Freie Universität Berlin
Fachbereich Physik
May 3, 2016

Prof. Dr. Roland Netz
Douwe Bonthuis
Jan Daldrop
Sadra Kashef

Statistical Physics and Thermodynamics (SS 2016)

Problem sheet 3

Hand in: Thursday, May 12 during the lecture

http://www.physik.fu-berlin.de/en/einrichtungen/ag/ag-netz/lehre/

1 Two normal distributions (10 points)

Consider the two independent normal distributions given by

$$P_1(x_1) \propto \exp\left(-\frac{(x_1-\mu_1)^2}{2\sigma_1^2}\right) \tag{1}$$

and

$$P_2(x_2) \propto \exp\left(-\frac{(x_2-\mu_2)^2}{2\sigma_2^2}\right). \tag{2}$$

We are interested in the variable

$$y = x_1 + x_2. \tag{3}$$

Remark: You don't have to normalize the distributions/characteristic functions for this exercise.

a) Calculate $P(y)$ explicitly by evaluating the integral

$$P(y) = \int dx_1 \int dx_2 \, P(x_1)P(x_2)\delta(x_1+x_2-y). \tag{4}$$

What is the resulting distribution? Read off the mean value and the variance. Assume for simplicity $\mu_1 = -\mu_2 \equiv \mu$ and $\sigma_1 = \sigma_2 \equiv \sigma$ (only for this task). You don't need to keep track of the normalization factor (which can be a bit lengthy). **(2 points)**

b) Show that $\langle x_1 x_2 \rangle = \langle x_1 \rangle \langle x_2 \rangle$. **(1 point)**

c) Now calculate the mean value $\langle y \rangle$ and the variance $\langle y^2 \rangle - \langle y \rangle^2$ directly by expressing y in terms of x_1 and x_2 and by using $\langle f+g \rangle = \langle f \rangle + \langle g \rangle$ as well as your result from (b). Express the result in terms of $\mu_1, \mu_2, \sigma_1, \sigma_2$ and compare to your result from (a). **(2 points)**

d) Calculate the characteristic functions $g_i(k)$ for $P_i(x_i)$. **(2 points)**

e) Check your result from (d) by calculating $\langle x_i \rangle$ and $\langle x_i^2 \rangle_C$ from $g_i(k)$. **(1 point)**

f) What is the characteristic function $G(k)$ of $P(y)$? **(1 point)**

g) Calculate the mean value and the variance of y from the characteristic function $G(k)$. **(1 point)**

2 Uniform distributions (10 points)

Now consider the uniform probability distribution

$$P(x) = \begin{cases} 1 & 0 \leq x \leq 1, \\ \\ 0 & \text{otherwise.} \end{cases} \tag{5}$$

a) Calculate the characteristic function $g(k)$ for $P(x)$. **(2 points)**

b) Now consider m independent variables x_i, with probabilities $P(x_i)$ according to eq. (5). Calculate the characteristic function $G_m^z(k)$ for the variable

$$z = \sum_{i=1}^{m} x_i. \tag{6}$$

(1 point)

c) Calculate the mean value from the characteristic function for the distribution with $m = 2$. **(3 points)**

d) Introduce $y = z/m$. What is the characteristic function $G_m^y(k)$ for y? **(1 point)**

e) Expand $\exp(-ik/m)$ to second order and use the identity

$$\exp(x) = \lim_{m\to\infty} \left(1 + \frac{x}{m}\right)^m \tag{7}$$

to calculate the characteristic function

$$G_\infty^y(k) \equiv \lim_{m\to\infty} G_m^y(k). \tag{8}$$

Interpret your result. **(3 points)**