

# Übungsblatt 1      Lösungen

①

## Aufg. 1

$$a) \frac{128}{84} = \frac{2^7}{2^2 \cdot 3 \cdot 7} = \frac{2^5}{3 \cdot 7} = \frac{32}{21} \quad (= 1 \frac{11}{21})$$

$$b) \frac{210}{135} = \frac{2 \cdot 3 \cdot 5 \cdot 7}{3^3 \cdot 5} = \frac{2 \cdot 7}{3^2} = \frac{14}{9} \quad (= 1 \frac{5}{9})$$

$$c) \frac{84}{210} = \frac{2^2 \cdot 3 \cdot 7}{2 \cdot 3 \cdot 5 \cdot 7} = \frac{2}{5}$$

$$d) \frac{231}{210} = \frac{3 \cdot 7 \cdot 11}{2 \cdot 3 \cdot 5 \cdot 7} = \frac{11}{10} \quad (= 1 \frac{1}{10})$$

## Aufg. 2

$$a) \frac{7}{12} - \frac{7}{16} = \frac{28}{48} - \frac{21}{48} = \frac{7}{48}$$

$$b) \frac{5}{6} - \frac{4}{15} = \frac{25}{30} - \frac{8}{30} = \frac{17}{30}$$

$$c) \frac{11}{18} + \frac{7}{24} = \frac{44}{72} + \frac{21}{72} = \frac{65}{72}$$

$$d) \frac{25}{27} - \frac{11}{18} = \frac{50}{54} - \frac{33}{54} = \frac{17}{54}$$

$$e) \frac{21}{25} + \frac{9}{20} = \frac{84}{100} + \frac{45}{100} = \frac{129}{100}$$

$$f) \frac{25}{28} - \frac{11}{21} = \frac{75}{84} - \frac{44}{84} = \frac{31}{84}$$

## Aufg. 3

$$a) 8 \frac{7}{9} + 3 \frac{2}{27} = 8 + 3 + \frac{7}{9} + \frac{2}{27} = 11 + \frac{21}{27} + \frac{2}{27} = 11 \frac{23}{27}$$

$$b) 14 \frac{4}{5} + 4 \frac{8}{15} = 14 + 4 + \frac{4}{5} + \frac{8}{15} = 18 + \frac{12}{15} + \frac{8}{15} = 18 \frac{20}{15} = 19 \frac{1}{3}$$

$$c) 17 \frac{1}{12} - 8 \frac{5}{6} = 17 - 8 + \frac{1}{12} - \frac{5}{6} = 16 - 8 + \frac{13}{12} - \frac{5}{6} = 8 + \frac{13}{12} - \frac{10}{12} \\ = 8 \frac{3}{12} = 8 \frac{1}{4}$$

Aufg. 4

$$a) \frac{17}{18} : \frac{11}{18} = \frac{17}{18} \cdot \frac{18}{11} = \frac{17}{11} \quad (= 1 \frac{6}{11})$$

$$b) \frac{4}{9} : \frac{5}{6} = \frac{4}{9} \cdot \frac{6}{5} = \frac{4 \cdot 2}{3 \cdot 5} = \frac{8}{15}$$

$$c) \frac{5}{14} : \frac{9}{20} = \frac{5}{14} \cdot \frac{20}{9} = \frac{5 \cdot 10}{7 \cdot 9} = \frac{50}{63}$$

$$d) \frac{3}{4} : 1 \frac{2}{3} = \frac{3}{4} : \frac{5}{3} = \frac{3}{4} \cdot \frac{3}{5} = \frac{9}{20}$$

$$e) 11 \frac{5}{9} : 9 \frac{1}{9} = \frac{104}{9} : \frac{82}{9} = \frac{104}{9} \cdot \frac{9}{82} = \frac{52}{41} \quad (= 1 \frac{11}{41})$$

$$f) 4 \frac{1}{7} : 1 \frac{4}{7} = \frac{29}{7} : \frac{11}{7} = \frac{29}{7} \cdot \frac{7}{11} = \frac{29}{11} \quad (= 2 \frac{7}{11})$$

$$g) 6 : \frac{5}{7} = \frac{6}{1} \cdot \frac{7}{5} = \frac{42}{5} \quad (= 8 \frac{2}{5})$$

$$h) 4 \frac{3}{4} : 5 = \frac{19}{4} \cdot \frac{1}{5} = \frac{19}{20}$$

Aufg. 5

$$a) (x-3)^2 = x^2 - 3(x+1)$$

$$\Leftrightarrow x^2 - 6x + 9 = x^2 - 3x - 3$$

$$\Leftrightarrow 12 = 3x$$

$$\Leftrightarrow x = 4$$

$$b) (x+2)^2 - (x-4)^2 = 11x - 8$$

$$\Leftrightarrow x^2 + 4x + 4 - x^2 + 8x - 16 = 11x - 8$$

$$\Leftrightarrow 12x - 12 = 11x - 8$$

$$\Leftrightarrow x = 4$$

$$c) (z-4)^2 - (z+8)^2 + 23z + 45 = 0$$

$$\Leftrightarrow z^2 - 8z + 16 - z^2 - 16z - 64 + 23z + 45 = 0$$

$$\Leftrightarrow -z - 3 = 0$$

$$\Leftrightarrow z = -3$$

$$d) (a-7)^2 + 9 = (a+3)^2 + 49$$

$$\Leftrightarrow a^2 - 14a + 49 + 9 = a^2 + 6a + 9 + 49$$

$$\Leftrightarrow -14a + 58 = 6a + 58$$

$$\Leftrightarrow -14a = 6a$$

$$\Leftrightarrow 0 = 20a$$

$$\Leftrightarrow a = 0$$

### Aufg 6

$$a) 1 - 4 + 7 - 10 + 13 - 16 + 19 - 22 = \sum_{k=0}^7 (-1)^k (3k+1)$$

$$b) \frac{1}{4 \cdot 6} + \frac{1}{5 \cdot 7} + \frac{1}{6 \cdot 8} + \frac{1}{7 \cdot 9} + \frac{1}{8 \cdot 10} + \dots + \frac{1}{11 \cdot 13} = \sum_{k=4}^{11} \frac{1}{k(k+2)}$$

$$= \sum_{k=1}^8 \frac{1}{(k+3)(k+5)}$$

$$c) 2^1 - 4^2 + 6^3 - \dots + 14^7 - 16^8 = \sum_{k=1}^8 (-1)^{k+1} (2k)^k = - \sum_{k=1}^8 (-2k)^k$$

### Aufg. 7

$$a) \sum_{n=2}^{100} \frac{n+1}{n-1} - \sum_{k=2}^{100} \frac{k+2}{k} = \sum_{k=1}^{99} \frac{k+2}{k} - \sum_{k=2}^{100} \frac{k+2}{k} = \frac{1+2}{1} - \frac{100+2}{100}$$

$$= 3 - \frac{102}{100}$$

$$b) 2 \sum_{m=1}^{50} m + \sum_{r=1}^{50} (r^2 + 1) \stackrel{1. \text{ Binom}}{=} \sum_{r=1}^{50} (r+1)^2 = \sum_{j=2}^{51} j^2$$

$$c) \sum_{j=1}^{200} \frac{1}{j(j+2)} - \sum_{i=2}^{200} \frac{1}{i^2-1} \stackrel{3. \text{ Binom}}{=} \sum_{j=1}^{200} \frac{1}{j(j+2)} - \sum_{j=1}^{199} \frac{1}{(j+2)j} = \frac{1}{200 \cdot 202}$$

3. Binom:  $i^2 - 1 = (i+1)(i-1)$