Freie Universität Berlin
Fachbereich Physik
Prof. Dr. Roland Netz
http://www.physik.fu-berlin.de/en/einrichtungen/ag/ag-netz/lehre

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Advanced Statistical Physics II – Problem Sheet 9

Problem 1 – **Inertial Markovian Langevin equation**

Consider the following set of Markovian equations of motions

$$m\frac{dv(t)}{dt} = -\gamma v(t) - kx(t) + F_R(t), \tag{1}$$

$$\frac{dx(t)}{dt} = v(t), \tag{2}$$

where the random force $F_R(t)$ is assumed to be Gaussian white noise, which has the properties $\langle F_R(t)\rangle = 0$ and $\langle F_R(t)F_R(t')\rangle = 2\gamma k_B T\delta(t-t')$.

a) (2P) Calculate $\left\langle \tilde{F}_R(\omega)\right\rangle$ and $\left\langle \tilde{F}_R(\omega)\tilde{F}_R(\omega')\right\rangle$. Why is $F_R(t)$ called "white noise"?

b) (1P) Calculate the response function $\tilde{\chi}(\omega)$ which is defined by the relation

$$\tilde{x}(\omega) = \tilde{\chi}(\omega)\tilde{F}_R(\omega), \tag{3}$$

and give $\tilde{\chi}'(\omega)$ and $\tilde{\chi}''(\omega)$.

c) (3P) Calculate $\langle x(t)\rangle$ and $\langle x^2(t)\rangle$ from the results of a) and b).
Hint: The calculation is simple when you realize that $\tilde{\chi}(\omega)\tilde{\chi}(-\omega) = \tilde{\chi}''(\omega)/(\gamma\omega)$ and use an appropriate Kramers-Kronig relation.

Problem 2 – **Non-Markovian Langevin equation**

a) (3P) From eqs. (1) and (2) derive a non-Markovian Langevin equation for $x(t)$ of the form

$$\frac{dx(t)}{dt} = -\int_0^\infty \Gamma_x(t')\; x(t-t')\; dt' + F_x(t). \tag{4}$$

Identify $F_x(t)$ and $\Gamma_x(t')$.

b) (1P) Why is this equation called non-Markovian? Discuss the "source" of non-Markovian effects in this system.
Hint: For what parameter values does the memory term vanish?

c) (4P) Calculate $\left\langle \tilde{F}_x(\omega)\right\rangle$ and $\left\langle \tilde{F}_x(\omega)\tilde{F}_x(\omega')\right\rangle$. Make a sketch of the random force correlation. Why is $F_x(t)$ called "colored noise"?

d) (1P) Calculate the response function $\tilde{\chi}_x(\omega)$ which is defined by the relation

$$\tilde{x}(\omega) = \tilde{\chi}_x(\omega)\tilde{F}_x(\omega), \tag{5}$$

starting from eq. (4).

e) (5P) Calculate $C_{F_xF_x}(t') = \langle F_x(t)F_x(t-t')\rangle$ by contour integration of the Fourier transform.