Freie Universität Berlin
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Statistical Physics and Thermodynamics (SS 2017)

Problem sheet 10

Hand in: Friday, June 30 during the lecture

http://www.physik.fu-berlin.de/en/einrichtungen/ag/ag-netz/lehre/

1 Efficiency of the ideal Otto and Diesel cycles (7 points)

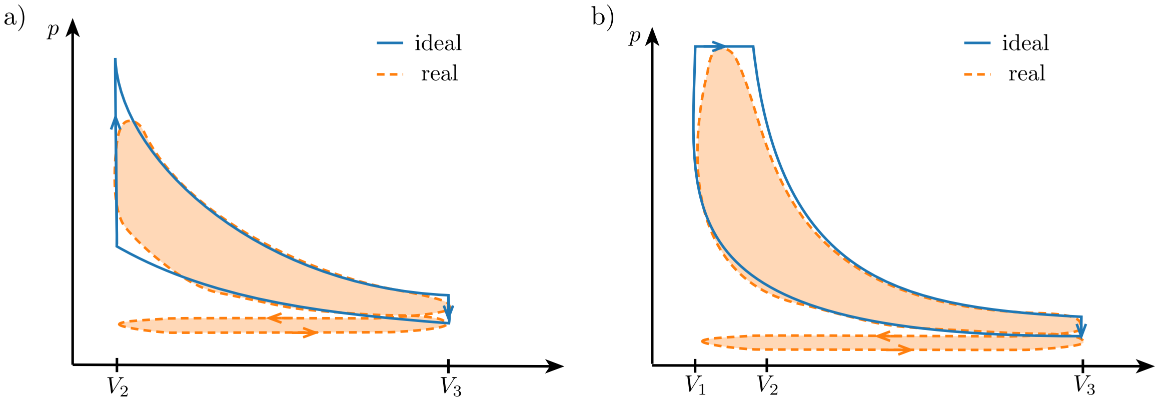

Figure 1: Schematic pV-diagram of the real and ideal Otto (a) and Diesel (b) cycle (color online).

The two most common internal combustion engines are the Otto engine created by Nikolaus Otto in 1876 and the Diesel engine created by Rudolf Diesel in 1892. Figure 1 shows both real cycles and their idealizations. Consider the gas in the engines as ideal and calculate the efficiency η for both cycles in terms of V_1, V_2 and V_3. As in the lecture, the efficiency is defined as the total work W_{tot} divided by the heat input $\eta = W_{tot}/Q_{in}$.

a) The idealized Otto cycle: A path with two adiabates and two isochores (V_2, V_3) (**3 points**)

b) The idealized Diesel cycle: A path with two adiabates, one isobar $(V_1 \to V_2)$ and one isochore (V_3). (**3 points**)

c) Which engine would you prefer in terms of efficiency using $V_1 = 1V$, $V_2 = 2V$ and $V_3 = 3V$? (**1 point**)

2 Stretching of an elastic rubber band (13 points)

In the following you will calculate the temperature and entropy change of stretching and relaxing an elastic rubber band. For a band of length L, at temperature T and under tension J it is found experimentally that

$$\left(\frac{\partial J}{\partial T}\right)_L = \frac{aL}{L_0}\left[1 - \left(\frac{L_0}{L}\right)^3\right] \tag{1}$$

$$\left(\frac{\partial J}{\partial L}\right)_T = \frac{aT}{L_0}\left[1 + 2\left(\frac{L_0}{L}\right)^3\right], \tag{2}$$

where L_0 is the length of the unstretched band which is assumed to be independent of temperature and a is a constant.

a) Write down the total differential $\mathrm{d}J(T,L)$ and obtain the equation of state for the rubber band $J(T,L)$. (**2 points**)

The work done on the band when its length is increased by an infinitisemal length $\mathrm{d}L$ is $\Delta W = J\mathrm{d}L$. Thus the first law of thermodynamics takes the form

$$\mathrm{d}U = \Delta Q + \Delta W = T\mathrm{d}S + J\mathrm{d}L. \tag{3}$$

b) Why is the sign of the work different from the usual usual $-p\mathrm{d}V$ term? (**1 point**)

c) Use the Helmholtz free energy $F(T,L)$ to derive the following Maxwell relation

$$\left(\frac{\partial S}{\partial L}\right)_T = -\left(\frac{\partial J}{\partial T}\right)_L. \tag{4}$$

(**2 points**)

d) Rewrite the first law of thermodynamics and use equation (4) to show that

$$\left(\frac{\partial U}{\partial L}\right)_T = 0. \tag{5}$$

(**2 points**)

Using the latter equation and assuming that the heat capacity at constant length of the band is a constant C_L, for any change of state the change in internal energy is $\mathrm{d}U = C_L\mathrm{d}T$ where $U(T,L)$ is a function of the temperature T and the length L.

e) Now the band is stretched adiabatically ($\Delta Q = 0$) and reversibly from an initial length L_i at an initial temperature T_i to a final length L_f. What is its final temperature T_f? (**2 points**)

f) How does the temperature change when the band is relaxed adiabatically and reversibly ($T_f > T_i$)? (**1 point**)

g) Now we assume an adiabatic contraction to its natural length L_0, where no external work is performed. That means, that the two ends of the rubber band relax freely (i.e. without being hold at a controlled force) in an isolated environment from an initially stretched configuration. Use the first law of thermodynamics to find the changes in temperature and entropy. (**3 points**)