Freie Universität Berlin
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Statistical Physics and Thermodynamics (SS 2016)

Problem sheet 5

Hand in: Thursday, May 26 during the lecture

http://www.physik.fu-berlin.de/en/einrichtungen/ag/ag-netz/lehre/

1 Particles in a gravitational potential (8 points)

Consider N non-interacting particles of mass m in a one-dimensional box located at $0 < z < L$. The particles are subject to a gravitational potential with energy mgz, with g being the strength of the potential.

a) Write down the Hamiltonian H including the kinetic term and calculate the 1-particle partition function Z_1 ($N = 1$). **(2 points)**

b) Write down the N-particle partition function Z_N for the case that the particles are distinguishable. **(1 point)**

c) Calculate the cumulants $c_n = \langle H^n \rangle_C$ for $n = 1, 2, 3$ by taking suitable derivatives, and determine their scaling with temperature. What physical quantities do the first and second cumulants correspond to? **(3 points)**

d) Determine the internal energy in the limits of high temperature ($\beta mgL \ll 1$ with $\beta = 1/(k_BT)$) and low temperature ($\beta mgL \gg 1$). (**2 points**)

2 Extensive & intensive functions (4 points)

a) Determine which of the following functions $F_i(S, V, N)$ of the entropy S, the volume V, and the number of particles N, are extensive or intensive:

$$F_1 = S^2/V + S^5/(VN^3) + S^7/(VN^5) \tag{1}$$

$$F_2 = S^5/V^3 \tag{2}$$

$$F_3 = S^4/N^4 \tag{3}$$

(**1 point**)

b) Explain whether the derivatives $\frac{\partial F_i}{\partial S}$ and $\frac{\partial^2 F_i}{\partial S^2}$ with respect to the entropy S are extensive or intensive for $i = 1, 2, 3$. (**3 points**)

3 Spins in a magnetic field (8 points)

Consider N non-interacting spins in a magnetic field B. Each spin i can be up ($s_i = 1$), zero ($s_i = 0$), or down ($s_i = -1$) with energy $\varepsilon s_i B$. The Hamiltonian is given by $H = \sum_{i=1}^{N} \varepsilon s_i B$.

a) Write down the N-particle partition function Z_N by summing over all microstates. (**1 point**)

b) What is the maximum energy of the system if it is not in thermodynamic equilibrium? Calculate the average energy $U = \langle H \rangle$, the fluctuations $\langle H^2 \rangle - \langle H \rangle^2$, and the heat capacity $\partial \langle H \rangle / \partial T$ in thermodynamic equilibrium. (**2 points**)

c) Calculate the free energy $F = -k_B T \ln Z_N$ and the entropy $S = -\partial F / \partial T$. Convince yourself that $\partial S / \partial U = 1/T$. (**2 points**)

d) Calculate the resulting temperature T_2 as a function of the original temperature T_1 when the magnetic field B is changed from B_1 to B_2 in an adiabatic process, *i.e.* a process in which the entropy remains constant. (**1 point**)

e) Derive the partition function again, now by sorting all microstates according to their energies, and summing over the energy states including their degeneracy. Show that the result is the same as in (a). *Hint:* You will need the trinomial coefficient $\frac{N!}{n!m!(N-m-n)!}$. (**2 points**)