Freie Universität Berlin
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April 19th 2016

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Statistical Physics and Thermodynamics (SS 2016)

Problem sheet 6

Hand in: Thursday, June 2 during the lecture

http://www.physik.fu-berlin.de/en/einrichtungen/ag/ag-netz/lehre/

1 Atoms in a harmonic potential (7 points)

In this exercise, you will derive the Dulong-Petit law for the heat capacity of a solid, which is valid for crystals at high temperature, in absence of collective lattice vibrations. Consider a model of a solid consisting of N independent, distinguishable, classical, atoms of mass m, each constrained in a 3-dimensional harmonic potential. Each atom is described by the Hamiltonian

$$H_1 = \frac{p_x^2 + p_y^2 + p_z^2}{2m} + \frac{m\omega^2}{2}\left(q_x^2 + q_y^2 + q_z^2\right).$$

The solid is isotropic, which means that the energy is identical in the x, y, and z directions.

a) Write down the single-particle partition function Z_1 ($N = 1$). **(1 point)**

b) Write down the N-particle partition function Z_N. **(2 points)**

c) Calculate the energy U for N atoms from a derivative of the partition function. Note that you do not need to calculate the Gaussian integral. What is the energy per degree of freedom? (A degree of freedom is defined as a fluctuating quantity that enters the Hamiltonian quadratically.) **(3 points)**

d) Calculate the heat capacity $C = (\partial U/(\partial T)$. What is its dependence on the temperature? **(1 point)**

2 Equipartition theorem for non-quadratic Hamiltonians (7 points)

Consider a system with a Hamiltonian $H = A|q|^m$, with A being a positive number.

a) Write down the partition function Z for a single positional degree of freedom q. **(2 points)**

b) Calculate the energy U from a derivative of $\ln Z$ using a suitable coordinate transform. What is the energy? **(3 points)**

c) Calculate the heat capacity of the system $C = (\partial U/\partial T)$. **(2 points)**

3 Maxwell-Boltzmann distribution (6 points)

A single particle moves in d dimensions at a velocity following the modified d-dimensional Maxwell-Boltzmann distribution.

a) Express the partition function Z as an integral over $\mathrm{d}^d p$. **(2 points)**

b) Calculate the expectation value $\langle p^2 \rangle = \langle \sum_{i=1}^{d} p_i^2 \rangle$ from a suitable derivative of the partition function for $d = 1, 2, 3, 4$, and determine the kinetic energy $\langle U_{kin} \rangle = \langle p^2 \rangle / (2m)$. What is the energy per degree of freedom? **(2 points)**

c) Calculate the most probable quadratic velocity $v_{max}^2 = \langle p^2 \rangle_{max} / m^2$ of the particle by maximizing the probability density of p^2 for $d = 1, 2, 3, 4$. **(2 points)**