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Fachbereich Physik
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Advanced Statistical Physics (WS11/12)
Problem sheet 3

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Problem 1: Phase Space

Sketch typical propagations of the following one-dimensional systems in the $\{q, p\}$ phase space:

a) free particle

b) free particle between two hard, reflecting walls at $\pm q_0$

c) particle in the homogeneously approximated gravitation field of the earth

d) undamped harmonic oscillator

e) damped harmonic oscillator

Problem 2: Lagrange Multipliers

We discuss the given functions $f(x, y) = ax + by$ and $g(x, y) = x^2 + y^2 - c^2$:

a) Minimise f with the side condition $g(x, y) = 0$ by solving the system of equations with the side condition and the extremal condition.

b) Minimise f with the side condition $g(x, y) = 0$ using Lagrange multipliers. Therefore minimise the function $h(x, y) = f(x, y) + \lambda g(x, y)$.

Hint: You should first minimise h with respect to x and y. Finally, minimise with respect to λ. Of course a) and b) will produce the same result.

Problem 3: Exact Differential

We discuss the differential $\mathrm{d}F = (x^2 - xy)\mathrm{d}x + x^2\mathrm{d}y$:

a) Show that $\mathrm{d}F$ is not an exact differential.

b) For which exponent n becomes $\mathrm{d}G = \mathrm{d}F/x^n$ an exact differential?

c) Integrate $\mathrm{d}G$ along two paths from (1,1) to (2,2) for the correct exponent n.

Problem 4: Classical harmonic oscillator

A classical harmonic oscillator of mass m and spring constant k is known to have the total energy of E, but the starting position and time are completely unknown.

Find the probability density function $\rho(x)$, where $\rho(x)\mathrm{d}x$ is the probability that the mass can be found in the interval $\mathrm{d}x$ at x.