Freie Universität Berlin
Fachbereich Physik
April 17th 2018

Prof. Roland Netz
Douwe Bonthuis
Jan Daldrop
Philip Loche

Statistical Physics and Thermodynamics (SS 2018)

Problem sheet 1

Hand in: Friday, April 27 during the lecture

http://www.physik.fu-berlin.de/en/einrichtungen/ag/ag-netz/lehre/

1 Random walker in 1D (6 points)

A drunken person starts in front of a lamp pole and takes N steps either to the left or to the right. All steps are of the same length and both direction probabilities p are equal.

a) What is the probability to for going to the right exactly 10 times? **(1 point)**

b) What is the probability $P_N(k)$ for going to the right exactly k times after N steps? **(1 point)**

c) Show that the probability distribution from task b) is properly normalized, i.e. that

$$\sum_{k=0}^{N} P_N(k) = 1. \tag{1}$$

(2 points)

Hint: Use the binomial formula.

d) What is the probability that the walker arrives back at the starting point after taking 10 steps? **(2 points)**

2 Poisson distribution (10 points)

The Poisson distribution describes the probability for the number of independent rare events, and in this exercise we will derive it as an approximation to the binomial distribution.

Assume you play in a football team. During a match your team scores goals at a rate α per unit time, so that the probability to score a goal in a time interval Δt is $p_{\Delta t} = \alpha \cdot \Delta t$. We assume that there are so few goals that we can neglect the possibility of two or more goals per time interval Δt. The total duration of a game is $T = N \cdot \Delta t$, where N is the number of intervals Δt during a game.

The probability to score exactly k goals during a game is then given by the binomial distribution

$$P_N(k) = \binom{N}{k} p_{\Delta t}^k (1 - p_{\Delta t})^{N-k}. \tag{2}$$

a) Show that in the limit $p_{\Delta t} \to 0$, $N \to \infty$ such that $p_{\Delta t} \cdot N = \alpha \cdot T$ remains constant, the binomial distribution can be approximated by the Poisson distribution, i.e.

$$\lim_{p_{\Delta t} \to 0} P_N(k) = \frac{\lambda^k}{k!} \exp(-\lambda) \tag{3}$$

with $\lambda = \alpha T$. **(3 points)**

Hint: Use that $\lim_{N\to\infty}(1 - \lambda/N)^N = \exp(-\lambda)$.

b) Show that the probability distribution (3) is properly normalized, i.e. that

$$\sum_{k=0}^{\infty} \frac{\lambda^k}{k!} \exp(-\lambda) = 1. \tag{4}$$

(1 point)

c) Show that the expectation value of the total number of goals is given by $\langle k \rangle = \alpha T$. **(1 point)**

d) Calculate the expectation value $\langle k^2 \rangle$ and use it to obtain a result for the variance $\Delta k^2 = \langle k^2 \rangle - \langle k \rangle^2$. **(2 points)**

Hint: Use the identity $k = (k-1) + 1$ to split the sum in the expectation value into two contributions.

Assume your football team scores on average two goals per game.

e) Calculate the probability that the team scores zero goals. **(1 point)**

f) Calculate the probability to score two goals. **(1 point)**

g) Calculate the probability to score more than two goals. **(1 point)**

3 Self-driving car (4 points)

Assume you earn enough money to buy a self-driving car and you consider getting one. A fancy Californian company offers a car A that is involved in a lethal crash on average every 100 000 km. A well-known German company has a special (hugely advertised) security system in their self-driving car of the model B that reduces the number of lethal crashes to 1 per 200 000 km. Unfortunately an illegal software is included that disables the system permanently in every second car. Without the security system the lethal crash rate increases to 1 per 50 000 km. Thus, the car you buy either has the system or not in its entire lifetime.

a) Calculate your survival probability as a function of driven kilometers for the Californian car A. **(1 point)**

Hint: Use the Poisson distribution and define a crash rate.

b) Assume you drive your for only 50 000 km. Which type of car (A or B) is the better bet for your safety? **(2 points)**

c) Is your result also the safer choice if you drive the car for 300 000 km? **(1 point)**