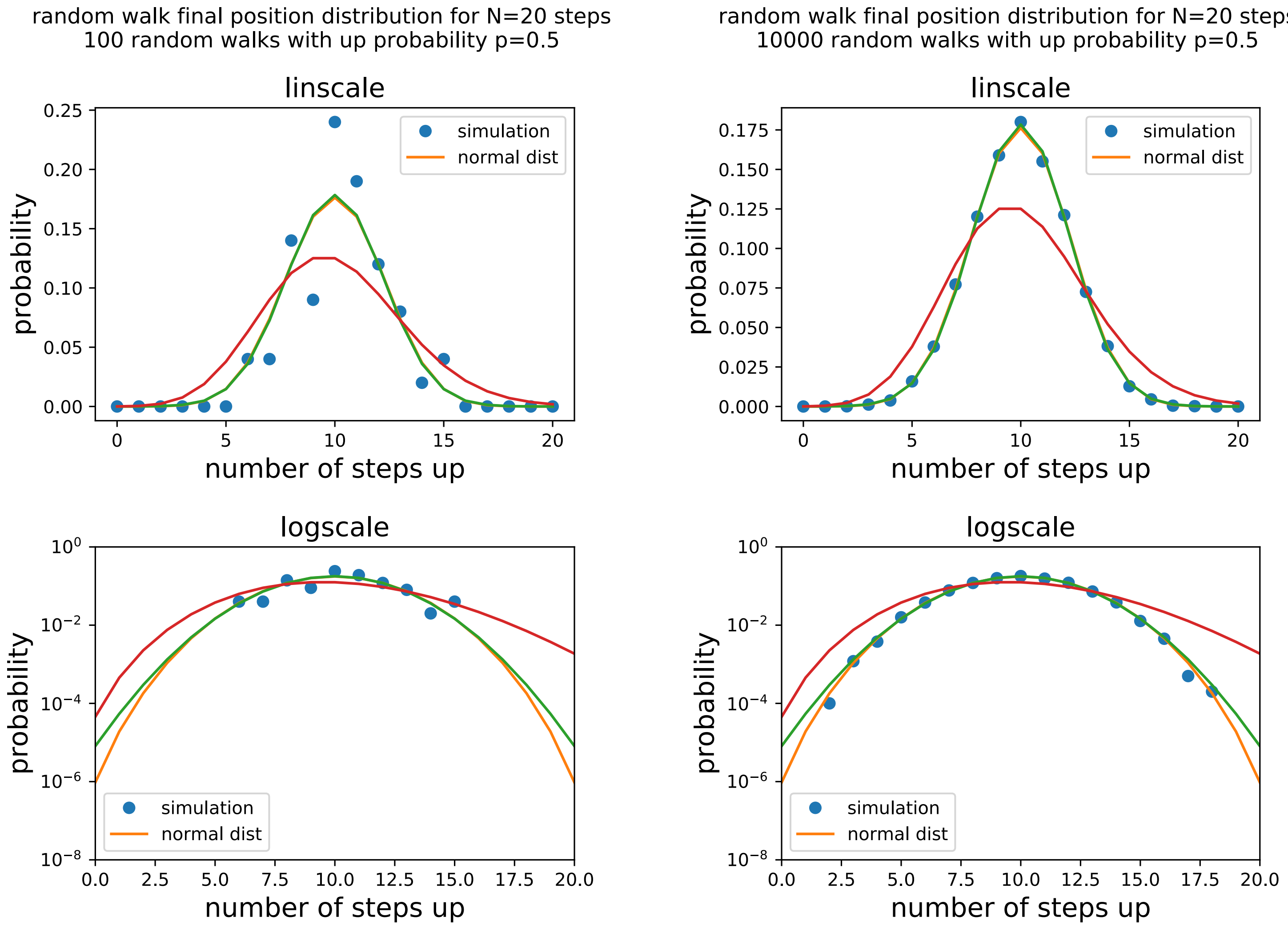
Binomial distribution, comparison of simulation data with analytic distributions



For N=20 steps with probability p=0.5 normal and binomial distributions are **very** close.
With 100 random walks we see substantial numerical deviations from the normal distribution, with 10,000 random walks deviations are small.
Poisson distribution does not describe the data.

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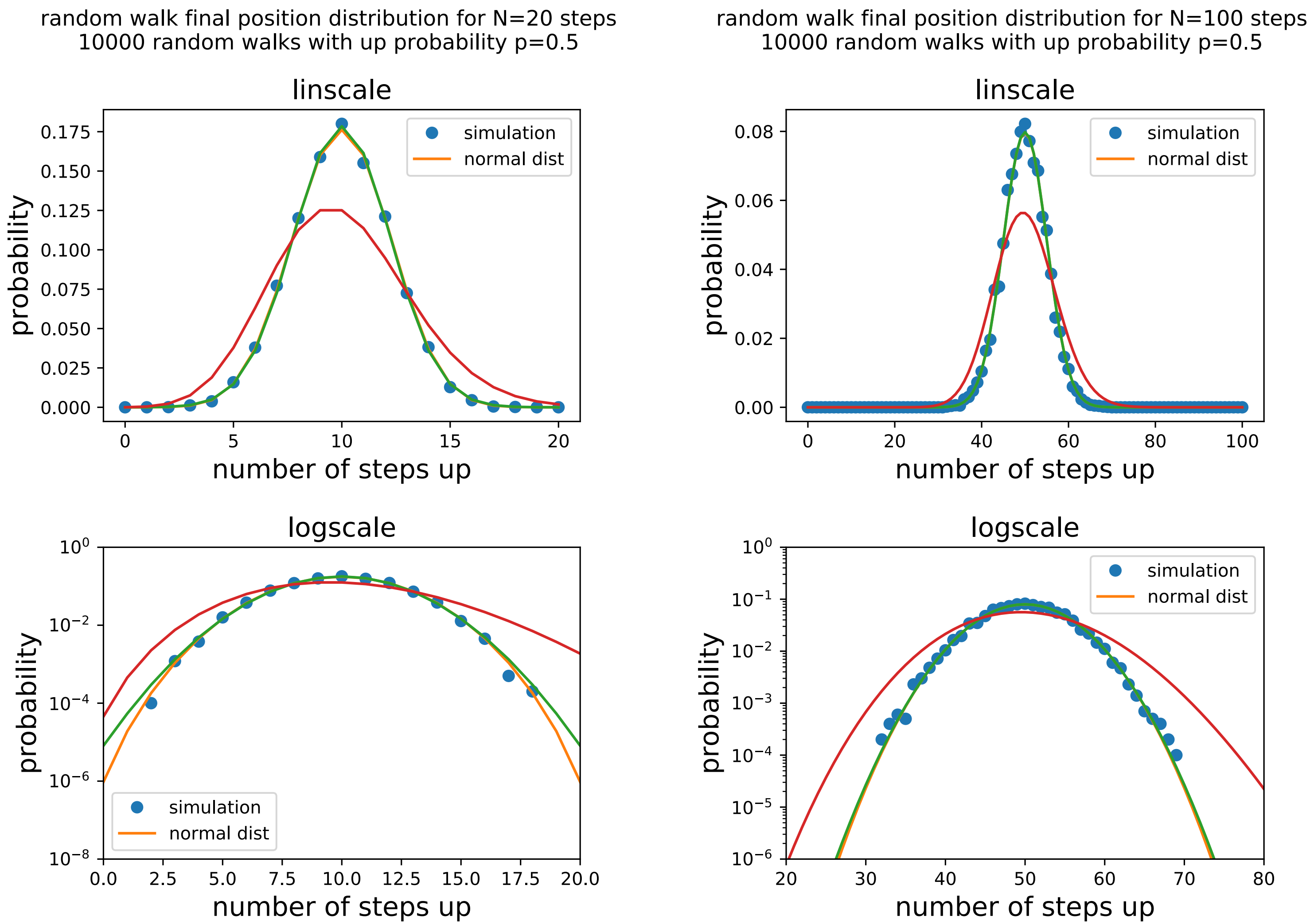

As we move from N=20 to N=100 steps at fixed probability p=0.5 the distribution becomes more sharply peaked. The mean deviation scales as $N^{1/2}$, while the relative mean deviation scales as $N^{-1/2}$

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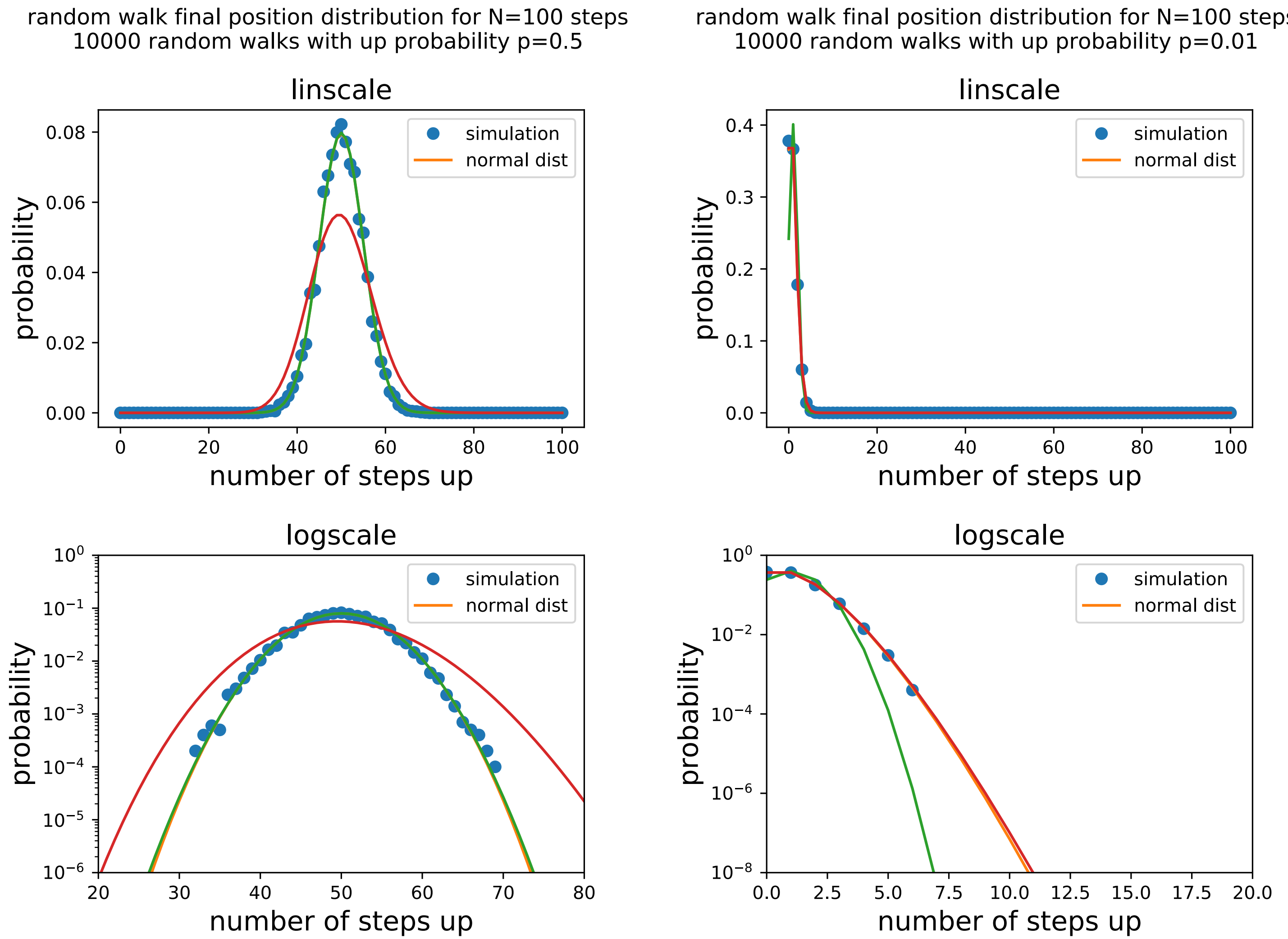
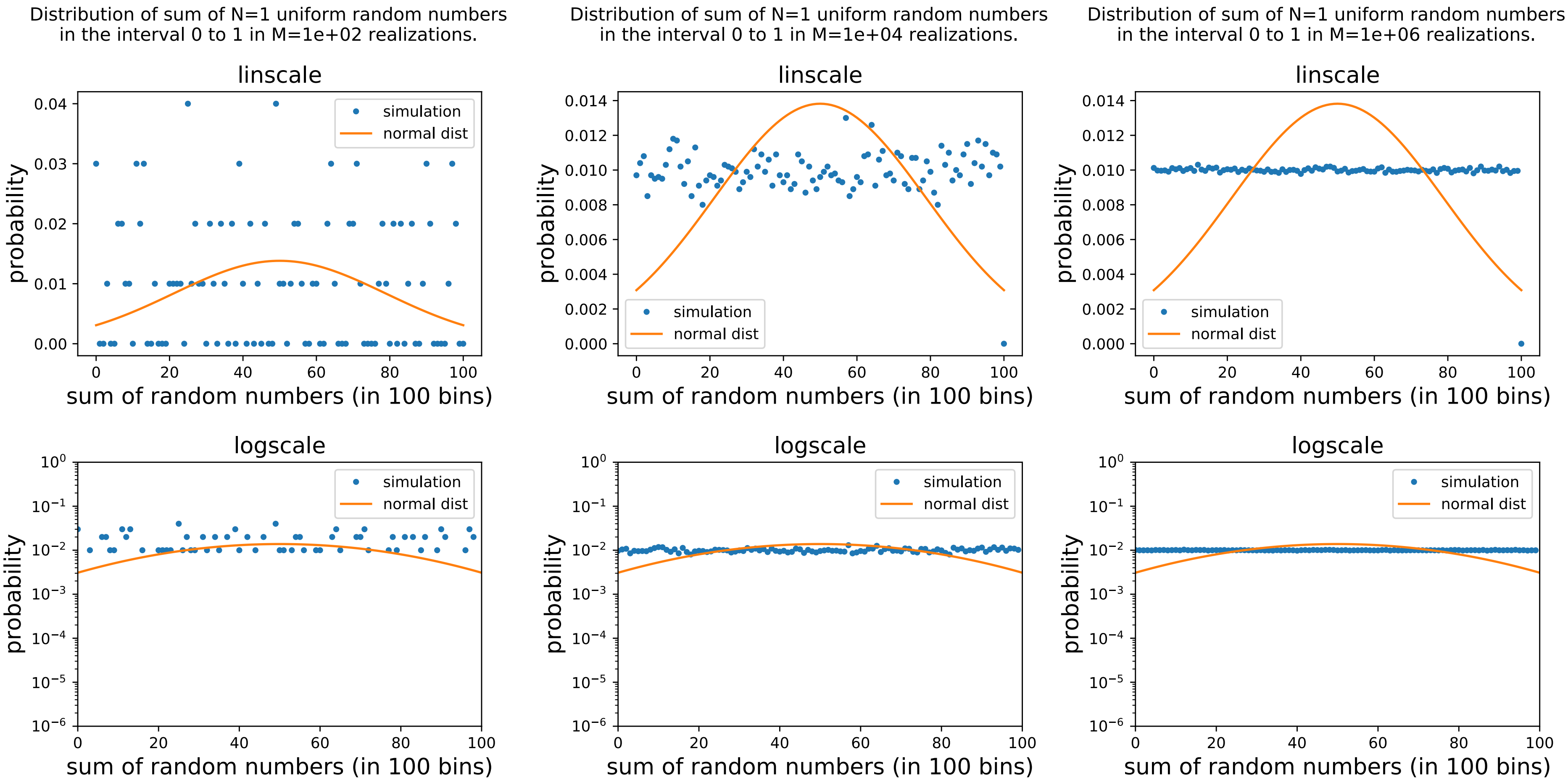


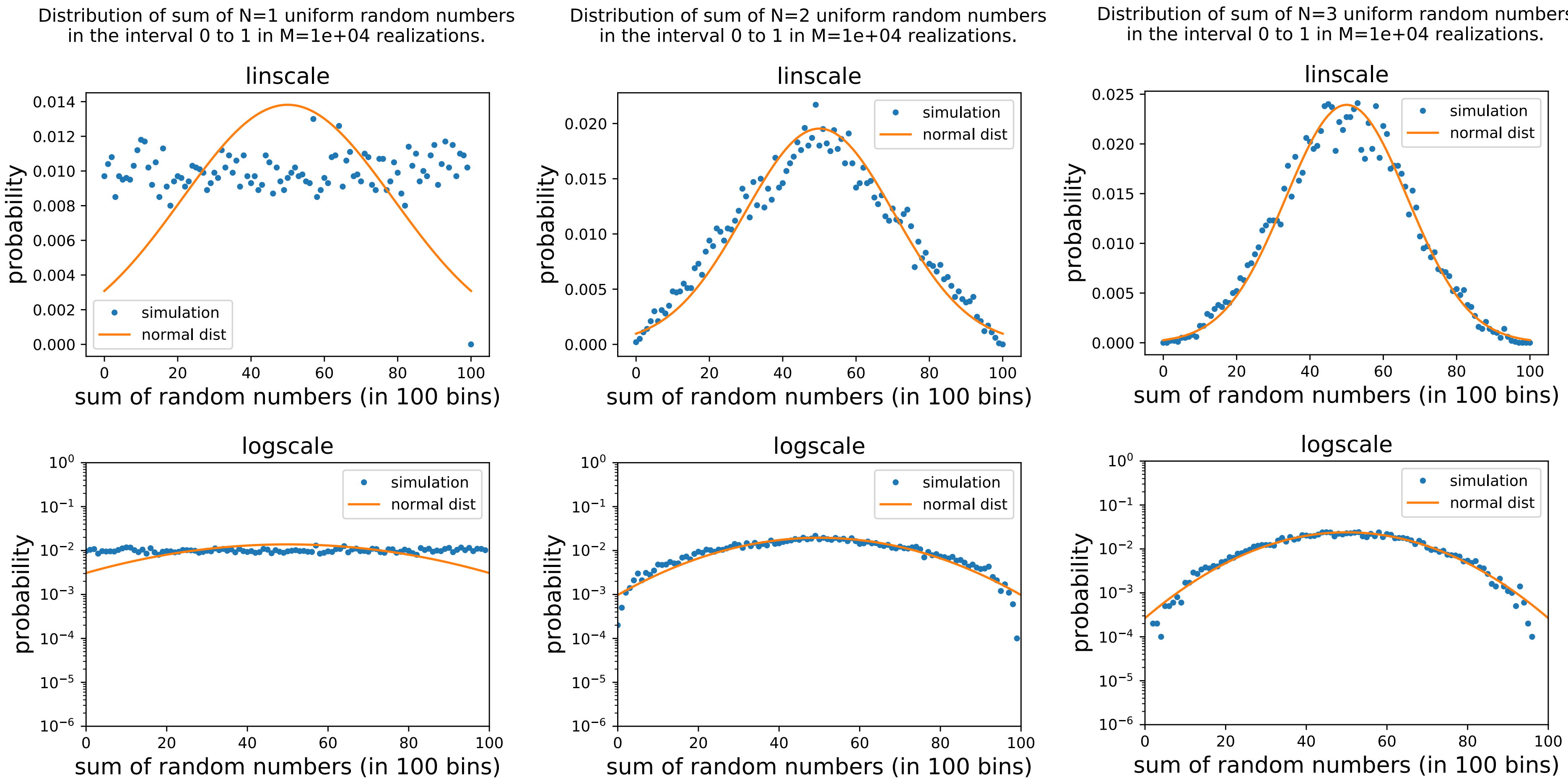
As we move from probability p=0.5 to p=0.01 for N=100 steps the distribution shifts from „normal“ to „Poisson-like“.

Central limit theorem: adding N uniform random numbers; first N=1



As we increase the number of realizations M from 10^2 over 10^4 to 10^6 the distribution becomes more uniform.

Central limit theorem: adding N uniform random numbers; now N=1, 2, 3



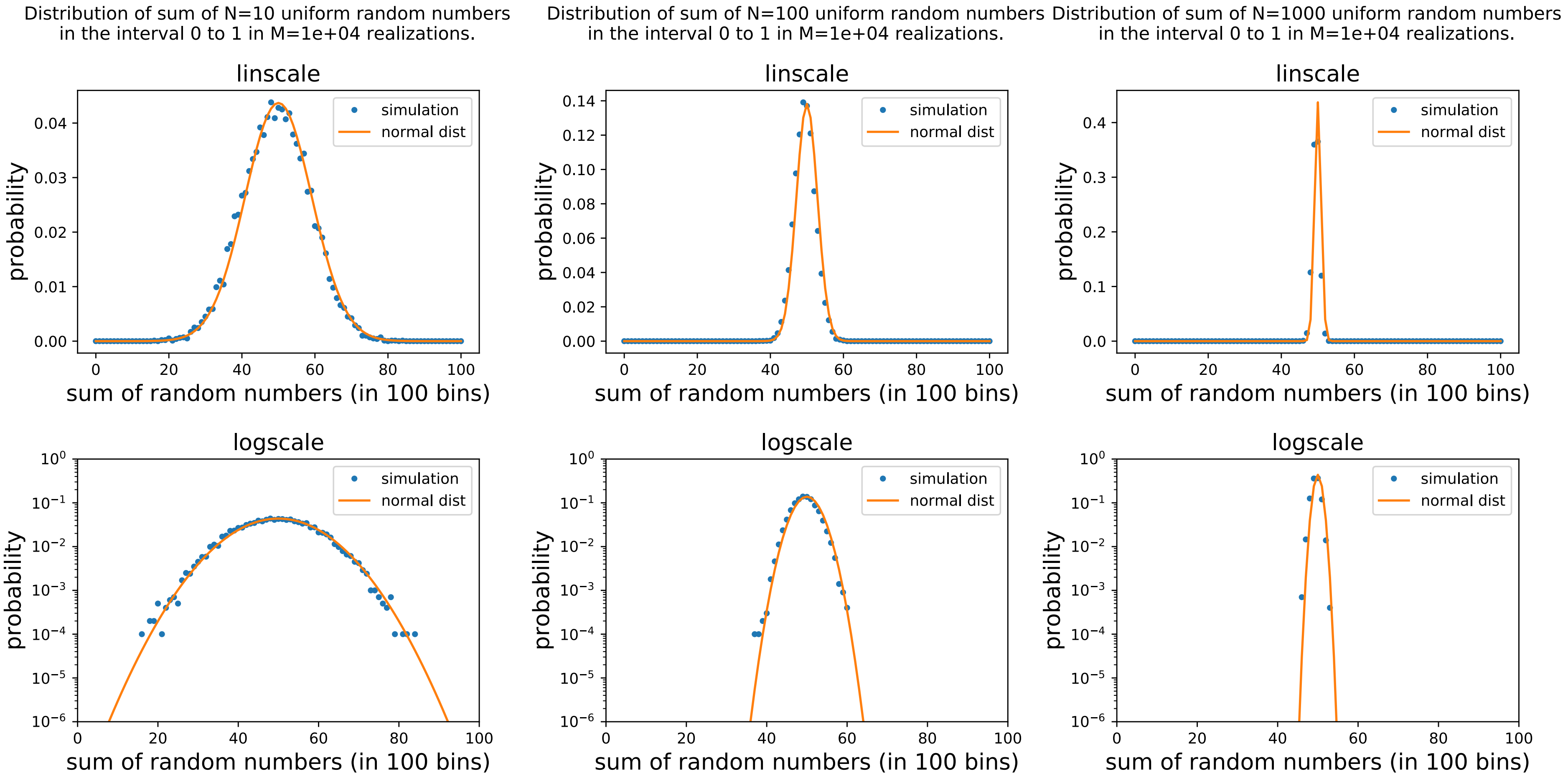
now we keep the number of realizations M fixed at 10^4

for N=2 the distribution is triangular (why?)

for N=3 the distribution already looks quite like a normal distribution.

Central limit theorem: adding N uniform random numbers; now N=10, 100, 1000

linear:



as N increases the distribution becomes sharper