Freie Universität Berlin
Fachbereich Physik
Prof. Dr. Roland Netz
http://www.physik.fu-berlin.de/en/einrichtungen/ag/ag-netz/lehre

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Advanced Statistical Physics II – Problem Sheet 0

Problem 1 – Gaussian integrals

a) Calculate

$$I \equiv \int_{-\infty}^{\infty} dx\, e^{-\frac{x^2}{2}} \tag{1}$$

by expressing I^2 in polar coordinates.

b) Now obtain a result for

$$\int_{-\infty}^{\infty} dx\, e^{-a\frac{x^2}{2}}. \tag{2}$$

c) Aditionally, evaluate the following integrals:

$$\int_{-\infty}^{\infty} dx\, x e^{-\frac{x^2}{2}}, \tag{3}$$

$$\int_{-\infty}^{\infty} dx\, x^2 e^{-\frac{x^2}{2}}. \tag{4}$$

d) A further generalization are integrals of the form

$$\int_{-\infty}^{\infty} dx\, e^{-\frac{x^2}{2}-bx}. \tag{5}$$

Find a solution by using your above results.

e) Now we want to consider the more interesting case of n-dimensional Gaussian integrals with arbitrary positive definite symmetric bilinear forms. We consider a symmetric matrix $G = (g_{ij})$ with positive eigenvalues (λ_i) and calculate the integral

$$\int d^n x\, e^{-\frac{1}{2}\vec{x}^T G \vec{x}} = \int d^n x\, e^{-\frac{1}{2}x_i g_{ij} x_j}. \tag{6}$$

Hint: Remember the spectral theorem.

f) Finally, solve the integral

$$\int d^n x\, e^{-\frac{1}{2}\vec{x}^T G \vec{x} - \vec{b}\cdot\vec{x}}. \tag{7}$$

g) Solve the double integral by changing the order of integration

$$\int_0^1 dx \int_x^1 dy\, e^{y^2}. \tag{8}$$

Problem 2 – Fourier Transform

Consider the definition of the Fourier transform

$$Y(k,t) = \int_{-\infty}^{\infty} y(x,t) e^{ikx} dx, \tag{9}$$

and the partial differential equation (PDE) of a vibrating string whose amplitude $y(x,t)$ satisfies the wave equation

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2}. \tag{10}$$

a) Convert the PDE Eq. (10) into an ordinary differential equation (ODE) for $Y(k,t)$.

b) Using the initial condition $y(x,t=0) = y_0(x)$ and its Fourier transform $Y_0(k)$, solve the ODE for $Y(k,t)$.

c) Calculate the Fourier transform of the following three functions

$$e^{(-t/\tau)}\Theta(t) \tag{11}$$

$$e^{(-|t|/\tau)} \tag{12}$$

$$e^{(-|t|/\tau)}(-\Theta(-t) + \Theta(t)) \tag{13}$$

d) Find a solution for $Y(k)$ in Fourier space for the following equation involving the functions $y(x), w(x)$ and $u(x)$.

$$\frac{\partial^2 y}{\partial x^2} = \int_{-\infty}^{\infty} w(x-x')y(x')dx' + u(x). \tag{14}$$