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Fachbereich Physik
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Prof. Dr. Roland Netz
Julius Schulz, Klaus Rinne

Advanced Statistical Physics (WS11/12)
Problem sheet 8

http://userpage.fu-berlin.de/krinne/

Problem 1: Air Conditioner

A home air conditioner operating on a reversible Carnot cycle between the inside, absolute temperature T_2, and the outside, absolute temperature $T_1 > T_2$, consumes the energy W from the power line during one cycle when operating continuously.

a) Develop a formula for the cooling efficiency ratio Q_2/W in terms of T_1 and T_2.

b) Now we want to consider a heat leakage into the house $Q = A(T_1 - T_2)$ during one operating cycle. Calculate the inside temperature T_2 as function of W, A and T_1.

c) Let us assume that the air conditioner is running with 30% of the maximal power to maintain $T_2 = 293\,\mathrm{K}$, if the outside temperature is $T_1 = 303\,\mathrm{K}$. Calculate the maximal outside temperature T_1, where the air conditioner can maintain $T_2 = 293\,\mathrm{K}$ inside the room.

d) In the winter the cycle is reversed and the air conditioner becomes a heat pump. Calculate the minimal outside temperature T_1, where the device can maintain $T_2 = 293\,\mathrm{K}$ inside the room.

Problem 2: Cyclic Process

An ideal gas with heat capacities C_V and C_P undergoes the following quasistatic cyclic process:

- $a \to b$: adiabatic expansion from V_1 to V_2,
- $b \to c$: isochoric cooling at V_2,
- $c \to d$: adiabatic compression from V_2 to V_1,
- $d \to a$: isochoric heating at V_1.

Calculate the efficiency η.

Problem 3: Ideal Gas, Equation of State

The entropy of a mono-atomic, ideal gas is

$$S(U,V,N) = \frac{N}{N_0} S(U_0,V_0,N_0) + k_B N \ln\left[\frac{V}{V_0}\left(\frac{U}{U_0}\right)^{3/2}\left(\frac{N}{N_0}\right)^{-5/2}\right],$$

where (U_0, V_0, N_0) is a reference state. Calculate the thermal equation of state $U(T,V,N)$ and the caloric equation of state $P(T,V,N)$.

Problem 4: Entropy, Thermal Expansion and Heat Capacity

Use the first law of thermodynamics, $đQ = \mathrm{d}U + P\,\mathrm{d}V$, and the differential of the entropy, $T\,\mathrm{d}S = đQ$, to derive the expression

$$T\,\mathrm{d}S = C_P \mathrm{d}T - \alpha T V\,\mathrm{d}P,$$

where

$$\alpha = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P$$

is the coefficient of thermal expansion and

$$C_P = \left(\frac{\partial (U + PV)}{\partial T}\right)_P$$

is the isobaric heat capacity.