Freie Universität Berlin
Fachbereich Physik
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Statistical Physics and Thermodynamics (SS 2016)

Problem sheet 2

Hand in: Latest on Monday, May 9 at 12:00 (note that Thursday May 5 is a holiday)

http://www.physik.fu-berlin.de/en/einrichtungen/ag/ag-netz/lehre/

1 Moments and cumulants (5 points)

Here you will algebraically calculate higher moments and cumulants of probability distributions.

a) Express the higher moments of the deviation from the mean, $\langle \Delta x^n \rangle \equiv \langle (x - \langle x \rangle)^n \rangle$, for $n = 1, 2, 3, 4$ as a function of the moments $\langle x^n \rangle$. **(2 points)**

b) Determine the cumulants $\langle x^n \rangle_C$, for $n = 1, 2, 3, 4$ as a function of the moments $\langle x^n \rangle$. For this use the characteristic function defined in the lecture as $G(k) = \int_{-\infty}^{\infty} \mathrm{d}x \mathrm{e}^{-ixk} P(x)$ and compare the series expansions of $G(k)$ and $\ln G(k)$. *Hint:* The series expansion $\log(1-x) = -\sum_{n=1}^{\infty} \frac{x^n}{n}$ ist useful. **(3 points)**

2 Higher moments and cumulants of the binomial distribution (5 points)

Using the same trick as in the lecture, you will calculate moments and cumulants of the binomial distribution $P_N(m)$.

a) Calculate $\langle m^3 \rangle$ and $\langle m^4 \rangle$. Before you do this, make sure you understand the derivation of $\langle m^2 \rangle$ in the lecture. **(3 points)**

b) Calculate the higher moments of the deviation from the mean, $\langle \Delta m^n \rangle \equiv \langle (m - \langle m \rangle)^n \rangle$, and the cumulants $\langle m^n \rangle_C$, both for $n = 1, 2, 3, 4$. **(2 point)**

3 Moments and cumulants of explicit continuous probability distributions (10 points)

Here you will consider four different (not necessarily normalized) probability distributions. These distributions constitute four different important classes that are encountered in many applications.

$$P_A(x) = \delta(x) \tag{1}$$

$$P_B(x) = 1 \qquad \text{for } |x| < 1, \text{ zero otherwise} \tag{2}$$

$$P_C(x) = \mathrm{e}^{-x^2/(2\Delta^2)} \tag{3}$$

$$P_D(x) = 1/(1+|x|)^2 \tag{4}$$

a) Normalize the probability distributions. Sketch them graphically. **(1 point)**

b) Calculate the moments $\langle x^n \rangle$ for $n = 1, 2, 3, 4$ for all distributions. **(5 points)**

(There are different ways of doing this. For example for the normal distribution, which essentially is a Gaussian distribution, one can look up the results in an integral table (which is not the most insightful way), one can derive the moments from the zeroth moment by differentiation, or one can use the characteristic function. By the way, do you know how to calculate the zeroth moment of the Gaussian distribution from scratch, i.e., without looking it up in a table?)

c) Calculate the deviations from the mean, $\langle \Delta x^n \rangle \equiv \langle (x - \langle x \rangle)^n \rangle$, for $n = 1, 2, 3, 4$. **(2 points)**

d) Calculate the cumulants $\langle x^n \rangle_C$, for $n = 1, 2, 3, 4$. **(2 points)**