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hand in on 19th June 2015 before lecture

Advanced Statistical Physics II – Problem Sheet 9

Problem 1 – **Response to time-dependent fields**

Calculate the response $\Delta A(t)$, for $t > 0$ and $C_{AB}(t) = C_0 e^{-t/\tau}$, using the following fields $h(t)$:

a) $h(t) = h_0 \Theta(t)$ where $\Theta(t)$ is the Heaviside step function.

b) $h(t) = h_0 \Theta(t) e^{-\alpha t}$ where α is positive real.

c) $h(t) = h_0 \Theta(t)(1 - \alpha t)$.

d) $h(t) = h_0 \Theta(t) \cos(\Omega t)$ where Ω is positive real.

Problem 2 – **Kramers-Kronig relation**

a) By using $\chi(t) = \chi_e(t) + \mathrm{sig}(t)\chi_e(t)$ derive the Kramers-Kronig relation for an imaginary part of a response function

$$\tilde{\chi}''(\omega) = \frac{1}{\pi} \mathcal{P} \int_{-\infty}^{\infty} d\omega' \frac{\tilde{\chi}'(\omega')}{\omega - \omega'}, \tag{1}$$

where $\mathcal{P}$ is the Cauchy principal value, $\chi_e(t) = (\chi(t) + \chi(-t))/2$ is an even-function contribution of the single-sided $\chi(t)$ [i.e. $\chi(t) = 0$ for $t < 0$], and $\mathrm{sig}(t)$ is the signum (or sign) function.

b) Derive alternative forms of the Kramers-Kronig relation

$$\tilde{\chi}'(\omega) = \frac{2}{\pi} \mathcal{P} \int_0^{\infty} d\omega' \frac{\omega' \tilde{\chi}''(\omega')}{\omega'^2 - \omega^2}, \tag{2}$$

$$\tilde{\chi}''(\omega) = \frac{2\omega}{\pi} \mathcal{P} \int_0^{\infty} d\omega' \frac{\tilde{\chi}'(\omega')}{\omega^2 - \omega'^2}. \tag{3}$$

Problem 3 – **Inhomogenous differential equations**

a) Consider the inhomogeneous differential equation

$$y' = p(x) \cdot y + r(x). \tag{4}$$

Derive a formula for the general solution of $y(x)$.

b) As an application consider a capacitor with capacity C and resistance R connected to a battery with voltage V_b.

$$R \frac{\mathrm{d}Q}{\mathrm{d}t} + \frac{Q}{C} - V_b = 0. \tag{5}$$

i) Use the above result to solve for $Q(t)$ when $Q(t = 0) = 0$.

ii) Use the Laplace transform to solve Eq. (5).

iii) How does the differential equation look when the capacitor is fully loaded and starting to get discharged at $t = 0$. How does the voltage change in time?