Freie Universität Berlin
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June 28 2016

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Statistical Physics and Thermodynamics (SS 2016)

Problem Sheet 11

Hand in: Thursday, July 7 during the lecture

http://www.physik.fu-berlin.de/en/einrichtungen/ag/ag-netz/lehre/

1. Carnot cycle in the $T - S$ diagram (8 points)

Consider the Carnot cyclic process operating between two reservoirs with temperatures T_1, T_2 with $T_1 > T_2$. Refer to the four states of the cycle as a, b, c, d, where a is the initial compressed state in the hot reservoir with $T = T_1$.

a) Sketch the $P - V$ diagram of the process, including the direction, and label the states. Which are the lines of constant temperature? **(1 point)**

b) Now sketch the process in a $T - S$ diagram. Mark the entropies S_1 and S_2 in the diagram with $S_2 > S_1$. **(1 point)**

c) What is the amount of heat ΔQ transferred in each part of the cycle in terms of T_1, T_2, S_1 and S_2? **(2 points)**

d) Obtain an expression for the work ΔW done by the heat engine during one cycle in terms of T_1, T_2, S_1 and S_2. **(1 point)**

e) Calculate the efficiency

$$\eta = \frac{\Delta W}{\Delta Q_1}, \tag{1}$$

where ΔQ_1 refers to the amount of heat transferred from the hot reservoir to the system during one cycle. Why is only ΔQ_1 used for the definition of η? **(1 point)**

f) Now consider an arbitrarily shaped cyclic curve in a $T - S$ diagram (see Fig. 1). Show that the efficiency of the Carnot process operating between $T_1 = T_{\max}$ and $T_2 = T_{\min}$ is larger. **(2 points)**

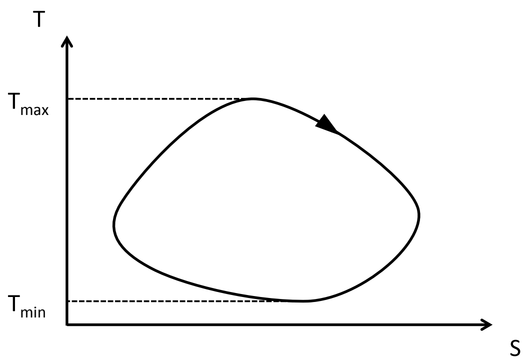

Figure 1: Example of an arbitrarily shaped cyclic process in a $T - S$ diagram.

2. Maximal efficiency of a heat engine I (4 points)

In this exercise we will derive the maximal efficiency η for an arbitrary cyclic process operating between two heat reservoirs T_1, T_2 with $T_1 > T_2$ from the second law of thermodynamics. Assume ΔQ_1 / ΔQ_2 to be the amounts of heat taken from / transferred to the hot / cold reservoir.

a) Express the efficiency η of the cycle in terms of ΔQ_1 and ΔQ_2. **(1 point)**

b) How does the entropy of each reservoir change in one cycle? What is hence the total entropy change of the two reservoirs after one cycle? **(1 point)**

c) Use the second law of thermodynamics to obtain an upper bound for η and compare to the efficiency η_C of the Carnot process. **(2 points)**

3. Maximal efficiency of a heat engine II (5 points)

In this exercise, we want to show in a different way that the Carnot cyclic process has the maximally possible efficiency. We consider therefore an arbitrary heat engine X with unknown efficiency η_X. We use the mechanic work ΔW done by the system X to operate an inverse Carnot machine I_C to pump back the heat from the cold to the hot reservoir, see Fig. 2.

Assume that ΔQ_1 is the amount of heat taken by the system X from the hot reservoir during one cycle.

a) Calculate ΔQ_2, which is the amount of heat transferred to the cold reservoir by X, $\Delta Q_1'$, i.e. the amount of heat transferred by the system I_C to the hot reservoir and $\Delta Q_2'$, which denotes the heat taken by I_C from the cold reservoir. Express your results in terms of ΔQ_1, η_X and $\eta_C = 1 - T_2/T_1$. **(3 points)**

b) Conclude that $\eta_X \leq \eta_C$ by regarding the direction of the total heat flux. **(2 points)**

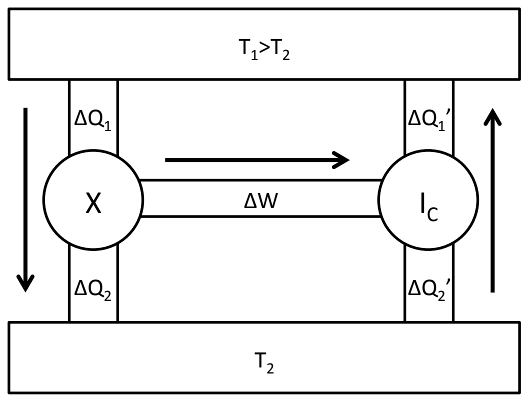

Figure 2: Visualization of the setup for exercise 3.

4. Stirling cycle (3 points)

The Stirling cycle consists of the following four steps

- isothermal expansion from V_1 to V_2 at the temperature of the hot reservoir ($T = T_1$)
- isochoric (constant volume) cooling to $T_2 < T_1$
- isothermal compression to $V_1 < V_2$ at the temperature of the cold reservoir ($T = T_2$)
- isochoric heating of the compressed system to T_1

a) Draw a $P - V$ diagram. **(1 point)**

b) Calculate the efficiency η_S of this cycle for an ideal gas. Is it an efficient cycle? **(2 points)**