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# Advanced Statistical Physics II – Problem Sheet 1

Problem 1 – **Integrating Factor**
Recall that a differential

$$df(\vec{x}) = a_1(\vec{x})dx_1 + a_2(\vec{x})dx_2 + \cdots + a_n(\vec{x})dx_n \tag{1}$$

is called *exact* if it can be integrated, i.e. if there exists a scalar potential $f(\vec{x})$ such that $a_i(\vec{x}) = \partial f/\partial x_i$ for $i = 1, \ldots, n$.

(a) (2P) Consider the following expression

$$\Delta Q(U, V, N) = dU + \frac{U}{V}dV + \frac{U}{N}dN \tag{2}$$

Convince yourself that it is *not* an exact differential (and that Q thus cannot define a thermodynamic potential) by showing that the curl of the vector field $(1, U/V, U/N)$ is non-zero.

(b) (2P) Given a non-exact differential $\Delta g(\vec{x})$, we might be able to find a function $c(\vec{x})$ such that $c(\vec{x})\Delta g(\vec{x})$ becomes an exact differential. Such a function $c$ is called *integrating factor*. Verify for our particular example that the entropy is a potential function by showing that for

$$T = \left(\frac{U^2}{aVN}\right)^{1/3} \tag{3}$$

$dS = \Delta Q/T$ is an exact differential.

(c) (1P) Integrate $dS$ to find the entropy $S$.

Problem 2 – **Legendre Transform**

(a) (2P) Compute the Helmholtz free energy $F(T, V, N)$ from the internal energy

$$U(S, V, N) = \frac{S^3}{27aVN} \tag{4}$$

(b) (1P) Do the back-transform

Problem 3 – **Gibbs-Duhem Equation**
(3P) Consider the thermodynamic potential $J(S, P, \mu)$. By using extensive/intensive arguments find an expression for $J$ and use it to derive the Gibbs-Duhem equation:

$$SdT - VdP + Nd\mu = 0 \tag{5}$$

Problem 4 – **Differential form of** $U(T, V, N)$

The aim of this exercise is to derive the differential form of the caloric equation of state mentioned in the lecture:

$$dU = C_V dT + \left(\frac{\alpha T}{\kappa_T} - P\right) dV + \left(\frac{U}{N} - \frac{V}{N}\left[\frac{\alpha T}{\kappa_T} - P\right]\right) dN \tag{6}$$

Intermediate steps:

(a) (3P) Show that

$$\left.\frac{\partial U}{\partial V}\right|_{T,N} = T \left.\frac{\partial P}{\partial T}\right|_{V,N} - P \tag{7}$$

Hint: Consider the differential of $S(U, V, N) - U/T$ and derive the appropriate Maxwell relation.

(b) (3P) Show that $\partial P/\partial T|_{V,N} = \alpha/\kappa_T$ by using the thermal equation of state $V(T, P, N)$ .

(c) (3P) $\partial U/\partial N|_{T,V}$ can be obtained by noting that the caloric equation of state may be written as

$$U(T, V, N) = Nu(T, V/N) \tag{8}$$

For this exercise, recall the definition of the following material constants:

$$C_V = \left.\frac{\partial U}{\partial T}\right|_{V,N}, \quad \alpha = \frac{1}{V} \left.\frac{\partial V(T, P, N)}{\partial T}\right|_{P,N}, \quad \kappa_T = -\frac{1}{V} \left.\frac{\partial V(T, P, N)}{\partial P}\right|_{T,N} \tag{9}$$