Probing the superfluid–Mott-insulating shell structure of cold atoms by parametric excitations

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We study the effect of parametric excitations on systems of confined ultracold Bose atoms in periodically modulated optical lattices. In the regime where Mott insulating and superfluid domains coexist, we show that the dependence of the energy absorbed by the system on the frequency of the modulation serves as an experimental probe for the existence and properties of Mott insulating-superfluid domains.

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The tunability of ultracold atom gases in optical lattices makes them ideal candidates for the investigation of quantum many-particle phenomena in different regimes. As a paradigmatic example, within the Bose-Hubbard (BH) model, bosonic atoms in optical lattices have been predicted to undergo a quantum phase transition from a compressible, phase-coherent superfluid (SF) phase to an incompressible incoherent Mott insulator (MI) induced by increasing the ratio of the on-site repulsion U to the hopping kinetic energy J [1]. For an infinite uniform system on a lattice the phase diagram in the $(J/U, \mu/U)$ plane is composed of lobelike MI regions with integer site occupation in an otherwise SF phase [1,2]. By controlling the ratio J/U via the intensity of the optical lattice, the MI-SF transition has been experimentally observed [3,4] together with an energy gap of the expected magnitude in the MI phase. Experiments are always performed in finite inhomogeneous systems with a fixed total number of atoms subject to an external confinement which has been numerically established to induce the coexistence of MI and SF domains [5–8].

The formation of MI shells of different site occupation have been recently detected by local probes [9–11] but still, an experimental probe of the SF domains and their properties is presently missing. It has been proposed [12] that the coexistence of MI and SF domains modifies the low-energy excitation spectrum of the system, finally affecting its thermodynamic properties.

In this paper we analyze the energy absorption by a trapped atomic cloud in an optical lattice due to a periodic time modulation of the lattice intensity. We show that the induced parametric resonances of the modes of the system can be used to probe the SF domains and provide a mean to detect their existence, estimate their size, and distinguish them from domains of normal fluid. We find that when the frequency of the modulation is lower than the MI gap the energy absorption is only due to excitations of the SF regions and is strongly peaked at frequencies related to the transverse size of the SF rings. An experimental observation of these peaks may then provide evidence for the coexistence of MI and SF domains and give an estimate for the width of these domains. The width of the peaks reflects the viscosity of the fluid and distinguishes superfluids from normal ones. Our proposal has the advantage of not requiring any additional lasers beyond those of the existing optical lattice.

A parametric excitation is a process in which the population of certain modes is exponentially enhanced as a result of a periodic temporal modulation of a parameter in the Hamiltonian. Parametric excitations have been studied in harmonically trapped BEC [13] and for condensates in optical lattices in different spatial dimensions [14]. Very recently a periodic variation of the optical lattice potential depth was used to probe the response of a one-dimensional (1D) condensate across the MI-SF quantum phase transition [15,16]. Here, the energy absorption versus the lattice modulation frequency Ω showed a broad spectrum in the SF phase and peaks in the gapped MI one. Theoretical works that followed analyzed the parametric instabilities of Bogoliubov modes in infinite superfluid systems [17,18]. In parallel, recent works considered the linear response energy absorption of 1D systems within the bosonization picture [19] or with numerical diagonalization of realistic systems [20–22]. In the presence of confinement, the low-energy form of the dynamic structure factor has been argued to yield signatures of the presence of SF-MI domains [22]. The main focus of our analysis here is the effect of parametric instabilities on the striped structure of MI-SF shells in a finite system as well as on a viscous fluid.

The BH Hamiltonian describing a system of bosons in a periodic optical lattice is

$$H = -J_0\sum_{\langle ij\rangle}(b_i^\dagger b_j + \text{H.c.}) + \frac{U_0}{2}\sum_i b_i^\dagger b_i(b_i^\dagger b_i - 1) + \sum_i(-\mu + V_i)b_i^\dagger b_i \quad (1)$$

with b_i the annihilation operator of a boson at site i. We assume nearest-neighbor hopping of strength J_0 and an on-site repulsion U_0. The values of J_0 and U_0 are given by $J_0 = 4/\sqrt{\pi}(V_0/E_R)^{3/4}\exp(-4\sqrt{V_0/E_R})E_R$ and $U_0 = \pi^2 a_0/3l(V_0/E_R)^{3/4}E_R$ [23], where V_0 and l are the optical lattice depth and spacing, E_R is the recoil energy and a_0 the scattering length. The chemical potential μ fixes the particle number and V_i is the potential bias at site i due to the external confinement.

We consider a two-dimensional atom cloud subjected to an optical lattice. In the 2D plane the confining potential V_i

grows from the center of the cloud towards its boundaries. For J_0/U_0 larger than the critical value inducing the MI-SF transition, the entire 2D system is a SF while below this critical ratio concentric alternated 2D ringlike MI and SF domains are formed [5].

In a compressible SF ring the average occupation per site $\bar{n}$ is in between the integer occupations of the neighboring MI domains. Assuming the length of the ring is much larger than its width L, we can view it as a SF stripe closed with periodic boundaries and consider momentum eigenstates along the stripe. For weak depletion, in momentum space the BH Hamiltonian (1) in the SF stripe reduces, at mean field [24], to

$$H=\sum_{\mathbf{k}\neq 0}(\epsilon_{\mathbf{k}}^0+\bar{n}U_0)a_{\mathbf{k}}^\dagger a_{\mathbf{k}}+\frac{\bar{n}U_0}{2}\sum_{\mathbf{k}\neq 0}(a_{\mathbf{k}}^\dagger a_{-\mathbf{k}}^\dagger+a_{\mathbf{k}}a_{-\mathbf{k}}), \quad (2)$$

where the ground state energy was subtracted and $a_{\mathbf{k}}=\sum_j b_j e^{i\mathbf{k}\cdot\mathbf{r}_j}$. Within the tight-binding approximation (valid for the deep lattices implied by the MI domain formation) we have, at small k, $\epsilon_{\mathbf{k}}^0=J_0 l^2|\mathbf{k}|^2$.

The Hamiltonian (2) is diagonalized via the Bogoliubov transformation $c_{\mathbf{k}}=u_{\mathbf{k}}a_{\mathbf{k}}+v_{\mathbf{k}}a_{-\mathbf{k}}^\dagger$ with $u_{\mathbf{k}}^2=1+v_{\mathbf{k}}^2=\frac{1}{2}[(\epsilon_{\mathbf{k}}^0+\bar{n}U_0)/E_{\mathbf{k}}^0+1]$ and results in

$$H_0=\sum_{\mathbf{k}\neq 0}E_{\mathbf{k}}^0 c_{\mathbf{k}}^\dagger c_{\mathbf{k}}, \quad (3)$$

where $c_{\mathbf{k}}^\dagger$ creates a Bogoliubov quasiparticle of 2D quasimomentum $\mathbf{k}$ and energy $E_{\mathbf{k}}^0=[(\epsilon_{\mathbf{k}}^0)^2+2\bar{n}U_0\epsilon_{\mathbf{k}}^0]^{1/2}$.

To the system described by Eq. (3) we introduce a periodic modulation of the lattice depth V_0 of the form $V_0(t)=V_0(1+A\sin\Omega t)$, with $A\ll 1$. Thus, J_0 and U_0 become time dependent, and to lowest order in A we have $\epsilon_{\mathbf{k}}(t)=\epsilon_{\mathbf{k}}^0(1+B\sin\Omega t)$ and $U(t)=U_0(1+C\sin\Omega t)$, where B and C are calculated from the relations below Eq. (1). The resulting Hamiltonian is

$$H=\sum_{\mathbf{k}\neq 0}\left[\frac{E_{\mathbf{k}}(t)}{2}(c_{\mathbf{k}}^\dagger c_{\mathbf{k}}+c_{-\mathbf{k}}^\dagger c_{-\mathbf{k}})+\frac{V_{\mathbf{k}}(t)}{2}(c_{\mathbf{k}}^\dagger c_{-\mathbf{k}}^\dagger+\text{H.c.})\right], \quad (4)$$

where

$$V_{\mathbf{k}}(t)=\frac{\epsilon_{\mathbf{k}}^0}{E_{\mathbf{k}}^0}\bar{n}U_0(C-B)\sin\Omega t,$$

$$E_{\mathbf{k}}(t)=E_{\mathbf{k}}^0+\Delta E_{\mathbf{k}}(t), \quad (5)$$

$$\Delta E_{\mathbf{k}}(t)=\frac{\epsilon_{\mathbf{k}}^0}{E_{\mathbf{k}}^0}[B(\epsilon_{\mathbf{k}}^0+\bar{n}U_0)+C\bar{n}U_0]\sin\Omega t.$$

The Hamiltonian (4) is no longer diagonal, and a time-dependent term creating pairs of excitations with opposite momenta is introduced, responsible for the parametric resonance. Following the method of Tozzo *et al.*, we introduce the operators $\alpha_{\mathbf{k}}\equiv c_{\mathbf{k}}\exp[i\int dt' E_{\mathbf{k}}(t')]$ and the Heisenberg equations on Eq. (4) yield (we set $\hbar=1$)

$$\dot{\alpha}_{\mathbf{k}}=\gamma_{\max}\alpha_{-\mathbf{k}}^\dagger[e^{-i(\Omega t-2\int E_{\mathbf{k}}dt)}-e^{i(\Omega t+2\int E_{\mathbf{k}}dt)}],$$

$$\dot{\alpha}_{-\mathbf{k}}^\dagger=\gamma_{\max}\alpha_{\mathbf{k}}[e^{i(\Omega t-2\int E_{\mathbf{k}}dt)}-e^{-i(\Omega t+2\int E_{\mathbf{k}}dt)}], \quad (6)$$

where $\gamma_{\max}=(C-B)\epsilon_{\mathbf{k}}^0\bar{n}U_0/2E_{\mathbf{k}}^0$. To lowest order in A we may replace $E_{\mathbf{k}}$ by its time independent part $E_{\mathbf{k}}^0$.

When the arguments of the exponents get large, the fast oscillations effectively kill the time dependence of $\alpha_{\mathbf{k}}$. In the regime $\Omega t\gg 1$ this is almost always the case, except on the slowly oscillating modes with $|\Omega-2E_{\mathbf{k}}^0|t\ll 1$ which fulfill the single second order equation

$$\ddot{\alpha}_{\mathbf{k}}+i(\Omega-2E_{\mathbf{k}}^0)\dot{\alpha}_{\mathbf{k}}-\gamma_{\max}^2\alpha_{\mathbf{k}}=0 \quad (7)$$

with solution $\alpha_{\mathbf{k}}=\eta_{\mathbf{k}}^+e^{i\omega_+t}+\eta_{\mathbf{k}}^-e^{i\omega_-t}$, where

$$\omega_\pm=-\frac{1}{2}[(\Omega-2E_{\mathbf{k}}^0)\pm\sqrt{(\Omega-2E_{\mathbf{k}}^0)^2-4\gamma_{\max}^2}]. \quad (8)$$

Here $\eta_{\mathbf{k}}^\pm$ are set by the initial conditions $\alpha_{\mathbf{k}}(t=0)=c_{\mathbf{k}}(t=0)$ and $\dot{\alpha}_{\mathbf{k}}(t=0)=\gamma_{\max}c_{-\mathbf{k}}^\dagger(t=0)$ and result in coherent superpositions of $c_{\mathbf{k}}(t=0)$ and $c_{-\mathbf{k}}^\dagger(t=0)$. In a narrow window of energies $|\Omega/2-E_{\mathbf{k}}^0|<|\gamma_{\max}|$ the frequencies $\omega_\pm$ gain an imaginary part and the occupation of the $\eta_{\mathbf{k}}^+$ modes grows exponentially with time (resonant modes), corresponding to the resonant excitation of pairs of Bogoliubov modes with opposite momenta, whose total energy is approximately equal to the external modulation frequency. Outside this window, the time evolution of the occupation is periodic. The occupation of the Bogoliubov modes is

$$\langle c_k^\dagger(t)c_k(t)\rangle=\begin{cases}\left(\dfrac{\gamma_{\max}}{\gamma}\right)^2\sinh^2(\gamma t)[2n_k(T)+1]+n_k(T), & 4\gamma_{\max}^2>(\Omega-2E_k^0)^2\\ \left(\dfrac{\gamma_{\max}}{\gamma}\right)^2\sin^2(\gamma t)[2n_k(T)+1]+n_k(T), & 4\gamma_{\max}^2<(\Omega-2E_k^0)^2,\end{cases} \quad (9)$$

where $\gamma\equiv[|\gamma_{\max}^2-\frac{1}{4}(\Omega-2E_{\mathbf{k}}^0)^2|]^{1/2}$ and $n_{\mathbf{k}}(T)=(e^{E_{\mathbf{k}}^0/k_BT}-1)^{-1}$ is the Bose distribution. This result is correct as long as the condensate depletion is small such that Eqs. (2) and (4) hold. The exponential growth of the resonant-modes occupation is compatible with the result of Tozzo *et al.* [18].

In order to calculate the energy absorbed by the system due to the time dependent lattice depth, we consider a modulation that is turned on at $t=0$ and switched off at $t=\tau$, bringing back the Hamiltonian to Eq. (3). The energy absorbed by the system during the time $0<t<\tau$ is

$$\mathcal{E}(\tau)=\sum_{\mathbf{k}\neq 0} E_{\mathbf{k}}[\langle c_{\mathbf{k}}^{\dagger}(\tau)c_{\mathbf{k}}(\tau)\rangle-\langle c_{\mathbf{k}}^{\dagger}(0)c_{\mathbf{k}}(0)\rangle], \qquad (10)$$

where we perform a thermal average with the unperturbed Hamiltonian H_0. The absorption is then obtained by inserting Eq. (9) into Eq. (10), with the summation over $\mathbf{k}$ limited to the range of energies where oscillations are slow.

The absorption is strongly affected by the density of states (DOS) of the system, which is crucially dependent on the shell structure. Indeed, if the width of the ring L is much smaller than its length L_x, transverse quantization of the 2D momenta yields a spectrum composed of separate 1D branches. The mth branch has a dispersion

$$E_{k,m}^{0}\simeq c\left[k^2+\left(\frac{\pi m}{L}\right)^2\right]^{1/2}, \qquad (11)$$

where k is the 1D wave number along the ring and we considered the low-energy phononic dispersion with sound velocity $c=(2\bar{n}U_0J_0l^2)^{1/2}$. The DOS is characterized by van Hove singularities near $k=0$ of the form $\nu(E)\sim E/[E^2-E_{g,m}^2]^{1/2}$, where $E_{g,m}=E_{k=0,m}^{0}$. In addition, in the presence of the SF rings, 1D surface modes exist at the interfaces between domains, with a linear dispersion at low momenta [12]. Their DOS is, however, smooth and shows no singularity, which makes their detection difficult with the technique proposed here.

We now examine the dependence of the energy absorption (10) on the modulation frequency Ω. The typical parameters used in recent experiments [15] are $\tau\sim 30$ ms and $\Omega\leqslant 8$ kHz such that $\Omega\tau\gg 1$ and the occupation of the modes is determined by Eq. (7). At a given Ω, modes with energies in a "resonant window" of width $2\gamma_{\max}$ around $\Omega/2$ are exponentially populated in time. Thus, by varying Ω and considering the energy absorbed by the system we can get information about its spectrum. For example, in the presence of SF rings, changing Ω in the vicinity of $2E_{g,1}$ the resonant window moves across the van Hove singularity and the absorption shows a peak. The absorption is then suppressed as Ω is further increased past the singularity. A similar enhancement is predicted every time the modulation frequency meets the condition $2(E_{g,m}-|\gamma_{\max}|)<\Omega<2(E_{g,m}+|\gamma_{\max}|)$ for some m. Figure 1 shows the energy absorbed by the system versus Ω for a modulation of amplitude $A=0.005$ in the lattice intensity V_0. For a typical $J_0\approx U_0/50\approx 0.15$ KHz (characteristic to the $\bar{n}=2$ MI lobe) this corresponds to the amplitudes $B=0.015$ in the hopping strength and $C\sim 0.005$ in the on-site repulsion.

The plot in Fig. 1 is the main result of this paper; a measurement of the energy absorption as function of the modulation frequency supplies a direct evidence for the existence of narrow SF shells, as can be seen from the difference between the dashed and the solid lines. We point out that the enhanced absorption shown in the latter is a result of the divergence in the DOS together with the fact that the exponent $\gamma_{\max}$ does not vanish at $k=0$. In the low-energy regime $2\bar{n}U_0\gg\epsilon_{k,m}^{0}$ we get $\gamma_{\max}\simeq(C-B)E_{k,m}^{0}/4$ which is finite for $k=0$ if $m\neq 0$.

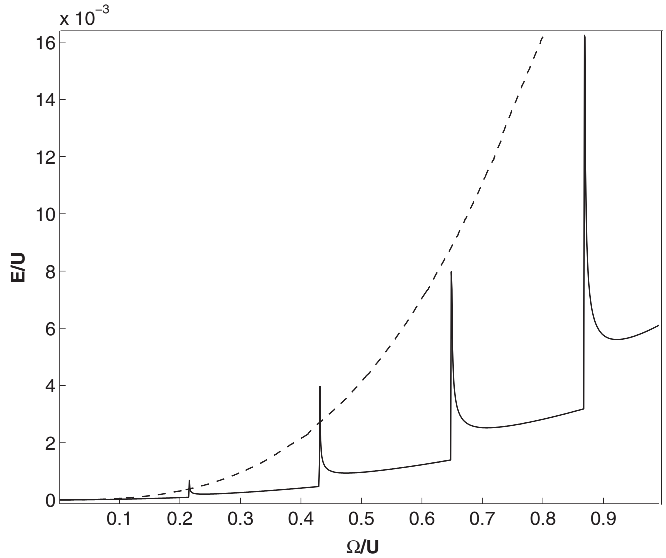

FIG. 1. The total energy absorption after a modulation of the lattice intensity $V_0\simeq 14E_R$ with $A=0.005$ during 10 ms, plotted as function of the modulation frequency Ω/U_0 for sodium atoms in the mixed MI-SF regime at $U_0/J_0=45$ (solid line) for the inner ring, of average density $\bar{n}=1.5$ particles per site, and for the 2D SF phase at $U_0/J_0=20$ (dashed line) and average density of $\bar{n}=1.5$. We considered $l=257$ nm and $L=10l$.

The energy difference between the transverse branches resulting from one SF ring, $\Delta E=E_{g,m}-E_{g,m-1}\simeq\pi c/L$ is inversely proportional to its width L. Therefore the spacing between adjacent absorption peaks in Fig. 1 is a signature of the ringlike SF domains, and their typical width may be estimated from it, given J_0, U_0, and l.

This result should be compared with the energy absorption in the absence of SF rings. In case J_0/U_0 is very small, the SF rings get thinner than one lattice spacing, and the SF domains effectively disappear. Then, only modes above the MI gap can be excited yielding no signature in low energy absorption. Otherwise, when J_0/U_0 is larger than the critical ratio for the MI-SF transition, the entire 2D system is SF with a smooth DOS. The dashed line in Fig. 1 shows the energy absorption as function of the modulation frequency in this regime.

For a three-dimensional cloud of atoms the arguments above hold in essentially the same form. The presence of SF shells with thickness L yields 2D Bogoliubov branches with interband separation $\propto 1/L$. The two-dimensional nature of the modes, however, does not lead to van Hove singularities but rather to a staircase profile in the DOS. The absorption vs Ω will not show peaks, as in Fig. 1, but steps, from the existence and spacing of which one can infer the presence and size of domains. In contrast, a pure 3D SF phase would show a smooth absorption. In a one-dimensional system the Mott insulating and superfluid regions are expected to be pointlike, and therefore cannot be observed in parametric excitation experiments such as those performed in Refs. [15,16]. A two-dimensional system is therefore optimal for observing the signature of the shells by parametric excitation.

We stress that the sharp peaks in the energy absorption survive also in the presence of additional low energy excitations with no singularities in the DOS. As a matter of fact, in Fig. 1 the absorption in the regime where MI and SF domains coexist has been calculated including Bogoliubov branches as well as surface modes at the MI-SF interfaces [12]. After 30 ms (a typical modulation duration in experiments [15]) the ratio between the absorption due to the Bogoliubov branch and the one due to the surface mode at energy $E_{g,1}$ is around 50. This large ratio, owing to the smooth DOS of surface modes highlights the importance of the transverse quantization of modes in the bulk 2D SF.

Our result is also insensitive to small modifications in the boundary conditions, such as a smooth boundary between the MI and SF shells. Numerical analysis of confined atoms in optical lattices [6] showed a smooth crossover of the local compressibility across the interface between MI and SF shells. Such a smooth boundary, of the order of one to three lattice sites, does not affect the existence of a gapless mode in the superfluid but only the precision of the transverse width L as estimated from the peak separation in the absorption. As long as the crossover region is small compared with the SF shell width L, an estimate for L can be inferred from the plot of Fig. 1. Based on Ref. [6] the error we anticipate is at most of 10%.

We now analyze the applicability of our results to experiments, choosing parameters close to those in Ref. [5]. For sodium atoms in an optical lattice of wavelength $2l = 514$ nm we get $E_R = 2\pi\hbar \times 32$ kHz. For the generation of two MI rings we need $V_0 \simeq 14E_R$, yielding $U_0 \simeq 0.24E_R$ and $J_0 \simeq 0.0046E_R$. The single band BH approximation holds, since $\frac{1}{2}U_0\bar{n}(\bar{n}-1) \ll \hbar\nu$, where $\hbar\nu = \sqrt{4E_R V_0}$ is the gap to the first excited Bloch band. A typical ring of width $L \simeq 10l$ and $\bar{n} = 1.5$ yields a transverse gap of $E_{g,1} \simeq 0.1U_0$, allowing one to probe about ten branches below the MI gap U_0. In order to obtain a good visibility of the parametric resonance peaks we need the timescale for exponential amplification of the first subband (i.e., $1/2\gamma_{\max}$ at energy $E_{g,1}$) to be shorter than the typical duration of the lattice modulations (about 30 ms, see Ref. [15]). This can be achieved by choosing $A \geqslant 0.005$, still significantly smaller than modulation amplitudes used in experiments. We point out that due to the energy scaling of $\gamma_{\max}$, the higher subbands with $m > 1$ are exponentially enhanced with an even shorter time scale.

The enhancement in the population of modes at zero temperature is triggered by the presence of the zero point fluctuations, or "quantum seeds," augmented by thermal fluctuations at finite temperature. A second important fact enabling the amplification is the bosonic character of the atoms. In the "fermionic version" of Hamiltonian (2), i.e., the BCS mean-field one, the periodic modulation yields no exponential growth of the occupation, due to the fermionic nature of the excitations.

The sharp peaks in Fig. 1 result from the confinement of the SF to narrow stripes and from the long-living modes that characterize superfluids. To separate the contribution of these two sources, we consider the effect of viscosity on the parametric resonances in a fluid stripe. In the absence of parametric modulations the hydrodynamic equation of motion for sound waves in a viscous fluid is of the form [25]

$$\ddot{\rho}_k + \eta k^2 \dot{\rho}_k + \omega_k^2 \rho_k = 0, \tag{12}$$

where $\omega_k = c_s k$ and η is the viscosity coefficient. Equation (12) can be derived using the Euler-Lagrange equation [26]

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{\rho}_k} = \frac{\partial \mathcal{L}}{\partial \rho_k} - \frac{\partial F}{\partial \dot{\rho}_k} \tag{13}$$

with the Lagrangian

$$\mathcal{L} = \sum_k \left(\frac{1}{2} M \dot{\rho}_k^2 - \frac{1}{2} U k^2 \rho_k^2 \right) \tag{14}$$

and the dissipative function

$$F = \frac{1}{2}\eta M \sum_k k^2 \dot{\rho}_k^2. \tag{15}$$

Here M is the effective mass and U the interaction constant, yielding the sound velocity $c_s = \sqrt{U/M}$. The introduction of the parametric modulations of the effective mass $M(t) = M_0(1 + B \sin \Omega t)$ and of the interaction constant $U(t) = U_0(1 + C \sin \Omega t)$ into the Lagrangian (14) yields the equation of motion

$$\ddot{\rho}_k + \zeta_k(t)\dot{\rho}_k + \omega_k^2(t)\rho_k = 0, \tag{16}$$

where $\omega_k(t) = c_s(t)k$, $\zeta_k(t) \equiv \dot{M}(t)/M(t) + \eta k^2$, and $c_s(t) = \sqrt{U(t)/M(t)}$. The transformation $\rho_k(t) = y_k(t) \times \exp[-\int_0^t \zeta_k(t')/2dt']$ maps Eq. (16) onto a parametrically excited oscillator

$$\ddot{y}_k + \tilde{\omega}_{k,0}^2[1 + \tilde{A} \sin \Omega(t - t_0)]y_k = 0, \tag{17}$$

where $\tilde{A} = \{[C - (1 - \Omega^2/2\omega_{k,0}^2)B]^2 + (\eta k^2 \Omega B/2\omega_{k,0}^2)^2\}^{1/2}$ and $\tilde{\omega}_{k,0}^2 = \omega_{k,0}^2 - (\eta k^2/2)^2$ to lowest order in B and C. The resonant y_k's are exponentially enhanced, $y_k(t) \sim \exp(\tilde{\gamma}_{\max} t)$ with $\tilde{\gamma}_{\max} \simeq \frac{1}{4}\tilde{A}\tilde{\omega}_{k,0}$. The resulting dynamics of the sound modes has the form $\rho_k \sim \exp[(\tilde{\gamma}_{\max} - \eta k^2/2)t]\exp(-B \sin \Omega t/2)$. Density fluctuations will grow exponentially in time only if the modulation amplitude is such that $\tilde{\gamma}_{\max}$ is real, with $\tilde{\gamma}_{\max} > \eta k^2/2$, and the width of the peaks is $2\tilde{\gamma}_{\max}$. Thus, when the modulation is large enough for the formation of peaks, these acquire a finite width, which for the m'th peak is larger than $\eta(\pi m)^2/2L^2$. When the viscosity is large enough, the peak width is larger than their spacings and the peaks are fully smeared.

In conclusion, we considered an ultracold cloud of Bose atoms in a periodically modulated optical lattice, inducing parametric amplification of its excitations. The energy absorption vs modulation frequency may serve as a tool to detect features in the DOS of the system, that are induced by a transition from a uniform SF phase for weak lattice intensities to a mixed MI-SF regime for stronger ones. This enables the experimental detection of the coexistence of MI-SF domains and the measurement of their typical size. In addition, we demonstrated the broadening of absorption peaks due to damping in normal fluids which is absent in case the domains are superfluids.

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