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Advanced Statistical Physics II – Problem Sheet 0

Problem 1 – Gaussian Integrals

a) Calculate

$$I \equiv \int_{-\infty}^{\infty} dx\, e^{-\frac{x^2}{2}} \tag{1}$$

by expressing I^2 in polar coordinates.

b) Now obtain a result for

$$\int_{-\infty}^{\infty} dx\, e^{-a\frac{x^2}{2}}. \tag{2}$$

c) Aditionally, evaluate the following integrals:

$$\int_{-\infty}^{\infty} dx\, x e^{-\frac{x^2}{2}}, \tag{3}$$

$$\int_{-\infty}^{\infty} dx\, x^2 e^{-\frac{x^2}{2}}. \tag{4}$$

d) A further generalization are integrals of the form

$$\int_{-\infty}^{\infty} dx\, e^{-\frac{x^2}{2}-bx}. \tag{5}$$

Find a solution by using your above results.

Problem 2 – Euler Theorem

The Euler theorem for homogeneous functions says that if a differentiable function $f(x_1, ..., x_k)$ is homogeneous of degree m, i.e. if

$$f(\lambda x_1, ..., \lambda x_k) = \lambda^m f(x_1, ..., x_k), \tag{6}$$

then it follows that

$$\frac{\partial f}{\partial x_1} x_1 + ... + \frac{\partial f}{\partial x_k} x_k = mf. \tag{7}$$

(This can easily be proven by taking the derivative with respect to λ on both sides of eq. 6 and then setting $\lambda = 1$.)

a) Use the Euler homogeneous function theorem to derive the following version of the internal energy

$$U = TS - PV + \mu N \tag{8}$$

b) Similar show that

$$\Omega = -PV \tag{9}$$

c) From (b) you can derive why the pressure P in the grand potential does not deppend on the volume V or the particle number N:

$$P = P(\mu, T). \tag{10}$$

Problem 3 – Fourier Transform

Consider the definition of the Fourier transform

$$Y(k,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} y(x,t) e^{ikx} dx, \tag{11}$$

a) Consider a vibrating long string whose amplitude $y(x,t)$ satisfies the wave equation

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2}. \tag{12}$$

b) Convert the partial differential equation (PDE) Eq. (12) into an ordinary differential equation (ODE) of Y.

c) Using the initial condition $y(x, t = 0) = y_0(x)$ and its Fourier transform $Y_0(k)$, solve the ODE for $Y(k,t)$.

d) Calculate the Fourier transform of the following three functions

- $\exp(-t/\tau)\Theta(t)$
- $\exp(|t|/\tau)$
- $\exp(|t|/\tau)(-\Theta(-t) + \Theta(t))$