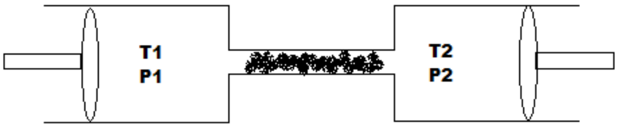
Freie Universität Berlin
Fachbereich Physik
January 11th 2012, discussed in tutorials on January 19th

Prof. Dr. Roland Netz
Julius Schulz, Klaus Rinne

# Advanced Statistical Physics (WS11/12)
# Problem sheet 11

**http://userpage.fu-berlin.de/krinne/**

## Problem 1: Joule-Thomson Process




The Joule-Thomson process consists of an adiabatic expansion of a gas. It can be realised by a system of two gas reservoirs. The reservoirs are hold at different pressures $p_1 > p_2$ by two pistons and are connected by a throttle valve. The gas flows slowly from the reservoir with higher pressure to the reservoir with lower pressure.

a) Show that the total enthalpy $H = U + pV$ of the system is conserved during this process.

b) The sign of the *Joule-Thomson coefficient* $\mu_{\mathrm{JT}} = (\partial T/\partial P)_H$ determines if a pressure reduction leads to cooling or warming of the gas. Show that $(\partial T/\partial P)_H = \left[T\left(\partial V/\partial T\right)_p - V\right]/C_p$.

c) Calculate the inversion curves, $T = T_i(V)$ and $P = P_i(T)$, defined by $\mu_{JT} = 0$, for the van der Waals-gas. Sketch $P = P_i(T)$.

## Problem 2: Grand Canonical 1D-Lattice Gas

Calculate the grand canonical partition function of the one dimensional lattice gas by using the transfer matrix method. Assume that the total energy of a configuration, characterised by occupation numbers $\{n_j\}$, on a lattice with $N$ sites, is given by $E = -\mu \sum_{j=1}^{N} n_j - \epsilon \sum_{j=1}^{N} n_j n_{j+1}$, where $n_j = 0, 1$ and $\mu$ is the chemical potential. Use periodic boundary conditions ($n_1 = n_{N+1}$).

Furthermore, calculate the average occupation number $n = \langle \sum_j n_j / N \rangle$ for $N \to \infty$. What result will we get in the limit of vanishing interaction $\epsilon$?

## Problem 3: Variational Method

We want to consider a system of $N$ interacting, magnetic spins in an external magnetic field $h$ at temperature $T$ with the Hamiltonian

$$H = -h \sum_{j=1}^{N} s_j - \frac{\epsilon}{2} \sum_{i,j} s_i s_j,$$

where $s_j = -1,\ 0,\ +1$. The double sum only involves nearest neighbours, where each spin has $z$ nearest neighbours. Use the variational method with the variational Hamiltonian $H_0 = -\tilde{h} \sum_{j=1}^{N} s_j$ to calculate the phase transition temperature and the leading term of the magnetisation $\langle s_j \rangle$ close to the phase transition point in the limit $h \to 0$.

## Problem 4: Correlation Length

Consider an interacting 1D spin chain with the Hamiltonian $H = -\epsilon \sum_{j=1}^{N} s_j s_{j+1}$ in absense of an external field in the thermodynamic limit $N \to \infty$. The correlation length $\xi$ is defined by the spin correlation function $\langle s_1 s_{n+1} \rangle = e^{-n/\xi}$. Calculate $\xi$ by using the transfer matrix methode. Use $\langle s_1^2 \rangle = 1$ for $n = 0$. Also discuss $\xi$ in the limit $T \to 0$, where $T$ is the temperature.