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Hand in on October 24th 2016 before the lecture

Advanced Statistical Physics II – Problem Sheet 1

Problem 1 – **Thermodynamic Potentials and State Variables**

a) (2P) Convince yourself that a generic function $f(u, v)$ fulfills the relations:

$$\left(\frac{\partial f}{\partial u}\right)_v \left(\frac{\partial u}{\partial f}\right)_v = 1 \quad \text{and} \quad \left(\frac{\partial f}{\partial u}\right)_v \left(\frac{\partial u}{\partial v}\right)_f \left(\frac{\partial v}{\partial f}\right)_u = -1 \tag{1}$$

In the following, a thermodynamic system with a constant particle number is considered:

b) (1P) Using the results from subtask a), express the isochoric pressure change with temperature

$$\left(\frac{\partial p}{\partial T}\right)_V \tag{2}$$

by the response functions

$$\alpha = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p, \qquad \kappa_{\mathrm{T}} = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T. \tag{3}$$

What is the total differential of $p(T, V)$?

c) (1P) Derive the differential forms of the caloric equations of state $U(T, V)$ and $U(T, p)$. Express the appearing partial derivatives by standard response functions.

d) (1P) Find the following Maxwell relation

$$\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial p}{\partial T}\right)_V \tag{4}$$

and then derive the relation

$$-p + T\left(\frac{\partial p}{\partial T}\right)_V = \left(\frac{\partial U}{\partial V}\right)_T. \tag{5}$$

Hint: V and T are the natural variables of the free energy F.

e) (1P) Show that the functional determinant

$$\frac{\partial(T, S)}{\partial(p, V)} = \begin{vmatrix} \left(\frac{\partial T}{\partial p}\right)_V & \left(\frac{\partial T}{\partial V}\right)_p \\ \left(\frac{\partial S}{\partial p}\right)_V & \left(\frac{\partial S}{\partial V}\right)_p \end{vmatrix} = 1. \tag{6}$$

Hint: You can use the identity $\frac{\partial(T,S)}{\partial(p,V)} = \frac{\partial(T,S)}{\partial(A,B)}\frac{\partial(A,B)}{\partial(p,V)}$, where A and B are any state variables.

Problem 2 – **Thermodynamic calculus**

The internal energy of a system in its natural variables is given by

$$U(S,V) = (\sigma V)^{-m/n}\, N^{(m-1)/n} \left(\frac{nS}{n+1}\right)^{(n+1)/n} \tag{7}$$

a) (4P) Calculate $U(T,V)$.
b) (2P) Calculate $p(T,V)$.
c) (3P) Calculate the free energy F, the enthalpy H and the Gibbs free energy G in their natural variables.
d) (4P) Compute the response functions α and κ_T from problem 1 b). Compute also the quotient α/κ_T.
e) (1P) Choose the parameter m such that p in b) is independent of the Volume V. What do you obtain for for the results of a)-c) when additionally choosing $n = 3$? Which physical system is described by these equations?