Freie Universität Berlin
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May 10th 2016

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Statistical Physics and Thermodynamics (SS 2016)

Problem Sheet 4

Hand in:Thursday, May 19 during the lecture

http://www.physik.fu-berlin.de/en/einrichtungen/ag/ag-netz/lehre/

1. Classical Harmonic Oscillator (8 points)

The potential energy term of a harmonic oscillator can be written as $V(x) = \frac{1}{2}kx^2$, the frequency of such an oscillator is $\nu = \sqrt{\frac{k}{m}}$:

a) Write down the Hamiltonian of such a system in terms of x and p. Use the Hamiltonian to calculate the time derivatives $\dot{x}$ and $\dot{p}$. **(2 points)**

b) Assuming the initial conditions x(0)=x_0 and p(0)=p_0, solve the equations from part (a) and find explicit expressions for x(t) and p(t). (**Hint:** you need to take the time derivative of $\dot{x}$ and $\dot{p}$!). **(2 points)**

c) Now consider the time dependence of the Hamiltonian. For this you should use your result for x(t) and p(t) obtained in part (b) and plug it into the Hamiltonian equation from part (a). Explain how this Hamiltonian evolves in time. **(2 points)**

d) Sketch the time dependent trajectory of this oscillator in the $\{q, p\}$ phase space. **(1 point)**

e) Now sketch the time dependent trajectory of a damped harmonic oscillator and compare it to the one from part (d).**(1 point)**

2. Total Differential in Two-Dimensions (5 points)

The differential $dF(x, y) = A(x, y)dx + B(x, y)dy$ is exact if it satisfies $(\frac{dA}{dy})_x = (\frac{dB}{dx})_y$ and there is a function F(x,y), so that $(\frac{dF}{dx})_y = A(x, y)$ and $(\frac{dF}{dy})_x = B(x, y)$

Consider $dF(x, y) = (\cos x - x\sin x + y^2)dx + 2xy\ dy$.

a) Show that this differential is exact. **(1 point)**

b) Find the function $F(x, y)$. **(2 points)**

c) For the range $-2\pi < x < 2\pi$, plot the curve $y(x)$ determined by the equation $F(x, y) = F(\pi, 1)$. **(2 points)**

2. Total Differential in Three-Dimensions (7 points)

The differential $dF(x, y, z) = A(x, y, z)dx + B(x, y, z)dy + C(x, y, z)dz$ is exact only if: $(\frac{dA}{dy})_{x,z} = (\frac{dB}{dx})_{y,z}$; $(\frac{dA}{dz})_{x,y} = (\frac{dC}{dx})_{y,z}$; $(\frac{dB}{dz})_{x,y} = (\frac{dC}{dy})_{x,z}$ and there must be a function $F(x, y, z)$ such that $\nabla F = \vec{R}$, where $\vec{R} = A(x, y, z)\hat{i} + B(x, y, z)\hat{j} + C(x, y, z)\hat{k}$.

a) Show that we can also say $dF(x, y, z)$ is exact if $\nabla \times \vec{R} = 0$. **(1 points)**

Now consider $dF(x, y, z) = ydx + (x + 2y \sin z)dy + y^2 \cos(z)dz$

b) Verify that it is an exact differential by using part (a). **(1 points)**

c) Find the function $F(x, y, z)$ in terms of an arbitrary function $g(y, z)$. (**Hint:** $(\frac{dF(x,y,z)}{dx})_{y,z} = y$, then integrate this to obtain $F(x, y, z)$. Remember to include the arbitrary function $g(y, z)$). **(1 point)**

d) Solve the resulting equation and find both $g(y, z)$ and $F(x, y, z)$ in terms of an arbitrary function $h(z)$. (**Hint:** Calculate $(\frac{dF(x,y,z)}{dy})_{x,z}$ by using the F(x,y,z) that you obtained in part (c) and set it equal to $x + 2y \sin(z)$). **(2 points)**

e) Set $(\frac{dF(x,y,z)}{dz})_{x,y} = y^2 \cos z$ to find $h(z)$ in terms of an arbitrary constant C and write the final expression for $F(x, y, z)$. **(2 points)**