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Hand in on October 31th 2016 before the lecture

Advanced Statistical Physics II – Problem Sheet 2

Problem 1 – **Computing thermodynamic potentials**

For an one component system we know the isothermal compressibility κ_T, the thermal expansion coefficient α and the heat capacity at constant volume C_V. These are given by

$$\kappa_T = \frac{2}{p} \qquad , \qquad \alpha = \frac{3}{T} \qquad , \qquad \frac{C_V}{N} = \frac{3\sigma T^2}{2p} \, , \tag{1}$$

where σ is a numerical constant. With the assumption that the entropy is zero at zero temperature, $S(T = 0) = 0$, we want to compute the internal energy $U(S, V)$ up to a constant U_0.

a) (3P) Use the first two equations and calculate $V(T, p)$. You will be left with a constant prefactor c that does not depend on T and p.
b) (2P) Using your result in a) show that the free energy is of the form $F(T, V) = f_1(T, V) + f_2(T)$. Determine $f_1(T, V)$.
c) (2P) Using your result in b) compute C_V. Compare to the given function for this quantity to obtain the constant c. What can you say about $f_2(T)$ now?
d) (1P) With the help of $S(T = 0) = 0$ argue that $f_2(T) = U_0$ is a constant.
e) (2P) Compute the internal energy $U(S, V)$.

Problem 2 – **Thermodynamics of macromolecular deformation**

For a rubber band of length z the following relation between temperature T, pulling force F in $z-$direction and length z is given by

$$z = z_o + \frac{\alpha F}{T}, \quad \alpha, z_0 > 0. \tag{2}$$

Further, it is known that in order to heat the rubber band at fixed length z the constant heat capacity $C_z > 0$ is needed, which is independent of temperature T.

a) (3P) Show that the internal energy U is independent of z.
Hint: In this scenario the internal energy is given by $dU = TdS + Fdz$. Why? Reformulate $U(S, z)$ as $U(T, z)$ and remember that S is a total differential $\frac{\partial^2 S}{\partial z \partial T} = \frac{\partial^2 S}{\partial T \partial z}$.
b) (1P) Show that the heat capacity at constant z is not a function of z or T.

c) (3P) Derive the heat capacity at constant pulling force F

$$C_F = \left(\frac{\Delta Q}{\Delta T} \right)_F \tag{3}$$

Hint: First, derive an expression for $dS(T, F)$

d) (3P) The rubber band is pulled from z_1 to $z_2 > z_1$. Derive an expression for $\left(\frac{\partial T}{\partial z}\right)_S$. Does the temperature increase or decrease?