Freie Universität Berlin
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Statistical Physics and Thermodynamics (SS 2016)

Problem sheet 1

Hand in: Thursday, April 28 during the lecture

http://www.physik.fu-berlin.de/en/einrichtungen/ag/ag-netz/lehre/

1 Yahtzee (5 points)

We want to consider a simplified version of the game of dice Yahtzee: 5 ideal dice are thrown at the same time only once per move. Calculate:

a) The probability of a Yahtzee, which is all five dice showing the same face. **(1 point)**

b) The probability of a Large Straight, which is five sequential dice (1-2-3-4-5 or 2-3-4-5-6). **(1 point)**

c) The probability of a Full House, i. e. a three-of-a-kind and a pair with a different face. **(2 points)**

d) The probability of exactly one Yahtzee (at any time) in ten moves, i.e. after 10 times throwing the 5 ideal dice. **(1 point)**

2 Random walker (5 points)

A drunken person starts in front of a lamp pole and can only take steps in two directions (right/left). All steps are of the same length and both direction probabilities are equal. Calculate:

a) The probability of 10 steps into the same direction. **(2 points)**

b) The probability of reaching the starting point after taking 10 steps. **(3 points)**

3 Football team (10 points)

The Poisson-distribution is a probability distribution which describes the number of events m in a fixed interval of time (or space, etc.) if they occur with a fixed probability. It is a good approximation for the binomial distribution for a large number of experiments N and a small probability p. A typical example is radioactive decay. The (discrete) Poisson-distribution is given by

$$P(m) = \frac{\lambda^m}{m!} \exp\left(-\lambda\right). \tag{1}$$

a) Show that

$$\sum_{m=0}^{\infty} P(m) = 1. \tag{2}$$

(1 point)

b) Show that the expectation value $\langle m \rangle$ of this distribution is equal to λ. **(3 points)**

Now, we want to use the Poisson distribution, to gain insight about statistical distributions of goals in football matches. Assume a football team that scores on average two goals per game and use your result from (b) to calculate

c) the probability that the team scores zero goals **(1 point)**

d) the probability to score two goals **(1 point)**

e) the probability to score more than two goals **(2 points)**

f) Calculate the expectation value $\langle m^2 \rangle$ and use it to obtain a result for the variance

$$\Delta m^2 = \langle m^2 \rangle - \langle m \rangle^2 . \tag{3}$$

Hint: Use the identity $m = (m-1) + 1$ *to split the sum in the expectation value into two contributions.*
(3 points)