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Advanced Statistical Physics II – Problem Sheet 8

Problem 1 – **Time Scale**

(6P) Consider the linear differential equation

$$m\frac{dv(t)}{dt} = -\gamma v(t) \tag{1}$$

where γ is the friction coefficient. Consider Stokes law $\gamma = 6\pi\eta R$, where η is the viscosity and R is the radius of the particle.
The water's viscosity is $\eta_{H_2O} = 10^{-3} Pa \cdot s$, the density of sucrose is $\rho = 1,5879 \quad g/cm^3$ and its formula is $C_{12}H_{22}O_{11}$. From these, estimate the radius of a sucrose molecule.
Using these material constants, calculate the intertial time scale $\tau_m = m/\gamma$ for a molecule of water and for a molecule of sucrose in water.

Problem 2 – **Mean Square Displacement (MSD)**

The Langevin equation is

$$m\frac{\mathrm{d}}{\mathrm{d}t}v(t) = -\gamma v(t) + F_R(t), \tag{2}$$

where m denotes the particle mass, $v(t)$ the velocity, γ the friction coefficient and $F_R(t)$ the random force. It is an inhomogenous linear differential equation of first order. As in sheet 5, the solution of this type of equations reads:

$$v(t) = v(0)\mathrm{e}^{-\gamma t/m} + \frac{1}{m}\int_0^t \mathrm{d}t' \mathrm{e}^{-\gamma(t-t')/m} F_R(t'). \tag{3}$$

The random force F_R is assumed to be Gaussian white noise, which has the properties

$$\langle F_R(t)\rangle = 0 \qquad \text{and} \qquad \left\langle F_R(t)F_R(t')\right\rangle = 2B\delta(t-t'), \tag{4}$$

where $\langle \dots \rangle$ denotes the average over the random force.

(a) (5P) Calculate the mean squared displacement

$$\left\langle \Delta x(t)^2 \right\rangle = \left\langle (x(t) - x(0))^2 \right\rangle , \tag{5}$$

where $x(t)$ is the position of the particle at time $t > 0$.
Use the solution of the Langevin equation by writing the MSD in terms of the velocity.
Hint: Remember the change of the order of integration

(b) (2P) Discuss the limits $t \gg m/\gamma$ and $t \ll m/\gamma$.

(c) (3P) Using the results of problem (1) calculate the diffusion length $l = \sqrt{\langle \Delta x(t)^2 \rangle}$ for $t = 1s$.

Problem 3 – **Diffusion coefficient**

(4P) Start from the Einstein equation $\left\langle \Delta x(t)^2 \right\rangle = 2Dt$, where D is the diffusion constant. Show that the diffusion coefficient can be written as

$$D = \int_0^t \mathrm{d}t' C_{vv}(t') \tag{6}$$

where $C_{vv}(t) = \langle v(t)v(0) \rangle$ is the velocity autocorrelation function.