

Langevin dynamics of a heavy particle and orthogonality effectsMark Thomas,¹ Torsten Karzig,² and Silvia Viola Kusminskiy¹¹*Dahlem Center for Complex Quantum Systems and Fachbereich Physik, Freie Universität Berlin, 14195 Berlin, Germany*²*Department of Physics, California Institute of Technology, Pasadena, California 91125, USA*

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The dynamics of a classical heavy particle moving in a quantum environment is determined by a Langevin equation which encapsulates the effect of the environment-induced reaction forces on the particle. For an open quantum system, these include a Born-Oppenheimer force, a dissipative force, and a stochastic force due to shot and thermal noise. Recently, it was shown that these forces can be expressed in terms of the scattering matrix of the system by considering the classical heavy particle as a time-dependent scattering center, allowing to demonstrate interesting features of these forces when the system is driven out of equilibrium. At the same time, it is well known that small changes in a scattering potential can have a profound impact on a fermionic system due to the Anderson orthogonality catastrophe. In this work, by calculating the Loschmidt echo, we relate Anderson orthogonality effects with the mesoscopic reaction forces for an environment that can be taken out of equilibrium. In particular, we show how the decay of the Loschmidt echo is characterized by fluctuations and dissipation in the system and discuss different quench protocols.

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I. INTRODUCTION

Understanding the effect of fluctuations and dissipation in nonequilibrium settings is essential for an ultimate control of quantum systems. Dissipation is, on one hand, unavoidable in realistic systems, and known to play an important role in their dynamics (a paradigm is the exponential suppression of quantum tunneling out of a metastable state as modeled by Caldeira and Leggett [1,2]) while nonequilibrium can provide new levels of tunability. This is a topic of renewed interest in view of current experiments which explore the possibility of quantum information processing, by embedding a qubit degree of freedom in a mesoscopic system [3,4]. The coupling of the qubit to an environment causes decoherence and consequently loss of information, which is closely related to the fluctuations and dissipation in the system [5].

In this context, the quantum Loschmidt echo, also known as *fidelity*, is a useful quantity that indicates the sensitivity of the system to small perturbations [6–8]. In its generalization to many-body systems [9], the Loschmidt echo corresponds to the off-diagonal element (norm-squared) of the reduced density matrix for the qubit degree of freedom, and its decay in time characterizes the environment-induced decoherence [10]. For a fermionic environment, this decay is directly related to the Anderson orthogonality catastrophe, which describes the response of the fermionic system to a sudden perturbation [11]. In his seminal work [12], Anderson showed that the many-body ground state of a fermionic system is, in the thermodynamic limit, orthogonal to that of the same system in which a local scattering potential is introduced. More precisely, the overlap of the two states decays as a power law with the system size, with an orthogonality exponent characterized by the scattering phase shift produced by the scattering potential. The orthogonality catastrophe plays an essential role in describing the so-called “impurity problems” in which a local degree of freedom interacts with a fermionic environment [13–19].

Solid-state systems as well as cold-atom systems provide experimental realizations of impurity problems. Aside

from the above-mentioned experiments embedding qubits in mesoscopic systems, devices consisting of noninteracting quantum dots connected to electronic leads, where parameters of the system can be tuned to generate local time-dependent potentials (used, for example, to pump charge through the system [20]) can be thought as an impurity scattering potential acting on a fermionic system. In turn, the high control that has been achieved over cold-atomic systems [21] allows to design and manipulate atomic mixtures, that can serve to model time-dependent impurities in a fermionic environment and even simulate the presence of a bias [22]. Here, the Anderson orthogonality exponent and the Loschmidt echo are key quantities which reveal intrinsic time scales and dynamical properties of the system and measure the response of the environment after a change in the system’s variables. The Loschmidt echo was measured early on in the form of spin echo in nuclear magnetic resonance experiments, where a time-reversal protocol by radio-frequency pulses was implemented and the decay of the echo used as a measure of decoherence [23]. Nowadays, variations of this technique are used to measure some incarnation of the Loschmidt echo in both solid-state [24] and cold-atomic systems [25]. Meanwhile, in quantum transport experiments the orthogonality catastrophe manifests itself in the tunneling density of states, typically as power-law singularities at the Fermi level [26].

In this paper, we consider a case of an impurity problem which consists of a “heavy particle” embedded in a quantum environment [27]. The impurity in this case is heavy compared to those particles comprising the environment, and can be treated as a classical degree of freedom with semiclassical dynamics dictated by the back-action of the environment. In a concrete experimental setup of current relevance, the heavy particle can represent the classical vibrational degrees of freedom of a molecule or suspended carbon nanotube connected to conducting leads [28,29]. The dynamics of the heavy particle can be described in terms of a Langevin equation, which is a stochastic equation of motion that describes at an effective, macroscopic level, the effects of dissipation and fluctuations induced by the environment on the heavy particle. An

interesting question is how the dynamics of the heavy particle and orthogonality effects of its environment are related. For a quantum environment in equilibrium, the Anderson orthogonality catastrophe exponent was conjectured by Sols and Guinea [30] to be proportional to the dissipation coefficient a heavy particle experiences when moving in a metallic environment. This relation was later proved to be valid, in the small-distance limit,¹ for a heavy particle moving in a quantum environment at zero temperature [31]. For nonequilibrium fermionic systems, this problem has been studied in the context of some concrete models [16,17,19,32,33], while the complementary problem of how decoherence in the environment affects the dissipation coefficient has been studied recently [34].

Motivated by these findings, in this work we calculate the Loschmidt echo for small changes of a scattering potential, in a fermionic open system which is taken out of equilibrium by imposing a voltage bias. With the aim of exploring the relation between the orthogonality exponent and the dissipation coefficient in this case, we express the decay in time of the Loschmidt echo in terms of the coefficients of the corresponding Langevin equation, in particular, in terms of the dissipation and noise coefficients. To this effect we make use of the recent developed formalism that describes the effective forces in the Langevin equation in terms of scattering theory [35,36]. Our results apply generally to systems for which changes in the scattering potential can be treated perturbatively. The paper is organized as follows. We start in Sec. II by presenting the Langevin equation in terms of current-induced forces, and the associated force-force noise correlator. In Sec. III, we perform a perturbative expansion of the Loschmidt echo and show that it can be expressed in terms of the noise correlator, and discuss different quench protocols. In Sec. IV, we make use of the results of Secs. II and III to show that, in equilibrium and for zero temperature, the decay of the Loschmidt echo is a power law with an exponent dictated by the dissipation coefficient a heavy particle experiences in the fermionic environment, in agreement with the known orthogonality results. Finite temperatures, however, render the decay exponential. In Sec. V, we turn to the nonequilibrium case for which we calculate the decay of the Loschmidt echo within linear response in the applied bias. In this case we show that the decay of the Loschmidt echo cannot be expressed solely in terms of the dissipation coefficient, providing a general expression for the decay in terms of the macroscopic Langevin parameters. We then discuss different time scales for which the results can be cast in a simple form. In Sec. VI, we apply our results to a simple example and check the limits of validity of our approximations, while we list our main conclusions in Sec. VII. Quite a few calculations in this work are rather lengthy. To improve readability, and at the same time to make this paper self-contained, we have included some details of these calculations in the appendixes.

II. LANGEVIN EQUATION AND NOISE CORRELATOR

In this section we briefly review the elements of the Langevin equation that governs the stochastic dynamics of

¹The small-distance limit corresponds to small variations in the classical coordinates.

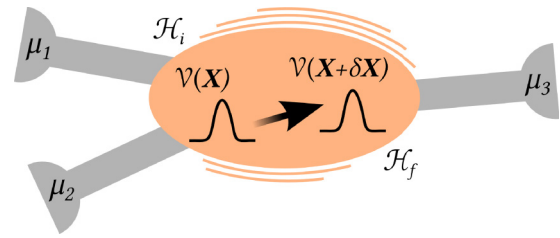


FIG. 1. (Color online) In the scattering region, electrons couple \mathbf{X} via the scattering potential $\mathcal{V}_{\mathbf{X}}$. When the scatterer moves by $\delta\mathbf{X}$, the scattering potential changes accordingly, i.e., $\mathcal{V}_{\mathbf{X}+\delta\mathbf{X}}$. This gives rise to the two different Hamiltonians $\mathcal{H}_i = \mathcal{H}_0 + \mathcal{V}_{\mathbf{X}}$ and $\mathcal{H}_f = \mathcal{H}_0 + \mathcal{V}_{\mathbf{X}+\delta\mathbf{X}}$. The scattering region is finite and the dwell time τ_D gives the time scale the electrons spend within.

a heavy particle in an open quantum environment. Throughout this paper we will consider a fermionic quantum environment that can be taken out of equilibrium by a difference of chemical potential in the leads as illustrated in Fig. 1. The heavy particle is represented through classical degrees of freedom which are coupled to the quantum environment, and disturb it as they evolve in time. The back-action of this disturbance onto the heavy particle gives rise to reaction forces [37], also called current-induced forces in a quantum transport setup. In the adiabatic limit, for which the dynamics of the heavy particle is much slower than that of the quantum environment, this effect is well described semiclassically at the level of a Langevin equation obtained by tracing out the quantum environment. If we denote the degrees of freedom of the heavy particle by $\mathbf{X}(t)$, the Langevin equation reads as (in what follows, we omit the time dependence for notational simplicity)

$$\dot{P}_{\alpha} - F_{\alpha}^{\text{cl}}(\mathbf{X}) = F_{\alpha}(\mathbf{X}) - \sum_{\beta} \Gamma_{\alpha\beta}(\mathbf{X}) \dot{X}_{\beta} + \xi_{\alpha}(\mathbf{X}). \quad (1)$$

On the left-hand side, P_{α} denotes the canonical momentum of coordinate X_{α} ($\alpha = 1, \dots, N$), and we have included the possibility of an external classical force $\mathbf{F}^{\text{cl}}(\mathbf{X})$ (throughout the text we will indicate matrices and vectors in the space spanned by the X_{α} with bold letters). The right-hand side of Eq. (1) contains the forces due to the quantum environment. $\mathbf{F}(\mathbf{X})$ is the usual Born-Oppenheimer force, while the symmetric and antisymmetric parts of the tensor $\Gamma(\mathbf{X})$ represent a dissipative and Lorentz-type (and therefore nondissipative) force, respectively. Fluctuations due to shot and thermal noise are taken into account by the stochastic Langevin force $\xi(\mathbf{X})$.

When the fermionic system is taken out of equilibrium, the current-induced forces present qualitative differences with respect to the equilibrium situation [35,38–40]. The Born-Oppenheimer force \mathbf{F} is nonconservative in this case, and therefore provides a way of exchanging energy between the classical field and the quantum environment which is nondissipative. The tensor Γ constitutes the first-order correction in an adiabatic expansion to the Born-Oppenheimer force. It can be split into symmetric and antisymmetric components. The antisymmetric component is a Lorentz-type term, which can be interpreted as an effective magnetic field acting on the space spanned by \mathbf{X} ; this term is not relevant for the Loschmidt echo (which involves only symmetric components as we will see in the following) and hence will be not dealt further

with within this work. We denote the symmetric dissipative term of Γ by $\boldsymbol{\gamma}$. It is convenient to express the latter as $\boldsymbol{\gamma} = \boldsymbol{\gamma}^{\text{eq}} + \boldsymbol{\gamma}^{\text{neq}}$, where $\boldsymbol{\gamma}^{\text{neq}}$ represents a pure nonequilibrium contribution while $\boldsymbol{\gamma}^{\text{eq}}$ is a straightforward generalization of the equilibrium contribution evaluated in a nonequilibrium environment. Explicit expressions for these quantities are given in Appendix A. According to our definition, $\boldsymbol{\gamma}^{\text{eq}}$ connects to the equilibrium results, but it also contains nonequilibrium terms for finite bias. The pure nonequilibrium term $\boldsymbol{\gamma}^{\text{neq}}$ can take negative values and, moreover, render the full dissipative term negative.

Of particular importance for the following discussion is the stochastic component $\boldsymbol{\xi}$, characterized by the force-force noise correlator. We will see in the next section that the Loschmidt echo can be, perturbatively for small displacements $\delta\mathbf{X}$, written in terms of the noise correlator

$$D_{\alpha\beta}(t, t') = \{ \langle \hat{\xi}_\alpha(t) \hat{\xi}_\beta(t') \rangle \}_s, \quad (2)$$

where $\{M_{\alpha\beta}\}_s = (M_{\alpha\beta} + M_{\beta\alpha})/2$ indicates the symmetric component of a generic matrix \mathbf{M} . To give a concrete expression for this noise correlator we consider a generic, albeit noninteracting, many-body Hamiltonian which depends parametrically on time *via* the potential $\mathcal{V}_\mathbf{X}$, $\mathcal{H}_\mathbf{X} = \mathcal{H}_0 + \mathcal{V}_\mathbf{X}$. This potential represents the coupling between the heavy particle and the fermionic environment. The current-induced force operator is given by

$$\mathcal{F}(t) = -\nabla_{\mathbf{X}} \mathcal{H}_\mathbf{X}(t). \quad (3)$$

The Langevin Eq. (1) is obtained by calculating the quantum-statistical average $\langle \mathcal{F}(t) \rangle$ within an adiabatic expansion to linear order in the velocity $\dot{\mathbf{X}}$, together with the quantum and thermal fluctuations given by

$$\hat{\xi}(t) = \mathcal{F}(t) - \langle \mathcal{F}(t) \rangle. \quad (4)$$

The coefficients of this expansion are instantaneous: the noise is assumed to be delta correlated $\mathbf{D}(t, t') \rightarrow \mathbf{D}(\mathbf{X}) \delta(t - t')$, and there is no retardation kernel for the dissipative term $\boldsymbol{\gamma}(\mathbf{X})$. These are the zero-frequency limit, respectively, of the force-force correlator (2) and force susceptibility

$$\chi_{\alpha\beta}^{FF}(t, t') = -i \theta(t - t') \langle [\mathcal{F}_\alpha(t), \mathcal{F}_\beta(t')] \rangle, \quad (5)$$

where $\theta(t)$ is the usual step function. In equilibrium and assuming steady state, so the time dependence in the relevant quantities is through the time difference $(t - t')$, we have

$$\gamma_{\alpha\beta}^{\text{eq}}(\omega) = -\frac{\text{Im} \chi_{\alpha\beta}^{FF}(\omega)}{\omega}, \quad (6)$$

where $\gamma_{\alpha\beta}^{\text{eq}}(\omega)$ denotes the Fourier transform of the friction kernel $\gamma_{\alpha\beta}^{\text{eq}}(t - t')$.² The correlator of the fluctuating Langevin force and the friction tensor are related in equilibrium via the

²We note that we assume the initial Hamiltonian $\mathcal{H}_\mathbf{X} = \mathcal{H}_0 + \mathcal{V}_\mathbf{X}$ to be time independent. The Green's functions depend therefore only on the difference of the time arguments. Throughout the paper, we define the Fourier transform of a function $f(t)$ as $f(\omega) = \int_{-\infty}^{\infty} dt e^{i\omega t} f(t)$ with its inverse $f(t) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} e^{-i\omega t} f(\omega)$.

finite-frequency fluctuation-dissipation theorem [41]

$$\mathbf{D}(\omega) = \omega \coth\left(\frac{\omega}{2T}\right) \boldsymbol{\gamma}^{\text{eq}}(\omega) \quad (7)$$

(we take the Boltzmann constant $k_B = 1$), where $\mathbf{D}(\omega)$ is the real part of the Fourier transform of the fluctuating force correlator in Eq. (2). In the limit $\omega \ll T$, Eq. (7) reduces to the classical identity

$$\mathbf{D}_{\text{cl}} = 2T \boldsymbol{\gamma}^{\text{eq}}, \quad (8)$$

where $\boldsymbol{\gamma}^{\text{eq}}$ is evaluated at zero frequency, while for $\omega \gg T$

$$\mathbf{D}_q(\omega) = |\omega| \boldsymbol{\gamma}^{\text{eq}}(\omega). \quad (9)$$

Out of equilibrium the fluctuation-dissipation relation (7) does not hold. However, within linear response in the applied bias $\Delta\mu$ (we consider two leads and without loss of generality $\Delta\mu > 0$), we can write an expression relating the noise correlator and the dissipative matrix $\boldsymbol{\gamma}$ that generalizes Eq. (7) in the limit of low frequencies (as compared to the characteristic energy scales of the quantum environment, this statement will be made more precise in the following sections). We state here the result which will be proven later in the text:

$$\begin{aligned} \mathbf{D}(\omega) = & \omega \coth\left(\frac{\omega}{2T}\right) \left(\boldsymbol{\gamma}^{\text{eq}} - \frac{\mathbf{D}_{[0, \Delta\mu]}}{\Delta\mu} \right) \\ & + \left[\frac{\omega_+}{2} \coth\left(\frac{\omega_+}{2T}\right) + \frac{\omega_-}{2} \coth\left(\frac{\omega_-}{2T}\right) \right] \frac{\mathbf{D}_{[0, \Delta\mu]}}{\Delta\mu} \\ & + \frac{\omega}{2} \left[\omega_+ \coth\left(\frac{\omega_+}{2T}\right) - \omega_- \coth\left(\frac{\omega_-}{2T}\right) \right] \frac{\boldsymbol{\gamma}^{\text{neq}}}{\Delta\mu}, \end{aligned} \quad (10)$$

where we defined $\omega_\pm = \omega \pm \Delta\mu$, and all remaining quantities are evaluated at zero frequency and to linear order in the applied bias. In general, the noise correlator depends on both temperature and bias, which at zero frequency we denote by $\mathbf{D}_{[T, \Delta\mu]}$. It is easy to see that for $\Delta\mu = 0$, Eq. (10) reduces to the equilibrium identity (7), and in particular $\mathbf{D}_{[T, 0]} = \mathbf{D}_{\text{cl}}$. For zero temperature but finite bias accordingly we obtain

$$\mathbf{D}(\omega) = \begin{cases} \mathbf{D}_{[0, \Delta\mu]} + |\omega| \left(\boldsymbol{\gamma}^{\text{eq}} - \frac{\mathbf{D}_{[0, \Delta\mu]}}{\Delta\mu} \right), & |\omega| \ll \Delta\mu \\ |\omega| \boldsymbol{\gamma}^{\text{eq}}, & |\omega| \gg \Delta\mu. \end{cases} \quad (11)$$

The explicit expression for $\mathbf{D}_{[0, \Delta\mu]}$ is given later in the text in Eq. (47).³ One should note the similarities and subtle differences between the finite-temperature expressions (8), (9), and finite bias (11) in the respective limits of high and low bias and temperatures when identifying $\Delta\mu \leftrightarrow 2T$. All these expressions are evaluated to first order in ω . From this we note that the first correction to Eq. (8) is of order $O(\omega^2)$. That the linear term in ω vanishes in equilibrium is easy to see by looking at the corresponding finite-bias result and identifying $\mathbf{D}_{[0, \Delta\mu]} \Delta\mu \leftrightarrow \mathbf{D}_{[T, 0]}/2T = \boldsymbol{\gamma}^{\text{eq}}$ in the first line of Eq. (11).

In the next section, we show that the Loschmidt echo, within a perturbative expansion in the change of the scattering potential, is directly related to the noise correlator of Eq. (2).

³Note that the quantities \mathbf{D} and $\boldsymbol{\gamma}$ depend on \mathbf{X} . In order to simplify the notation, we do not write this dependence explicitly.

III. LOSCHMIDT ECHO: GENERAL RESULTS FOR SMALL DISPLACEMENTS

The Loschmidt echo in quantum systems is given by the (squared) overlap of eigenstates of an initial system which evolved in time with two different many-body Hamiltonians. Alternatively, it can be seen as measure of how close to a given initial state a system comes back to, when the evolution on the time-reversed path is determined by a different Hamiltonian from the forward evolution one. This can be generalized for initial states which are a quantum statistical mixture [9]. Denoting the Loschmidt echo by the function $\mathcal{L}(\tau)$, this is given by $\mathcal{L}(\tau) = |\mathcal{A}(\tau)|^2$, with

$$\mathcal{A}(\tau) = \langle e^{i\mathcal{H}_i\tau} e^{-i\mathcal{H}_f\tau} \rangle, \quad (12)$$

where $\langle \dots \rangle$ is the quantum-statistical average characterized by the initial Hamiltonian \mathcal{H}_i ($\hbar = 1$) and \mathcal{H}_f denotes the perturbed Hamiltonian. The overlap $\mathcal{A}(\tau)$ is denominated fidelity amplitude.

The relation between the Loschmidt echo and the orthogonality catastrophe is seen by treating Anderson's orthogonality as a dynamical process [14]. In the problem of x-ray absorption spectrum of a metal, the creation of a deep hole produces a "shakeup" of the Fermi sea that causes a suppression of Mahan's power-law divergence at threshold frequency (known as the Fermi edge or x-ray singularity) [13], with an exponent that can be identified directly with Anderson's orthogonality exponent [15]. This is captured by the hole propagator which can be calculated by evaluating the overlap of the fermionic ground state evolved with a Hamiltonian including the core hole, with that of the ground state evolved without the hole. This is therefore nothing else than the Loschmidt echo where the two Hamiltonians \mathcal{H}_i , \mathcal{H}_f correspond to considering the system with or without the potential of the core hole. Beyond the original problem of the Fermi edge singularity in metals, different problems in which some local, time-varying degree of freedom interacts with a fermionic environment, can be treated with the same methodology and hence some incarnation of the fidelity amplitude and Loschmidt echo appears naturally. Examples include the absorption spectrum of Luttinger liquids [42-44] and beyond [45], the single-channel Kondo problem [46] or time-dependent impurities in cold-atom systems [22].

In order to make the connection with the Langevin equation discussed above, in this section we obtain an expression for the Loschmidt echo *via* a perturbative expansion for small changes in the potential $\mathcal{V}_{\mathbf{X}}$, by considering $\mathcal{H}_i = \mathcal{H}_0 + \mathcal{V}_{\mathbf{X}}$ and $\mathcal{H}_f = \mathcal{H}_0 + \mathcal{V}_{\mathbf{X}+\delta\mathbf{X}}$ (cf. Fig. 1), with $\delta\mathcal{H}_{\mathbf{X}} = \mathcal{V}_{\mathbf{X}+\delta\mathbf{X}} - \mathcal{V}_{\mathbf{X}}$ small with respect to \mathcal{H}_i . An important factor to determine the time dependence of the Loschmidt echo is *how rapidly* the change in the coupling potential occurs. This rapidity is determined by what is called the "quench protocol." Here, we consider this is given externally by an arbitrary function $g(t)$ such that

$$\mathcal{H}(t) = \mathcal{H}_0 + \mathcal{V}_{\mathbf{X}} + g(t)\delta\mathcal{H}_{\mathbf{X}}. \quad (13)$$

Initially, $g(0) = 0$ so that we obtain \mathcal{H}_i . We impose the quench is completed at some time τ by setting $g(\tau) = 1$. We consider an open quantum system in which the electrons spend on average some finite time τ_D in the scattering region. τ_D is referred to as the *dwelt time* and we consider it to be

TABLE I. Protocol-dependent constants used throughout the text. Note that while α_A is universal and valid for any adiabatic quench, the rest of the "adiabatic" constants are valid exclusively for the linear quench; they are actually dependent on the adiabatic quench protocol.

Symbol	Sudden ($P = S$)	Adiabatic ($P = A$)
λ_P	1	2
α_P	2	1
β_P	1	$\frac{1}{3}$
δ_P	2	$\frac{1}{2}$

the smallest time scale in the system, in the spirit of the Born-Oppenheimer approximation. This defines the time scale for the quench. In particular, in this work we will study two complementary protocols: sudden and adiabatic quenches. In the *sudden quench*, the scattering potential is changed suddenly, which is realized by a step-function shape such that $g(t) = 1$ for $t > 0$. For the *adiabatic quench* instead, the potential is ramped up slowly, where slow refers to the dwell time. For the adiabatic quench, we choose a linear ramping protocol $g(t) = t/\tau$. We will show below that, in the limit $\tau \gg \tau_D$, our results in equilibrium are independent of this choice, while for an imposed bias our results are characterized by coefficients that depend on the specifics of the adiabatic protocol. In what follows, we will treat in parallel both above-mentioned protocols, and we list in Table I the protocol-dependent parameters as used in the text.

The Hamiltonian defined in Eq. (13) is time dependent through $g(t)$. To treat this time dependence, we write the fidelity amplitude in terms of evolution operators

$$\mathcal{A}(\tau) = \langle U_0^\dagger(\tau, 0) U(\tau, 0) \rangle. \quad (14)$$

The operator U_0 is the time-evolution operator of the (constant) initial Hamiltonian $\mathcal{H}_0 + \mathcal{V}_{\mathbf{X}}$ and U that of the (time-dependent) Hamiltonian $\mathcal{H}(t)$. In the case of a sudden quench, we recover the usual expression (12). We introduce now the interaction picture with respect to the initial Hamiltonian. This allows us to write

$$\mathcal{A}(\tau) = \left\langle \hat{T} \exp \left(-i \int_0^\tau dt g(t) \delta\mathcal{H}_{\mathbf{X}}(t) \right) \right\rangle \quad (15)$$

with time-ordering operator \hat{T} and $\delta\mathcal{H}_{\mathbf{X}}(t) = e^{i\mathcal{H}_i t} \delta\mathcal{H}_{\mathbf{X}} e^{-i\mathcal{H}_i t}$. The expression given in Eq. (15) is the starting point for the perturbative expansion given in the following.

We now perform a perturbative expansion of the fidelity amplitude in the displacement $\delta\mathbf{X}$ up to the first nonzero terms both in imaginary and real parts. We assume the scattering potential to be well behaved, such that small changes in \mathbf{X} correspond to small changes in \mathcal{V} .⁴ The corresponding change

⁴In a scattering approach, "small changes in the potential" can be understood as $\tau_D \delta\mathcal{H}_{\mathbf{X}} \ll 1$ since the electronic Hamiltonian is of the order of $1/\tau_D$. For the adiabatic quench, the change in the potential is time dependent such that most of the scattering electrons see a change smaller than $\delta\mathcal{H}_{\mathbf{X}}$, hence the condition is also satisfied.

in the Hamiltonian is

$$\delta\mathcal{H}_{\mathbf{X}} = \sum_{\alpha} \partial_{\alpha} \mathcal{V}_{\mathbf{X}} \delta X_{\alpha} = \sum_{\alpha} \partial_{\alpha} \mathcal{H}_{\mathbf{X}} \delta X_{\alpha}. \quad (16)$$

We therefore obtain

$$\begin{aligned} \ln \mathcal{A}(\tau) = & -i \int_0^{\tau} dt g(t) \langle \delta\mathcal{H}_{\mathbf{X}}(t) \rangle - \frac{1}{2} \int_0^{\tau} dt \int_0^{\tau} dt' \\ & \times \sum_{\alpha\beta} g(t) g(t') D_{\alpha\beta}(t, t') \delta X_{\alpha} \delta X_{\beta}, \end{aligned} \quad (17)$$

where $D_{\alpha\beta}(t, t')$ is the noise correlator given in Eq. (2) and we have used Eqs. (3) and (4). As anticipated in the previous section, due to the sum over the indices α and β , only the symmetric component of the noise correlator is relevant. The second term in Eq. (17) is a real quantity, while the first-order term is purely imaginary and hence contributes as an overall phase. This phase is directly related to the infinitesimal work made by the Born-Oppenheimer force, which is consistent with the shift in the dynamical phase of the system's eigenstates which is acquired due to the change in the potential $\delta\mathcal{H}_{\mathbf{X}}$. This can be seen by expressing the quantum statistical average $\langle \delta\mathcal{H}_{\mathbf{X}} \rangle$ in terms of scattering states [36]. For the adiabatic ($P = A$) and sudden ($P = S$) quenches we obtain

$$\mathcal{A}_P(\tau) = e^{i \frac{\tau}{\lambda_P} \mathbf{F}(\mathbf{X}) \cdot \delta \mathbf{X}} |\mathcal{A}_P(\tau)|, \quad (18)$$

with $\lambda_A = 2$ and $\lambda_S = 1$, where the reduction by a factor of 2 of the adiabatic fidelity phase with respect to the sudden quench results from integrating over the linear ramp up of the potential.

The Loschmidt echo, in turn, is given solely in terms of the integrated two-time noise correlation function

$$\ln \mathcal{L}(\tau) = - \int_0^{\tau} dt \int_0^{\tau} dt' g(t) g(t') \delta \mathbf{X}^{\dagger} \cdot \mathbf{D}(t, t') \cdot \delta \mathbf{X}, \quad (19)$$

where the function $g(t)$ enters as a weight factor. We note that assuming Gaussian white noise, where $\mathbf{D}(t, t')$ is delta correlated in time, it immediately follows from Eq. (19) that the Loschmidt echo decays exponentially with a strength proportional to $\delta \mathbf{X}^{\dagger} \cdot \mathbf{D}(\mathbf{X}) \cdot \delta \mathbf{X}$ in the large-time limit. For now we keep the results general and discuss the regime of applicability for the white-noise limit later. Expressing the fluctuating force correlator by its Fourier transform we readily observe that the decay of the Loschmidt echo is determined by the symmetric noise correlator $\mathbf{D}(\omega)$,⁵

$$\ln \mathcal{L}(\tau) = - \int_0^{\infty} \frac{d\omega}{\pi} \left| \int_0^{\tau} dt g(t) e^{i\omega t} \right|^2 \delta \mathbf{X}^{\dagger} \cdot \mathbf{D}(\omega) \cdot \delta \mathbf{X}. \quad (20)$$

The time integral above can be performed once the quench dynamics $g(t)$ is specified

$$\ln \mathcal{L}_P(\tau) = - \int_0^{\infty} \frac{d\omega}{\pi} B_P(\omega, \tau) \delta \mathbf{X}^{\dagger} \cdot \mathbf{D}(\omega) \cdot \delta \mathbf{X}, \quad (21)$$

where the function $B_P(\omega, \tau)$ is protocol dependent:

$$\begin{aligned} B_S(\omega, \tau) &= 2 \frac{1 - \cos(\omega \tau)}{\omega^2}, \\ B_A(\omega, \tau) &= \frac{2[1 - \cos(\omega \tau) - \omega \tau \sin(\omega \tau)] + \omega^2 \tau^2}{\tau^2 \omega^4}. \end{aligned} \quad (22)$$

The dwell time τ_D is a characteristic time scale for the scattering of the fast (electronic) degrees of freedom and provides a high-energy cutoff $1/\tau_D$ for the energy integrals in Eq. (21). At the same time, the function $B_P(\omega, \tau)$ selects frequencies $\omega \lesssim 1/\tau$. This allows us, in the limit of large $\tau \gg \tau_D$, to neglect the dynamics of the fast degrees of freedom and evaluate the fluctuating force correlator $\mathbf{D}(\omega)$ in Eq. (21) in the limit of small frequencies $\omega \sim 1/\tau \sim 0$; we will use this fact in the next sections.

The expressions obtained in this section are, within the limit of validity of the perturbative approach, quite general. In particular, they hold for out-of-equilibrium situations. To investigate these results we start in the next section with the equilibrium case, for which we can straightforwardly apply the fluctuation-dissipation theorem as given in Eq. (7). The out-of-equilibrium regime is treated later in Sec. V.

IV. EQUILIBRIUM

We consider here the equilibrium case for which all leads are kept at a same chemical potential denoted by μ . In equilibrium the Anderson orthogonality exponent has been shown in Ref. [31] to be proportional to the friction coefficient of the noninteracting fermionic environment for finite systems. This corresponds to the $\tau \rightarrow \infty$ limit of the Loschmidt echo [47]. We generalize this result here to the case of an open system with a continuous energy spectrum by calculating the decay of the Loschmidt echo for finite times τ .

We continue with evaluating Eq. (21) at $T = 0$ and we discuss the effect of finite temperature later. We can therefore use the fluctuation-dissipation theorem as given in Eq. (9). We conclude that, to leading order in τ/τ_D ,

$$\ln \mathcal{L}_P(\tau) = - \frac{1}{\pi} \int_0^{1/\tau_D} \omega d\omega B_P(\omega, \tau) \delta \mathbf{X}^{\dagger} \cdot \boldsymbol{\gamma}^{\text{eq}} \cdot \delta \mathbf{X}, \quad (23)$$

where $\boldsymbol{\gamma}^{\text{eq}}$ is the equilibrium friction coefficient evaluated at zero frequency. We see therefore that, as expected, in equilibrium the Loschmidt echo is closely related to dissipation. We obtain

$$\ln \mathcal{L}_P(\tau) = - \frac{\alpha_P}{\pi} \left[\ln \left(\frac{\tau}{\tau_D} \right) + \gamma_e \right] \delta \mathbf{X}^{\dagger} \cdot \boldsymbol{\gamma}^{\text{eq}}(\mathbf{X}) \cdot \delta \mathbf{X}, \quad (24)$$

where α_P is protocol dependent, with $\alpha_S = 2$, $\alpha_A = 1$ for the sudden and adiabatic quenches, respectively, and $\gamma_e = 0.5772$ is the Euler-Mascheroni constant.

It is interesting to note that the value $\alpha_A = 1$ is independent of the assumption of linearity for the adiabatic quench protocol. To show this we specify a general adiabatic quench dynamics as a power law $g(t) = (\frac{t}{\tau})^n$, with $n \geq 1$ integer. Using the fluctuation-dissipation theorem (9), we obtain for the adiabatic

⁵Note that $\mathbf{D}(\omega)$ is not the Fourier transform of $\mathbf{D}(t - t')$. It is rather the symmetrized Fourier transform (in frequency ω).

Loschmidt echo in this case

$$\ln \mathcal{L}_A(\tau) = -\frac{1}{\pi} \int_0^{1/\tau_D} d\omega \omega \left| \int_0^\tau dt \left(\frac{t}{\tau}\right)^n e^{i\omega t} \right|^2 \times \delta \mathbf{X}^\dagger \cdot \boldsymbol{\gamma}^{\text{eq}}(\mathbf{X}) \cdot \delta \mathbf{X}. \quad (25)$$

For large τ/τ_D , the lower bound of the frequency integral can be set to $1/\tau$ since the integral for $0 \leq \omega \leq 1/\tau$ gives a (n -dependent) constant and is hence irrelevant in this limit.⁶ To evaluate the t integral we integrate by parts and for large τ/τ_D we approximate to leading order

$$\int_0^\tau dt \left(\frac{t}{\tau}\right)^n e^{i\omega t} = \frac{e^{i\omega\tau}}{i\omega}. \quad (26)$$

Inserting this into Eq. (25) we obtain

$$\ln \mathcal{L}_A(\tau) = -\frac{1}{\pi} \sum_{\alpha\beta} \ln \left(\frac{\tau}{\tau_D}\right) \delta \mathbf{X}^\dagger \cdot \boldsymbol{\gamma}^{\text{eq}}(\mathbf{X}) \cdot \delta \mathbf{X} \quad (27)$$

up to an irrelevant constant for all $g(t) = (\frac{t}{\tau})^n$ with $n \geq 1$ integer.

Therefore, we conclude

$$\mathcal{L}_P(\tau) \propto \left(\frac{\tau}{\tau_D}\right)^{-\frac{\alpha_P}{\pi} \delta \mathbf{X}^\dagger \cdot \boldsymbol{\gamma}^{\text{eq}}(\mathbf{X}) \cdot \delta \mathbf{X}}, \quad (28)$$

so that the decay of the Loschmidt echo in equilibrium, both in the sudden and adiabatic quench scenarios, is a power law controlled by the friction coefficient of the fermionic system with universal coefficients α_P . The power-law decay of the Loschmidt echo is consistent with known literature results [14,31] and reflects the Anderson orthogonality catastrophe [12]. For a finite system the $\tau \rightarrow \infty$ limit of the Loschmidt echo can be obtained, up to prefactors, by replacing τ/τ_D by the number of particles of the system and the power law takes the usual Anderson's form. This is justified since the ratio τ/τ_D can be taken as an estimate of how many particles have been scattered up to time τ ; for $\tau \rightarrow \infty$, all particles in the system have participated in the scattering.

Note that the power-law decay of the Loschmidt echo is inconsistent with a delta-correlated noise; recall that white noise implies an *exponential* decay of the Loschmidt echo. The power-law decay signals the breakdown of the Markovian, semiclassical Langevin equation (1) in equilibrium and at zero temperature, for which case the classical noise correlator is zero. In other words, the system loses its memory as a power law in time instead of exponentially, which renders the Markovian approximation inapplicable. An exponential decay of the Loschmidt echo is recovered either by imposing finite temperature or a finite-bias voltage. In the following, we comment on the finite-temperature case, and we reserve the next section for out-of-equilibrium effects.

For temperatures such that $T \gg 1/\tau$, we can use the classical version of the fluctuation-dissipation theorem Eq. (8) in Eq. (19) to obtain

$$\ln \mathcal{L}_P(\tau) = -2\beta_P \tau T \delta \mathbf{X}^\dagger \cdot \boldsymbol{\gamma}^{\text{eq}} \cdot \delta \mathbf{X}, \quad (29)$$

and therefore we recover, for $\tau \gg \tau_D$, an exponential decay governed by the thermal noise

$$\mathcal{L}_P(\tau) = e^{-\tau \beta_P \delta \mathbf{X}^\dagger \cdot \mathbf{D}_{[T,0]} \cdot \delta \mathbf{X}}, \quad (30)$$

where β_P is protocol dependent, with $\beta_S = 1$, $\beta_A = \frac{1}{3}$ for the sudden and adiabatic (linear) quench, respectively. Note that in this case the coefficient β_A depends on the nature of the adiabatic protocol.

As a last remark of this section, we observe that in equilibrium and zero temperature the adiabatic and sudden Loschmidt echo are related by a simple exponent.⁷ For zero temperature, from Eq. (24) we obtain

$$\mathcal{L}_S(\tau) = \mathcal{L}_A(\tau)^2. \quad (31)$$

As pointed out before, $\alpha_A = 1$ is independent of the adiabatic protocol, and therefore Eq. (31) holds generally. The relation given by Eq. (31) has been recently pointed out for particular examples in Refs. [48,49] for infinite τ in finite systems, and argued to be valid in more general situations [48]. For finite temperatures we obtain instead from Eq. (30)

$$\mathcal{L}_S(\tau) = \mathcal{L}_A(\tau)^{1/\beta_A}, \quad (32)$$

which is valid to leading order in τ_D/τ .

V. OUT OF EQUILIBRIUM

In this section, we take a step further and allow for the presence of an applied bias voltage, represented by different chemical potentials in the leads. For clarity we consider only two leads which are kept at a chemical potential difference $\Delta\mu > 0$. We obtain the decay of the Loschmidt echo in the limit of linear response, for which the applied bias is small as defined by the condition $\Delta\mu \tau_D \ll 1$. By evaluating Eq. (21) with the out-of-equilibrium noise correlator given in Eq. (10), we can obtain closed expressions for the time dependence of the Loschmidt echo in terms of the macroscopic coefficients appearing in the Langevin equation (1). These expressions are valid for all times longer than the dwell time, as detailed in the following. The derivation is lengthy and therefore we summarize here the main results and give a sketch of the calculation in the next subsections, while the details can be found in the corresponding appendixes as listed.

It is instructive to consider the long- and short-time dynamics of the Loschmidt echo as compared with the time scale determined by the inverse of the imposed bias since in these limits the expressions simplify considerably. We state here the results for zero temperature. The case of *short-time dynamics* (but still large times compared with the dwell time) is given by the condition $\Delta\mu \tau \ll 1$. For these short times the system is being probed at high energies and it is not sensitive to the applied bias. We therefore recover the equilibrium result

$$\mathcal{L}_P(\tau) \propto \left(\frac{\tau}{\tau_D}\right)^{-\frac{\alpha_P}{\pi} \delta \mathbf{X}^\dagger \cdot \boldsymbol{\gamma} \cdot \delta \mathbf{X}}. \quad (33)$$

Here, $\boldsymbol{\gamma}$ is the full dissipation matrix evaluated to first order in the bias. Ignoring this first-order correction due to the bias we

⁶This can be easily seen by performing the change of variables $x = \omega\tau$ and $y = t/\tau$ in the integral for $0 \leq \omega \leq 1/\tau$.

⁷Note that these relations are valid for $\tau/\tau_D \gg 1$.

recover exactly the equilibrium result obtained previously in Eq. (24).

In the opposite limit of *long-time dynamics* $\Delta\mu\tau \gg 1$, the system is more sensitive to the nonequilibrium imposed by the bias which results in a different qualitative behavior. The major effect due to bringing the system out of equilibrium is an exponential suppression of the Loschmidt echo in the long-time dynamics, compared with the equilibrium power-law decay in Eq. (24). We obtain

$$\mathcal{L}_P(\tau) \propto e^{-\beta_P \tau \delta \mathbf{X}^\dagger \cdot \mathbf{D}_{[0, \Delta\mu]} \cdot \delta \mathbf{X}} (\Delta\mu \tau_D)^{\frac{\alpha_P}{\pi} \delta \mathbf{X}^\dagger \cdot \frac{\mathbf{D}_{[0, \Delta\mu]}}{\Delta\mu} \cdot \delta \mathbf{X}} \times \left(\frac{\tau}{\tau_D} \right)^{-\frac{\alpha_P}{\pi} \delta \mathbf{X}^\dagger \cdot [\gamma^{\text{eq}} - \frac{\mathbf{D}_{[0, \Delta\mu]}}{\Delta\mu}] \cdot \delta \mathbf{X}}, \quad (34)$$

where the dissipation matrix and noise correlators are evaluated to first order in the bias. The exponential suppression of the Loschmidt echo is dictated by the shot-noise fluctuations in the system given by the noise correlator $\mathbf{D}_{[0, \Delta\mu]}$ to first order in the bias [50]. Note that this exponential decay is completely analogous to that for equilibrium and finite temperatures in the long-time limit ($T\tau \gg 1$) given in Eq. (30), which is dominated by the thermal noise.

The exponential decay in Eq. (34) comes on top of a power-law behavior, with an exponent that is also modified from equilibrium showing a competition between fluctuations and dissipation, and with the possibility of a change of sign in the exponent. This power-law correction is absent in the complementary case of equilibrium and finite temperatures given in Eq. (30), due to the vanishing of the linear order in frequency correction for the thermal noise correlator in Eq. (8). Note that the out-of-equilibrium power law crosses over to the equilibrium one at a time $\tau \approx 1/\Delta\mu$ as expected.

We proceed now with the derivation of these results.

A. Out-of-equilibrium noise correlator

We see from Eq. (19) that the force-force noise correlator is crucial to determine the behavior of the Loschmidt echo. In this section, we derive the expression given in Eq. (10) for the noise correlator within a scattering approach which highlights the connection to the current-induced forces in the Langevin equation (1). An equivalent derivation in terms of Keldysh Green's functions is given, for completeness, in Appendix C. Alternatively, the correlator can be calculated within a Feynman-Vernon influence functional approach [51].

We now proceed with evaluating $\mathbf{D}(t, t')$ as given in Eq. (2) in terms of single-particle scattering states. For this we introduce the notation

$$\partial_\alpha V_{\mathbf{X}}^{kn}(\varepsilon, \varepsilon') = \langle \psi_k^{\mathbf{X}+}(\varepsilon) | \partial_\alpha V_{\mathbf{X}} | \psi_n^{\mathbf{X}+}(\varepsilon') \rangle \quad (35)$$

for the matrix elements of the representation of $\nabla V_{\mathbf{X}}$ in the scattering basis (cf. Appendix A). Here, $|\psi_n^{\mathbf{X}+}(\varepsilon)\rangle$ is the single-particle retarded scattering state with combined channel-lead index n and energy ε . For notational convenience in what follows, we further define the function

$$K_{kn}^{\alpha\beta}(\varepsilon, \varepsilon') = \left\{ \partial_\alpha V_{\mathbf{X}}^{kn}(\varepsilon, \varepsilon') \partial_\beta V_{\mathbf{X}}^{nk}(\varepsilon', \varepsilon) \right\}_s. \quad (36)$$

Using Eq. (A1) to evaluate the quantum statistical expectation values [52] appearing in the noise correlator (2) we obtain

$$\mathbf{D}(t, t') = \int \frac{d\varepsilon}{2\pi} \int \frac{d\varepsilon'}{2\pi} \sum_{kn} f_k(\varepsilon) [1 - f_n(\varepsilon')] \times e^{i(\varepsilon - \varepsilon')(t - t')} \mathbf{K}_{kn}(\varepsilon, \varepsilon'). \quad (37)$$

After Fourier transforming, we obtain a general expression for the force-force noise correlator as a function of frequencies [53]

$$\mathbf{D}(\omega) = \int \frac{d\varepsilon}{2\pi} \sum_{kn} f_k\left(\varepsilon - \frac{\omega}{2}\right) \left[1 - f_n\left(\varepsilon + \frac{\omega}{2}\right) \right] \times \mathbf{K}_{kn}\left(\varepsilon - \frac{\omega}{2}, \varepsilon + \frac{\omega}{2}\right). \quad (38)$$

We observe that the function $\mathbf{K}_{kn}(\varepsilon, \varepsilon')$ contains overlaps of scattering states which are associated with scattering events including an energy transfer $\omega = \varepsilon - \varepsilon'$. We expect these overlaps to vary within energies up to the inverse dwell time $1/\tau_D$. We can expect our description of the Loschmidt echo in terms of scattering states to be valid in the limit $\tau \gg \tau_D$; a description on microscopic time scales smaller than τ_D is beyond an adiabatic scattering formulation. Hence, in the following we restrict our calculations to this limit, and evaluate $\mathbf{K}_{kn}(\varepsilon, \varepsilon')$ to first order in $\omega \ll 1/\tau_D$. We will see later in the text that this is enough to capture the leading behavior of the Loschmidt echo as a function of time. It is also convenient to define the function

$$\mathbf{K}_{kn}(\varepsilon) = \mathbf{K}_{kn}(\varepsilon, \varepsilon). \quad (39)$$

We note that the function $\mathbf{K}_{kn}(\varepsilon)$ is closely related to the dissipation matrix in equilibrium. At zero temperature and taking all leads to be at an equal chemical potential μ , as we show in Appendix B, it takes the simple form

$$\frac{1}{4\pi} \sum_{kn} \mathbf{K}_{kn}(\mu) = \gamma^{\text{eq}}, \quad (40)$$

in agreement with the result found previously by use of the fluctuation-dissipation theorem.

The second contribution to Eq. (38) is given by the product of Fermi functions $f_k(\varepsilon)[1 - f_n(\varepsilon')]$. Due to this product, the average energy $\bar{\varepsilon} = (\varepsilon + \varepsilon')/2$ is limited to a region of size $\Delta\mu_{kn} = \mu_k - \mu_n$ around the respective average chemical potential $\bar{\mu}_{kn} = 1/2(\mu_k + \mu_n)$. To make analytical progress, we limit our results to the linear-response regime $\Delta\mu_{kn} \tau_D \ll 1$, which allows a perturbative treatment of the function $\mathbf{K}_{kn}(\varepsilon, \varepsilon')$ for small deviations of $\bar{\varepsilon}$ around $\bar{\mu}_{kn}$.

Given these considerations, we calculate Eq. (38) to leading order in τ_D/τ , in the linear-response regime (linear order in $\Delta\mu \tau_D$). Expanding $\mathbf{K}_{kn}(\varepsilon, \varepsilon')$ to first order in ω and $\bar{\varepsilon} - \bar{\mu}_{kn}$ we obtain

$$\mathbf{K}_{kn}(\varepsilon, \varepsilon') = \mathbf{K}_{kn}(\bar{\mu}_{kn}) + 2(\bar{\varepsilon} - \bar{\mu}_{kn}) \partial_{\bar{\varepsilon}}^s \mathbf{K}_{kn}(\bar{\mu}_{kn}) + \omega \partial_{\bar{\varepsilon}}^a \mathbf{K}_{kn}(\bar{\mu}_{kn}), \quad (41)$$

where we have introduced

$$\partial_{\bar{\varepsilon}}^{s/a} \mathbf{K}_{kn}(x) = \frac{1}{2} (\partial_{\bar{\varepsilon}} \pm \partial_{\varepsilon'}) \mathbf{K}_{kn}(\varepsilon, \varepsilon') \Big|_{\varepsilon' = \varepsilon = x}, \quad (42)$$

which describes the symmetric and antisymmetric energy derivatives of \mathbf{K}_{kn} . We note in passing the following useful properties: $\mathbf{K}_{kn}(\varepsilon) = \mathbf{K}_{nk}(\varepsilon)$, $\partial_\varepsilon^s \mathbf{K}_{kn}(\varepsilon) = \partial_\varepsilon^s \mathbf{K}_{nk}(\varepsilon)$, and $\partial_\varepsilon^a \mathbf{K}_{kn}(\varepsilon) = -\partial_\varepsilon^a \mathbf{K}_{nk}(\varepsilon)$. Substituting Eq. (41) into Eq. (38), the energy integral can be performed to obtain

$$\mathbf{D}(\omega) = \frac{1}{2\pi} \sum_{kn} \frac{\omega + \Delta\mu_{kn}}{e^{(\omega + \Delta\mu_{kn})/T} - 1} \times e^{(\omega + \Delta\mu_{kn})/T} [\mathbf{K}_{kn}(\bar{\mu}_{kn}) - \omega \partial_\varepsilon^a \mathbf{K}_{kn}(\bar{\mu}_{kn})]. \quad (43)$$

To be consistent with the linear-response approximation, the functions \mathbf{K}_{kn} and $\partial_\varepsilon^{s/a} \mathbf{K}_{kn}^{\alpha\beta}$ have to be evaluated to first order in $\Delta\mu\tau_D$. We proceed with this expansion below for the case of two leads, for which the expressions are more transparent. The indices k, n describe hereafter (the two) lead indices only, where we implicitly assume a summation over the channel index. A generalization to an arbitrary number of leads is straightforward. Without loss of generality we write $\mu_R = \mu - \Delta\mu/2$, $\mu_L = \mu + \Delta\mu/2$ with $\Delta\mu > 0$. Explicitly, $\mathbf{K}_{LL}(\mu_L) = \mathbf{K}_{LL}(\mu) + \Delta\mu \partial_\varepsilon^s \mathbf{K}_{LL}(\mu)$ and $\mathbf{K}_{RR}(\mu_R) = \mathbf{K}_{RR}(\mu) - \Delta\mu \partial_\varepsilon^s \mathbf{K}_{RR}(\mu)$. Hence, within linear response we obtain (with $\omega_\pm = \omega \pm \Delta\mu$)

$$\begin{aligned} \mathbf{D}(\omega) &= \frac{1}{4\pi} \omega \coth\left(\frac{\omega}{2T}\right) \{ \mathbf{K}_{LL}(\mu) + \mathbf{K}_{RR}(\mu) \\ &\quad + \Delta\mu [\partial_\varepsilon^s \mathbf{K}_{LL}(\mu) - \partial_\varepsilon^s \mathbf{K}_{RR}(\mu)] \} \\ &\quad + \frac{1}{4\pi} \left[\omega_+ \coth\left(\frac{\omega_+}{2T}\right) + \omega_- \coth\left(\frac{\omega_-}{2T}\right) \right] \mathbf{K}_{LR}(\mu) \\ &\quad + \frac{\omega}{4\pi} \left[-\omega_+ \coth\left(\frac{\omega_+}{2T}\right) + \omega_- \coth\left(\frac{\omega_-}{2T}\right) \right] \\ &\quad \times \partial_\varepsilon^a \mathbf{K}_{LR}(\mu). \end{aligned} \quad (44)$$

The coefficients appearing in the expansion can be interpreted in terms of the different dissipative contributions with the help of the relations found in Appendix A. The connection to the friction tensor is found by expanding the dissipation tensor $\boldsymbol{\gamma}$ to first order in $\Delta\mu\tau_D$ as $\boldsymbol{\gamma} = \boldsymbol{\gamma}_0^{\text{eq}} + \boldsymbol{\gamma}_1^{\text{eq}} + \boldsymbol{\gamma}_1^{\text{neq}} + \dots$, where the subscript 0 (1) denotes the zeroth (first) order in the expansion respectively of the equilibrium (eq) and nonequilibrium (neq) contributions to the friction tensor. In Appendix B, we show the following identities:

$$\boldsymbol{\gamma}_1^{\text{eq}} = \frac{\Delta\mu}{4\pi} [\partial_\varepsilon^s \mathbf{K}_{LL}(\mu) - \partial_\varepsilon^s \mathbf{K}_{RR}(\mu)], \quad (45)$$

$$\boldsymbol{\gamma}_1^{\text{neq}} = \frac{\Delta\mu}{4\pi} [\partial_\varepsilon^a \mathbf{K}_{RL}(\mu) - \partial_\varepsilon^a \mathbf{K}_{LR}(\mu)], \quad (46)$$

$$\mathbf{D}_{[0, \Delta\mu]} = \frac{\Delta\mu}{4\pi} [\mathbf{K}_{LR}(\mu) + \mathbf{K}_{RL}(\mu)], \quad (47)$$

⁸Note that the linear-response expansion is indeed an expansion in $\Delta\mu\tau_D$ since the energy derivatives appearing in Eqs. (45) and (46) are of order the Wigner time delay, which can be estimated to be of the order of τ_D .

which together with Eq. (40) imply

$$\mathbf{K}_{LL}(\mu) + \mathbf{K}_{RR}(\mu) = 4\pi \left(\boldsymbol{\gamma}_0^{\text{eq}} - \frac{\mathbf{D}_{[0, \Delta\mu]}}{\Delta\mu} \right). \quad (48)$$

Plugging in these identities in Eq. (44) we obtain the anticipated result stated in Eq. (10).

In order to be sensitive to nonequilibrium effects, we need to impose temperatures smaller than the bias. We therefore take the zero-temperature limit of the noise correlator given in Eq. (10). For $|\omega| < \Delta\mu$ we then obtain

$$\mathbf{D}(\omega) = \mathbf{D}_{[0, \Delta\mu]} + |\omega| \left(\boldsymbol{\gamma}^{\text{eq}} - \frac{\mathbf{D}_{[0, \Delta\mu]}}{\Delta\mu} \right) + \frac{\omega^2}{2} \frac{\boldsymbol{\gamma}^{\text{neq}}}{\Delta\mu}, \quad (49)$$

while for $|\omega| > \Delta\mu$ we have

$$\mathbf{D}(\omega) = |\omega| \boldsymbol{\gamma}, \quad (50)$$

where we have used that $\lim_{x \rightarrow \pm\infty} \coth x = \pm 1$.

B. Short- and long-time dynamics

Inserting Eqs. (49) and (50) into (21) we can express the decay of the Loschmidt echo in terms of the mesoscopic coefficients that control the Langevin dynamics of a heavy particle embedded in the fermionic environment [cf. Eq. (1)]. We obtain

$$\begin{aligned} \ln \mathcal{L}_P(\tau) &= -\frac{\alpha_P}{\pi} \left[\gamma_e + \ln\left(\frac{\tau}{\tau_D}\right) \right] \delta \mathbf{X}^\dagger \cdot \boldsymbol{\gamma} \cdot \delta \mathbf{X} \\ &\quad + \frac{\alpha_P}{\pi} [\gamma_e + \ln(\Delta\mu\tau)] \delta \mathbf{X}^\dagger \cdot \left(\frac{\mathbf{D}_{[0, \Delta\mu]}}{\Delta\mu} + \boldsymbol{\gamma}_1^{\text{neq}} \right) \cdot \delta \mathbf{X} \\ &\quad - \beta_P \tau \delta \mathbf{X}^\dagger \cdot \mathbf{D}_{[0, \Delta\mu]} \cdot \delta \mathbf{X} - \frac{\delta_P}{\pi} \delta \mathbf{X}^\dagger \cdot \boldsymbol{\gamma}_1^{\text{neq}} \cdot \delta \mathbf{X} \\ &\quad + \frac{1}{\pi} (\alpha_P - 2\beta_P \cos(\Delta\mu\tau)) \delta \mathbf{X}^\dagger \cdot \frac{\mathbf{D}_{[0, \Delta\mu]}}{\Delta\mu} \cdot \delta \mathbf{X} \end{aligned} \quad (51)$$

with $\delta_S = 2$, $\delta_A = \frac{1}{2}$. Equation (51) gives the behavior of the Loschmidt echo at arbitrary times larger than the dwell time, up to quadratic order in $\delta \mathbf{X}$. In the following, we further investigate different time-scale regimes.

The short-time dynamics is given by the limit $\Delta\mu\tau \ll 1$: “short times” here should be considered as short with respect to the inverse bias time scale but long compared to the dwell time τ_D . In this regime, we conclude

$$\ln \mathcal{L}_P(\tau) = -\frac{\alpha_P}{\pi} \left[\gamma_e + \ln\left(\frac{\tau}{\tau_D}\right) \right] \delta \mathbf{X}^\dagger \cdot \boldsymbol{\gamma} \cdot \delta \mathbf{X}, \quad (52)$$

which yields Eq. (33). Note that the equilibrium friction term given by $\boldsymbol{\gamma}_0^{\text{eq}}$ constitutes the dominant contribution for the decay, and the full friction matrix $\boldsymbol{\gamma}$ is restricted to positive values for small $\Delta\mu\tau_D \ll 1$, which ensures that $\mathcal{L}_P(\tau) \leq 1$ for all times τ .

We turn now to evaluating Eq. (51) in the long-time limit $\Delta\mu\tau \gg 1$. Writing $\ln(\tau/\tau_D) = \ln[\Delta\mu\tau/(\Delta\mu\tau_D)]$ and observing that $\Delta\mu\tau_D \ln(\Delta\mu\tau_D)$ goes to zero for $\Delta\mu\tau_D \ll 1$,

we obtain

$$\begin{aligned}
 \ln \mathcal{L}_P(\tau) = & -\beta_P \tau \delta \mathbf{X}^\dagger \cdot \mathbf{D}_{[0, \Delta\mu]} \cdot \delta \mathbf{X} \\
 & - \frac{\alpha_P}{\pi} [\gamma_e + \ln(\Delta\mu\tau)] \delta \mathbf{X}^\dagger \cdot \left(\boldsymbol{\gamma}^{\text{eq}} - \frac{\mathbf{D}_{[0, \Delta\mu]}}{\Delta\mu} \right) \cdot \delta \mathbf{X} \\
 & + \frac{\alpha_P}{\pi} \ln(\Delta\mu\tau_D) \delta \mathbf{X}^\dagger \cdot \boldsymbol{\gamma}_0^{\text{eq}} \cdot \delta \mathbf{X} \\
 & - \frac{\delta_P}{\pi} \delta \mathbf{X}^\dagger \cdot \boldsymbol{\gamma}_1^{\text{neq}} \cdot \delta \mathbf{X} \\
 & + \frac{1}{\pi} (\alpha_P - 2\beta_P \cos(\Delta\mu\tau)) \delta \mathbf{X}^\dagger \cdot \frac{\mathbf{D}_{[0, \Delta\mu]}}{\Delta\mu} \cdot \delta \mathbf{X},
 \end{aligned} \tag{53}$$

from which we obtain Eq. (34) by keeping the dominant terms in large τ [32].

The exponential suppression of the Loschmidt echo is dictated by the shot-noise fluctuations in the system (note that $\mathbf{D}_{[0, \Delta\mu]}$ is positive definite) and it is consistent with a leading behavior of Gaussian white noise for the fluctuating force correlator $\mathbf{D}(t, t')$ in Eq. (2). Furthermore, the exponent of the power law shows a competition between fluctuations and dissipation, which is a clear signature of the departure from equilibrium; as remarked before, the corresponding power law is absent in the equilibrium, finite-temperature case. Since $\boldsymbol{\gamma}^{\text{eq}} = \mathbf{D}_{[T, 0]}/(2T)$ for zero bias, the sign of the exponent $-\delta \mathbf{X}^\dagger \cdot [\boldsymbol{\gamma}^{\text{eq}} - \mathbf{D}_{[0, \Delta\mu]}/(\Delta\mu)] \cdot \delta \mathbf{X}$ depends on the asymmetry between shot and Nyquist noise in the linear-response regime. In fact, this exponent can be positive for finite bias, which leads to an enhancement of the power law instead of the usual decay, which has been dubbed as ‘‘antiorthogonality’’ [33]. We note, however, that $-\delta \mathbf{X}^\dagger \cdot [\boldsymbol{\gamma}_0^{\text{eq}} - \mathbf{D}_{[0, \Delta\mu]}/(\Delta\mu)] \cdot \delta \mathbf{X}$ is always negative [see Eqs. (48) and (B4)]. The sign change of the exponent happens when the leading order $\mathbf{K}_{RR}(\mu) \approx -\mathbf{K}_{LL}(\mu)$ cancels out, such that the linear order correction $\boldsymbol{\gamma}_1^{\text{eq}}$ in the bias becomes dominant, which can then lead to a change of the sign of the exponent.

Since the out-of-equilibrium short-time dynamics is essentially the equilibrium one, the identity $\mathcal{L}_S(\tau) = \mathcal{L}_A(\tau)^2$ is still fulfilled in this limit, as can be seen directly from Eq. (52). On the other hand, in the large-time regime, we conclude from Eq. (53) that this identity is violated due to the factor β_P in the exponential. This difference, attributed to the structure of $g(t)$, can already be obtained by looking at Eq. (19). Assuming white noise, we immediately deduce from Eq. (19) an exponential decay of the Loschmidt echo with an exponent $-\beta_P \delta \mathbf{X}^\dagger \cdot \mathbf{D}_{[0, \Delta\mu]} \cdot \delta \mathbf{X} \tau$. The power-law decay in Eq. (53) constitutes minor correction terms to the white-noise assumption, and therefore to leading order in τ_D/τ we obtain $\mathcal{L}_S(\tau) = \mathcal{L}_A(\tau)^{1/\beta_A}$ as in the equilibrium, finite-temperature case given in Eq. (32).

In Sec. VI, we study these results for a specific example of a two-level model coupled to one vibrational mode.

VI. EXAMPLE: TWO-LEVEL MODEL WITH ONE VIBRATIONAL MODE

In this section, we analyze the sudden quench Loschmidt echo for the example of a system with one classical degree of freedom connected to two leads. This serves as a toy model

to illustrate the above results. The ‘‘heavy’’ classical degree of freedom $X = X(t)$ corresponds to a mechanical vibrational mode of the system. Accordingly, we consider the Hamiltonian

$$\mathcal{H} = \mathcal{H}_X + \mathcal{H}_L + \mathcal{H}_D + \mathcal{H}_T, \tag{54}$$

where the different terms are specified as

$$\mathcal{H}_X = \frac{P^2}{2M} + U(X), \tag{55}$$

$$\mathcal{H}_L = \int \frac{d\varepsilon}{2\pi} \sum_{\eta} (\varepsilon - \mu_{\eta}) c_{\eta}^{\dagger}(\varepsilon) c_{\eta}(\varepsilon), \tag{56}$$

$$\mathcal{H}_D = \sum_{mm'} d_m^{\dagger} [h_0(X)]_{mm'} d_{m'}, \tag{57}$$

$$\mathcal{H}_T = \int \frac{d\varepsilon}{\sqrt{2\pi}} \sum_{\eta m} [c_{\eta}^{\dagger}(\varepsilon) W_{\eta m}(\varepsilon) d_m + \text{H.c.}]. \tag{58}$$

Here, the operator $c_{\eta}^{\dagger}(\varepsilon)$ [$c_{\eta}(\varepsilon)$] creates (annihilates) electronic states $|\phi_{\eta}(\varepsilon)\rangle$, which are approaching the scattering region from lead $\eta = L, R$ with chemical potential $\mu_L \geq \mu_R$. \mathcal{H}_X describes the evolution of the parameter X with potential $U(X)$, mass M , and frequency ω_0 . \mathcal{H}_D models the two-level system (quantum dot) with states $|m\rangle$, created (annihilated) by the operators d_m^{\dagger} (d_m). \mathcal{H}_T represents tunneling between the leads and the system with tunneling amplitudes $W_{\eta m}(\varepsilon) = \langle \phi_{\eta}(\varepsilon) | W | m \rangle / \sqrt{2\pi}$. The coupling of the mechanical degree of freedom and the electrons in the dot is described by the matrix $h_0(X)$.

We consider a two-level system with degenerate energy levels ε_0 . The single oscillator mode X is assumed to couple to the difference in the energy-level occupation with a strength given by λ . Hence, we write

$$h_0(X) = \begin{pmatrix} \varepsilon_+ & t \\ t & \varepsilon_- \end{pmatrix} \tag{59}$$

with interdot tunneling amplitude t and $\varepsilon_{\pm} = \varepsilon_0 \pm \lambda X$. Tunneling from the left (right) lead to the two-level system and back is described by the amplitudes Γ_L (Γ_R) which for simplicity we take as $\Gamma_L = \Gamma_R = \Gamma/2$. In the wide-band approximation, these amplitudes are assumed to be energy independent. With these definitions, the coupling matrix $W_{\eta m}$ reads as

$$W = \begin{pmatrix} \sqrt{\Gamma/(2\pi)} & 0 \\ 0 & \sqrt{\Gamma/(2\pi)} \end{pmatrix}, \tag{60}$$

and the frozen retarded Green’s function takes the form

$$G_X^R(\varepsilon) = \frac{1}{\Delta_X(\varepsilon)} \begin{pmatrix} \varepsilon - \varepsilon_- + i\Gamma/2 & t \\ t & \varepsilon - \varepsilon_+ + i\Gamma/2 \end{pmatrix} \tag{61}$$

with $\Delta_X(\varepsilon) = (\varepsilon - \varepsilon_- + i\Gamma/2)(\varepsilon - \varepsilon_+ + i\Gamma/2) - t^2$. This model was studied in Ref. [40] in the context of current-induced forces. The frozen scattering matrix S and its first-order nonadiabatic correction A are given by

$$S_X(\varepsilon) = \mathbb{1} - \frac{i\Gamma}{L_X(\varepsilon)} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}, \tag{62}$$

$$A_X(\varepsilon) = \frac{\lambda\Gamma t}{\Delta_X(\varepsilon)^2} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \tag{63}$$

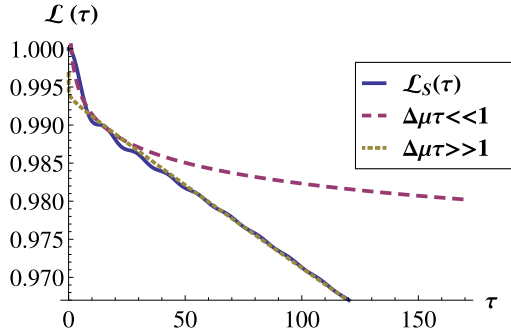


FIG. 2. (Color online) Sudden quench Loschmidt echo as given in Eq. (51) and comparison to the short- and long-time behavior Eqs. (33) and (34) (with the corresponding proportionality coefficients), for the example of a two-level system coupled to one vibrational mode with $\Gamma = t = 0.48$, $\tau_D = 1/\Gamma = 2.08$, $\varepsilon_0 = 0$, $\Delta\mu = 0.04$, $\delta X = 0.15$, $X = 0.4$, $T = 0$. All energies (and inverse times) are in units of $\lambda^2/(M\omega_0^2)$ and distances in units of $\lambda/(M\omega_0^2)$.

where $L_X(\varepsilon) = \varepsilon - \varepsilon_+ + i\Gamma$. With the expression of the S matrix and the A matrix, we can determine all the mesoscopic coefficients appearing in the Langevin equation (1), which determine the behavior of the Loschmidt echo. This is depicted for arbitrary times $\tau > \tau_D$ in Fig. 2 according to Eq. (51), and compared to the small- and large-time regimes expressions in Eqs. (33) and (34). The dwell time enters in this example as a time scale which is of the order of the inverse tunneling amplitudes.

The considered model allows us to analyze the Loschmidt echo also outside of the linear-response regime by directly evaluating the colored noise-noise correlator as given in Eq. (38). The matrix $\partial_\alpha V_X(\varepsilon, \varepsilon')$ [see Eq. (35)] is given by

$$\partial_\alpha V_X^{kn}(\varepsilon, \varepsilon') = 2\pi [W G_X^R(\varepsilon)^\dagger \partial_X h_0(X) G_X^R(\varepsilon') W^\dagger]_{kn}. \quad (64)$$

Equation (38) is neither restricted to $\Delta\mu\tau_D \ll 1$ nor to the regime $\tau_D/\tau \ll 1$ and hence is valid for arbitrary $\Delta\mu$ and all τ . Thus, we can study the Loschmidt echo for increasing bias voltages $\Delta\mu$. A comparison of the general solution, that is substituting Eq. (38) into (21), and the linear-response solution (51), is depicted in Fig. 3. The figure shows that the

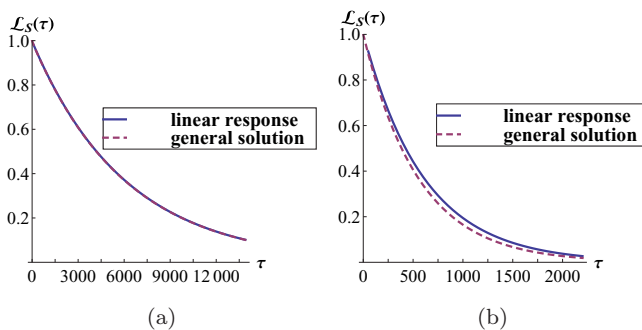


FIG. 3. (Color online) Linear-response solution of the sudden quench Loschmidt echo [Eq. (51)] vs general solution [i.e., Eq. (38) inserted into Eq. (21)] at different bias voltages (a) $\Delta\mu = 0.04$, (b) $\Delta\mu = 0.4$; other parameters as in Fig. 2.

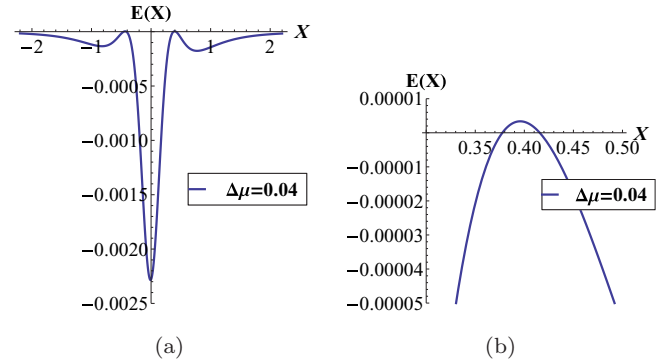


FIG. 4. (Color online) (a) The out-of-equilibrium power-law coefficient $E(X) = -\frac{2}{\pi}(\gamma_0^{\text{eq}} + \gamma_1^{\text{eq}} - \frac{D}{\Delta\mu})$ for $\Delta\mu = 0.04$ as a function of X ; other parameters as in Fig. 2. (b) Zoom-in of (a) which shows the change of sign in the exponent.

linear-response solution agrees very well with the general solution for small $\Delta\mu\tau_D$.

To close this section, we show that the power-law exponent $-\frac{2}{\pi}(\gamma^{\text{eq}} - \mathbf{D}_{[0, \Delta\mu]}/\Delta\mu)$ can present changes in sign. This is shown in Fig. 4 for a specific finite-bias voltage, where the exponent becomes positive for a small range of displacements.

VII. SUMMARY

In this paper, we calculated perturbatively the decay of the Loschmidt echo for an open noninteracting fermionic system in the presence of an external bias, which is subject to a scattering potential quench. We expressed our results in terms of the mesoscopic quantities describing the complementary problem of a heavy particle moving in a quantum environment, and showed that the Loschmidt echo decay is controlled by the noise correlator of the heavy particle.

This result allowed us to study Anderson orthogonality effects for the open quantum system. In the particular case of equilibrium and zero temperature, the decay of the Loschmidt echo is a power law controlled by the dissipation coefficient of the heavy particle, in agreement with the results found in Ref. [31] in which the exponent of the Anderson orthogonality was related to the dissipation of a heavy particle moving in a finite-sized quantum environment. For finite temperatures, in the limit of long times we recovered an exponential decay which reflects the classical version of the fluctuation-dissipation theorem.

When a small bias is imposed, we showed that in the long-time dynamics (as compared to the energy scale given by the bias), the Loschmidt echo is dictated by an exponential decay with a strength given by the shot-noise fluctuations. The exponential decay is consistent with a white-noise spectrum to leading order. As a correction term to white noise, the Loschmidt echo shows an algebraic behavior with a power-law exponent given by a competition between fluctuations and dissipation. This competition can give rise to changes in the sign of the power-law exponent. The power-law correction to the exponential decay is characteristic of nonequilibrium effects and is absent in equilibrium at finite temperatures. In the case of short-time dynamics, the system is mostly insensitive to the applied bias and the Loschmidt echo still presents

TABLE II. Loschmidt echo behavior for the different regimes studied in this work. These results represent the leading-order decay term for times $\tau \gg \tau_D$.

Bias regime	Temperature regime	$\mathcal{L}_P(\tau)$ decay
$\Delta\mu = 0$	$T \ll 1/\tau$	Power law
$\Delta\mu = 0$	$T \gg 1/\tau$	Exponential
	(classical regime)	
$\Delta\mu \ll 1/\tau$	$T = 0$	Power law
$\Delta\mu \gg 1/\tau$	$T = 0$	Exponential
		with power-law correction

a power-law decay, controlled by the full nonequilibrium dissipation coefficient.

The results summarized above are independent of the quench scenario and are shown in Table II. The dependence on the quench is only quantitative and it is represented by quench-dependent numerical coefficients, which are in turn given in Table I. In particular, we studied the complementary cases of sudden and slow quenches, and showed that in equilibrium the sudden quench Loschmidt echo is the square of the Loschmidt echo for the slow quench, independent of the functional form of the slow quench. This relation generalizes the relation found for finite quantum systems at infinite times in Refs. [48,49], where a Luttinger liquid subject to a linear slow quench was studied. We find that this correspondence breaks down out of equilibrium or for finite temperatures. To leading order, however, we can still establish a simple relation involving a nonuniversal exponent.

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APPENDIX A: ADIABATIC SCATTERING THEORY

In this Appendix, we outline the elements of scattering theory used in the main text. For $\mathcal{H}_X = \mathcal{H}_0 + \mathcal{V}_X$ assuming noninteracting particles, we can write

$$\mathcal{H}_X = \int \frac{d\varepsilon}{2\pi} \int \frac{d\varepsilon'}{2\pi} \sum_{kn} [H_X]_{kn}(\varepsilon, \varepsilon') a_k^{\dagger X}(\varepsilon) a_n^X(\varepsilon'), \quad (\text{A1})$$

with the *single-particle* Hamiltonian $H_X = H_0 + V_X$. The operators $a_m^X(\varepsilon)$ and $a_m^{\dagger X}(\varepsilon)$ annihilate and create, respectively, the retarded single-particle scattering states $|\psi_m^X(\varepsilon)\rangle$ of the Hamiltonian H_X with energy ε and combined channel and lead index m , hence, it follows

$$[H_X]_{kn}(\varepsilon, \varepsilon') = \langle \psi_k^X(\varepsilon) | H_X | \psi_n^X(\varepsilon') \rangle. \quad (\text{A2})$$

The corresponding advanced scattering states are indicated with the superscript $(-)$, that is $|\psi_m^X(\varepsilon)\rangle$. These scattering

states are solutions of the time-independent Schrödinger equation at every time t [note that we consider $\mathbf{X} = \mathbf{X}(t)$], and obey the Lippmann-Schwinger equation. The retarded and advanced scattering states are defined by their boundary conditions: $|\psi_m^X(\varepsilon)\rangle$ has incoming (outgoing) waves only in channel/lead m , and evolved from the free states $|\phi_m(\varepsilon)\rangle$ at $t \rightarrow \mp\infty$, which fulfill $H_0|\phi_m(\varepsilon)\rangle = \varepsilon|\phi_m(\varepsilon)\rangle$ with energy ε . We normalize the scattering states to unit flux. The overlap between retarded and advanced scattering states then defines the S matrix as

$$S_X^{lm}(\varepsilon) 2\pi \delta(\varepsilon - \varepsilon') = \langle \psi_l^X(\varepsilon) | \psi_m^X(\varepsilon') \rangle. \quad (\text{A3})$$

From this definition, it follows that $S_X^\dagger(\varepsilon)S_X(\varepsilon) = 1$ because of the scattering states’ normalization to unit flux.

For a Hamiltonian which parametrically depends on time via the classical parameter \mathbf{X} , Eq. (A3) gives the frozen S matrix of the system: the solution of the time-independent scattering problem at time t . The *exact* S matrix $S(\varepsilon, \varepsilon')$ is defined through the overlap of retarded and advanced scattering states’ solution of the full time-dependent problem. Assuming a slowly varying parameter \mathbf{X} , the exact S matrix can be expressed as an adiabatic expansion in the velocity $\dot{\mathbf{X}}_t$ [35,36,40,54,55]. In this context, ‘‘slowly varying’’ means that the dynamics of \mathbf{X} is much slower than the electronic time scales. To first order in the adiabatic expansion we can write, in the Wigner representation, $S(\varepsilon, t) = S_X(\varepsilon) + \sum_\alpha A_X^\alpha(\varepsilon) \dot{X}_\alpha$. The zeroth order is given by the frozen S matrix. The first nonadiabatic correction to the S matrix is given by [36]

$$A_X^{lm,\alpha}(\varepsilon) = \frac{1}{2} \langle \partial_\varepsilon \psi_l^X(\varepsilon) | \partial_\alpha V_X | \psi_m^X(\varepsilon) \rangle - \frac{1}{2} \langle \psi_l^X(\varepsilon) | \partial_\alpha V_X | \partial_\varepsilon \psi_m^X(\varepsilon) \rangle \quad (\text{A4})$$

with $\partial_\alpha = \partial/\partial X_\alpha$ and $|\partial_\varepsilon \psi_m^X(\varepsilon)\rangle \equiv \partial_\varepsilon |\psi_m^X(\varepsilon)\rangle$. Note that throughout the paper we work in the Heisenberg picture, so that there is no explicit time dependence on the states. Due to unitarity, the S and A matrices fulfill [36]

$$\langle \psi_n^X(\varepsilon) | \partial_\alpha V | \psi_k^X(\varepsilon) \rangle = i \partial_\alpha S_{nk}(\varepsilon), \quad (\text{A5})$$

$$\langle \psi_n^X(\varepsilon) | \partial_\alpha V | \partial_\varepsilon \psi_k^X(\varepsilon) \rangle = -A_{nk}^\alpha(\varepsilon) + \frac{i}{2} \partial_\varepsilon \partial_\alpha S_{nk}(\varepsilon). \quad (\text{A6})$$

These relations will be used in the next Appendix.

We can express the reaction forces in the Langevin equation (1) in terms of Eqs. (A3) and (A4), which we list here for completeness [35,36,40]. The Born-Oppenheimer force is given by

$$F_\alpha(\mathbf{X}) = \int \frac{d\varepsilon}{2\pi i} \sum_n f_n(\varepsilon) \text{tr}\{\Pi_n S_X^\dagger(\varepsilon) \partial_\alpha S_X(\varepsilon)\}, \quad (\text{A7})$$

where $\text{tr}\{\dots\}$ denotes a trace over scattering channels, $f_n(\varepsilon) = \{\exp[(\varepsilon - \mu_n)/T + 1]\}^{-1}$ is the fermionic distribution function in lead n with chemical potential μ_n , and $\partial_\alpha = \partial/\partial X_\alpha$, Π_n is a projector onto channel n . The two contributions to the dissipative force as discussed in the main text are in turn given

by

$$\gamma_{\alpha\beta}^{\text{eq}}(\mathbf{X}) = - \sum_n \int \frac{d\varepsilon}{4\pi} \partial_\varepsilon [f_n(\varepsilon)] \text{tr}\{\Pi_n \partial_\alpha S_{\mathbf{X}}^\dagger(\varepsilon) \partial_\beta S_{\mathbf{X}}(\varepsilon)\}_s, \quad (\text{A8})$$

$$\gamma_{\alpha\beta}^{\text{neq}}(\mathbf{X}) = \sum_n \int \frac{d\varepsilon}{2\pi i} f_n(\varepsilon) \text{tr}\{\Pi_n [\partial_\alpha S_{\mathbf{X}}^\dagger(\varepsilon) A_{\mathbf{X}}^\beta(\varepsilon) - A_{\mathbf{X}}^{\beta\dagger}(\varepsilon) \partial_\alpha S_{\mathbf{X}}(\varepsilon)]\}_s. \quad (\text{A9})$$

The white-noise fluctuating force correlator is given by

$$D_{\alpha\beta}(\mathbf{X}) = \int \frac{d\varepsilon}{2\pi} \sum_{k,m} f_m(\varepsilon) [1 \mp f_k(\varepsilon)] \text{tr}\{\Pi_m [S_{\mathbf{X}}^\dagger(\varepsilon) \partial_\alpha S_{\mathbf{X}}(\varepsilon)]^\dagger \cdot \Pi_k \cdot S_{\mathbf{X}}^\dagger(\varepsilon) \partial_\beta S_{\mathbf{X}}(\varepsilon)\}. \quad (\text{A10})$$

In equilibrium it is connected to the friction coefficient via $D_{\alpha\beta} = 2k_B T \gamma_{\alpha\beta}^{\text{eq}}$, where $\gamma_{\alpha\beta}^{\text{eq}}$ is evaluated in equilibrium. This agrees with the fluctuation-dissipation theorem in Eq. (7) in the classical limit $\omega \ll T$.

APPENDIX B: IDENTIFICATION OF FRICTION AND NOISE WITHIN SCATTERING THEORY

Following, we derive, within the framework of scattering theory, Eqs. (40) and (45)–(47) of the main text, which give the connection of the Loschmidt echo to fluctuations and dissipation. The friction tensor and the noise correlator are given in Eqs. (A8)–(A10). In linear response these quantities read as

$$(\boldsymbol{\gamma}_0^{\text{eq}})_{\alpha\beta} = \frac{1}{4\pi} \text{tr}\{\partial_\alpha S_t^\dagger(\mu) \partial_\beta S_t(\mu)\}_s, \quad (\text{B1})$$

$$(\boldsymbol{\gamma}_1^{\text{eq}})_{\alpha\beta} = \frac{\Delta\mu}{8\pi} \{\partial_\varepsilon [(\partial_\alpha S_t^\dagger(\varepsilon) \partial_\beta S_t(\varepsilon))_{LL} - (\partial_\alpha S_t^\dagger(\varepsilon) \partial_\beta S_t(\varepsilon))_{RR}]_{\varepsilon=\mu}\}_s, \quad (\text{B2})$$

$$(\boldsymbol{\gamma}_1^{\text{neq}})_{\alpha\beta} = \frac{\Delta\mu}{4\pi i} \{(\partial_\alpha S_t^\dagger(\mu) A_t^\beta(\mu) - A_t^{\beta\dagger}(\mu) \partial_\alpha S_t(\mu))_{LL} - (\partial_\alpha S_t^\dagger(\mu) A_t^\beta(\mu) - A_t^{\beta\dagger}(\mu) \partial_\alpha S_t(\mu))_{RR}\}_s. \quad (\text{B3})$$

We begin by expressing the function $K_{kn}^{\alpha\beta}(\varepsilon)$, defined in the main text in Eq. (36), in terms of the S matrix. By twice inserting the complete set of advanced scattering states $1 = \int \frac{d\varepsilon}{2\pi} \sum_m |\psi_m^-(\varepsilon)\rangle \langle \psi_m^-(\varepsilon)|$ we conclude from Eqs. (35) and (36) that

$$K_{kn}^{\alpha\beta}(\varepsilon) = \{ \langle \psi_k^{\mathbf{X}^+}(\varepsilon) | \partial_\alpha V_{\mathbf{X}} | \psi_n^{\mathbf{X}^+}(\varepsilon) \rangle \langle \psi_n^{\mathbf{X}^+}(\varepsilon) | \partial_\beta V_{\mathbf{X}} | \psi_k^{\mathbf{X}^+}(\varepsilon) \rangle \}_s = \{ (\partial_\alpha S_t^\dagger(\varepsilon) S_t(\varepsilon))_{kn} (S_t^\dagger(\varepsilon) \partial_\beta S_t(\varepsilon))_{nk} \}_s. \quad (\text{B4})$$

Because of the unitarity of the scattering matrix, it readily follows that

$$\frac{1}{4\pi} \sum_{kn} K_{kn}^{\alpha\beta}(\mu) = \frac{1}{4\pi} \text{tr}\{\partial_\alpha S_t^\dagger(\mu) \partial_\beta S_t(\mu)\}_s = (\boldsymbol{\gamma}_0^{\text{eq}})_{\alpha\beta} \quad (\text{B5})$$

at $T \rightarrow 0$ by referring to Eq. (B1). We thus have proven Eq. (40). We continue with the derivation of Eq. (45). Hereto we note that $\partial_\varepsilon^s K_{kn}^{\alpha\beta}(\varepsilon) = \frac{1}{2} \partial_\varepsilon K_{kn}^{\alpha\beta}(\varepsilon)$ and $\partial_\varepsilon^s K_{kn}^{\alpha\beta}(\varepsilon) = \partial_\varepsilon^s K_{nk}^{\alpha\beta}(\varepsilon)$. The latter property follows immediately from the relation $K_{kn}^{\alpha\beta}(\varepsilon) = K_{nk}^{\alpha\beta}(\varepsilon)$ and due to the symmetric summation in the indices α and β . Hence, we get

$$\begin{aligned} \frac{\Delta\mu}{4\pi} [\partial_\varepsilon^s K_{LL}^{\alpha\beta}(\mu) - \partial_\varepsilon^s K_{RR}^{\alpha\beta}(\mu)] &= \frac{\Delta\mu}{8\pi} \sum_{n=L,R} \partial_\varepsilon [K_{Ln}^{\alpha\beta}(\varepsilon) - K_{Rn}^{\alpha\beta}(\varepsilon)]_{\varepsilon=\mu} \\ &= \frac{\Delta\mu}{8\pi} \partial_\varepsilon \{ [(\partial_\alpha S_t^\dagger(\varepsilon) \partial_\beta S_t(\varepsilon))_{LL} - (\partial_\alpha S_t^\dagger(\varepsilon) \partial_\beta S_t(\varepsilon))_{RR}]_{\varepsilon=\mu} \}_s = (\boldsymbol{\gamma}_1^{\text{eq}})_{\alpha\beta} \end{aligned} \quad (\text{B6})$$

by referring to Eq. (B2) in the last step which proves Eq. (45). In order to derive the relation in Eq. (46), we first observe that $\partial_\varepsilon^a K_{kn}^{\alpha\beta}(\varepsilon)$ can be written in terms of the S and A matrices. Again, inserting two complete sets of advanced scattering states, we find

$$\begin{aligned} \partial_\varepsilon^a K_{kn}^{\alpha\beta}(\varepsilon) &= \frac{1}{2} (\partial_\varepsilon - \partial_{\varepsilon'}) K_{kn}^{\alpha\beta}(\varepsilon, \varepsilon') \Big|_{\varepsilon'=\varepsilon} = \frac{1}{2} \{ (\partial_\varepsilon \langle \psi_k^{\mathbf{X}^+}(\varepsilon) | \partial_\alpha V_{\mathbf{X}} | \psi_n^{\mathbf{X}^+}(\varepsilon) \rangle \langle \psi_n^{\mathbf{X}^+}(\varepsilon) | \partial_\beta V_{\mathbf{X}} | \psi_k^{\mathbf{X}^+}(\varepsilon) \rangle \\ &\quad + \langle \psi_k^{\mathbf{X}^+}(\varepsilon) | \partial_\alpha V_{\mathbf{X}} | \psi_n^{\mathbf{X}^+}(\varepsilon) \rangle \langle \psi_n^{\mathbf{X}^+}(\varepsilon) | \partial_\beta V_{\mathbf{X}} | \partial_\varepsilon \psi_k^{\mathbf{X}^+}(\varepsilon) \rangle - \langle \psi_k^{\mathbf{X}^+}(\varepsilon) | \partial_\alpha V_{\mathbf{X}} \partial_\varepsilon [| \psi_n^{\mathbf{X}^+}(\varepsilon) \rangle \langle \psi_n^{\mathbf{X}^+}(\varepsilon) |] \partial_\beta V_{\mathbf{X}} | \psi_k^{\mathbf{X}^+}(\varepsilon) \rangle \}_s \\ &= - \frac{1}{2} \sum_{ml} \{ \partial_\alpha S_t^{\dagger, km}(\varepsilon) \partial_\varepsilon [S_t^{mn}(\varepsilon) S_t^{\dagger, nl}(\varepsilon)] \partial_\beta S_t^{lk}(\varepsilon) \}_s - \frac{1}{i} \sum_{ml} \{ (\partial_\alpha S_t^\dagger(\varepsilon) S_t(\varepsilon))_{kn} (S_t^\dagger(\varepsilon) A_t^\beta(\varepsilon))_{nk} \\ &\quad - (A_t^{\beta\dagger}(\varepsilon) S_t(\varepsilon))_{kn} (S_t^\dagger(\varepsilon) \partial_\alpha S_t(\varepsilon))_{nk} \}_s. \end{aligned} \quad (\text{B7})$$

In order to identify the pure nonequilibrium friction tensor in linear response [cf. Eq. (B3)], we observe that $\partial_\varepsilon^a K_{kn}^{\alpha\beta}(\varepsilon) = -\partial_\varepsilon^a K_{nk}^{\alpha\beta}(\varepsilon)$, where we stress that $\partial_\varepsilon^a K_{nn}^{\alpha\beta}(\varepsilon) = 0$. Restricting to two leads $k, n = L, R$ we thus conclude

$$\begin{aligned} \frac{\Delta\mu}{4\pi} [\partial_\varepsilon^a K_{RL}^{\alpha\beta}(\mu) - \partial_\varepsilon^a K_{LR}^{\alpha\beta}(\mu)] &= \frac{\Delta\mu}{4\pi} \sum_{n=L,R} [\partial_\varepsilon^a K_{Rn}^{\alpha\beta}(\mu) - \partial_\varepsilon^a K_{Ln}^{\alpha\beta}(\mu)] \\ &= \frac{\Delta\mu}{4\pi i} \{ (\partial_\alpha S_t^\dagger(\mu) A_t^\beta(\mu) - A_t^{\beta\dagger}(\mu) \partial_\alpha S_t(\mu))_{LL} - (\partial_\alpha S_t^\dagger(\mu) A_t^\beta(\mu) - A_t^{\beta\dagger}(\mu) \partial_\alpha S_t(\mu))_{RR} \}_s \\ &= (\boldsymbol{\gamma}_1^{\text{neq}})_{\alpha\beta}. \end{aligned} \quad (\text{B8})$$

Here, we made use of $\text{tr}\{\partial_\alpha S_t^\dagger A_t^\beta - A_t^{\beta\dagger} \partial_\alpha S_t\} = 0$ to realize that the equilibrium contribution in $\boldsymbol{\gamma}_{\alpha\beta}^{\text{neq}}$ vanishes. We conclude with Eq. (46). Finally, we show Eq. (47). For two leads, we write the correlator of the fluctuating force as [36,40]

$$D_{\alpha\beta}(\mathbf{X}) = \int \frac{d\varepsilon}{2\pi} \sum_{k,m=L,R} f_m(\varepsilon)[1 - f_k(\varepsilon)] K_{mk}^{\alpha\beta}(\varepsilon). \quad (\text{B9})$$

Making use of $f_k(\varepsilon)[1 - f_k(\varepsilon)] = -T \partial_\varepsilon f_k(\varepsilon) = T \delta(\varepsilon - \mu_k)$ for small T , we can express the correlator in linear response to first order in the applied bias voltage as

$$D_{\alpha\beta}(\mathbf{X}) = \frac{T}{2\pi} \left(\sum_{k,m=L,R} K_{mk}^{\alpha\beta}(\mu) + \frac{\Delta\mu}{2} [\partial_\varepsilon K_{LL}^{\alpha\beta}(\varepsilon) - \partial_\varepsilon K_{RR}^{\alpha\beta}(\varepsilon)]_{\varepsilon=\mu} \right) + \frac{\Delta\mu}{4\pi} [K_{LR}^{\alpha\beta}(\mu) + K_{RL}^{\alpha\beta}(\mu)] \quad (\text{B10})$$

since $K_{km}^{\alpha\beta}(\varepsilon) = K_{mk}^{\alpha\beta}(\varepsilon)$. With $\frac{1}{2}\partial_\varepsilon = \partial_\varepsilon^s$ we identify the equilibrium friction tensor in Eq. (B1) and its nonequilibrium correction in Eq. (B2) in the above expression for the correlator. For $T \rightarrow 0$ we then obtain Eq. (47).

APPENDIX C: ALTERNATIVE KELDYSH APPROACH

This Appendix presents an alternative derivation of the results in the main text by using Keldysh Green's functions technique. We consider the generic Hamiltonian given in Eq. (54) and we define the *time-dependent* Green's functions of the dot

$$\mathcal{G}_{mm'}^R(t, t') = -i\theta(t - t') \langle \{d_m(t), d_{m'}^\dagger(t')\} \rangle, \quad (\text{C1})$$

$$\mathcal{G}_{mm'}^A(t, t') = i\theta(t' - t) \langle \{d_m(t), d_{m'}^\dagger(t')\} \rangle, \quad (\text{C2})$$

$$\mathcal{G}_{mm'}^>(t, t') = -i \langle d_m(t) d_{m'}^\dagger(t') \rangle, \quad (\text{C3})$$

$$\mathcal{G}_{mm'}^<(t, t') = i \langle d_{m'}^\dagger(t') d_m(t) \rangle, \quad (\text{C4})$$

where the curly brackets $\{\dots, \dots\}$ denote the anticommutator operation. We assume stationary states throughout this section so that the above-defined Green's functions depend on the difference of the time arguments. With these definitions the noise correlator defined in Eq. (2) reads as [35,40]

$$D_{\alpha\beta}(t - t') = \int \frac{d\varepsilon}{2\pi} \int \frac{d\varepsilon'}{2\pi} e^{i(\varepsilon - \varepsilon')(t - t')} \text{tr}\{\Lambda_\alpha \mathcal{G}^>(\varepsilon) \Lambda_\beta \mathcal{G}^<(\varepsilon')\}_s \quad (\text{C5})$$

with $\Lambda_\alpha = \partial_{X_\alpha} h_0(\mathbf{X})$. The functions $\mathcal{G}^>(\varepsilon)$ and $\mathcal{G}^<(\varepsilon')$ represent the Fourier transforms of the greater and lesser functions. It is sufficient to evaluate the noise correlator in the adiabatic limit as this already guarantees that the fluctuation-dissipation theorem is fulfilled. Hereto we introduce the Fourier transform of the adiabatic lesser and greater Green's functions $G^>(\varepsilon)$ and $G^<(\varepsilon)$ with respect to a frozen configuration \mathbf{X} and conclude for the Fourier transform of the fluctuating force

$$D_{\alpha\beta}(\omega) = \int \frac{d\varepsilon}{2\pi} \text{tr}\left\{ \Lambda_\alpha G^>\left(\varepsilon - \frac{\omega}{2}\right) \Lambda_\beta G^<\left(\varepsilon + \frac{\omega}{2}\right) \right\}_s. \quad (\text{C6})$$

Analogously, we introduce the adiabatic retarded and advanced Green's functions. Using Langreth rule, we can express the greater and the lesser Green's function, respectively, as

$$G^<(\varepsilon) = G^R(\varepsilon) \Sigma^<(\varepsilon) G^A(\varepsilon), \quad (\text{C7})$$

$$G^>(\varepsilon) = G^R(\varepsilon) \Sigma^>(\varepsilon) G^A(\varepsilon) \quad (\text{C8})$$

with greater and lesser self-energies

$$\Sigma^<(\varepsilon) = i \sum_k f_k(\varepsilon) W^\dagger(\varepsilon) \Pi_k(\varepsilon) W(\varepsilon), \quad (\text{C9})$$

$$\Sigma^>(\varepsilon) = -i \sum_k [1 - f_k(\varepsilon)] W^\dagger(\varepsilon) \Pi_k(\varepsilon) W(\varepsilon), \quad (\text{C10})$$

where $\Pi_k(\varepsilon) = |\phi_k(\varepsilon)\rangle\langle\phi_k(\varepsilon)|$ is a projector onto lead space and the coupling matrix W is defined via the Hamiltonian in Eq. (58).

Plugging Eqs. (C7) and (C8) into Eq. (C6) we can write the noise correlator as

$$D_{\alpha\beta}(\omega) = \int \frac{d\varepsilon}{2\pi} \sum_{kn} f_k\left(\varepsilon - \frac{\omega}{2}\right) \left[1 - f_n\left(\varepsilon + \frac{\omega}{2}\right)\right] \tilde{K}_{kn}^{\alpha\beta}\left(\varepsilon - \frac{\omega}{2}, \varepsilon + \frac{\omega}{2}\right), \quad (\text{C11})$$

where we defined the function $\tilde{K}_{kn}^{\alpha\beta}(\varepsilon, \varepsilon')$ as

$$\tilde{K}_{kn}^{\alpha\beta}(\varepsilon, \varepsilon') = \text{tr}\{\Lambda_\alpha G^R(\varepsilon) W^\dagger(\varepsilon) \Pi_k(\varepsilon) W(\varepsilon) G^A(\varepsilon) \Lambda_\beta G^R(\varepsilon') W^\dagger(\varepsilon') \Pi_n(\varepsilon') W(\varepsilon') G^A(\varepsilon')\}_s. \quad (\text{C12})$$

The expression in Eq. (C11) has the same structure as the expression in the main text in Eq. (38). Here, we proceed with the analogous steps to get to the result for the noise correlator in Eqs. (49) and (50) with all its consequences for the Loschmidt echo and the fidelity amplitude. In order to show their equivalence, we are thus left with identifying the coefficients

$$(\mathcal{Y}_0^{\text{eq}})_{\alpha\beta} = \frac{1}{4\pi} \sum_{kn} \tilde{K}_{kn}^{\alpha\beta}(\mu), \quad (\text{C13})$$

$$(\mathcal{Y}_1^{\text{eq}})_{\alpha\beta} = \frac{\Delta\mu}{4\pi} (\partial_\varepsilon^s \tilde{K}_{LL}^{\alpha\beta}(\mu) + \partial_\varepsilon^s \tilde{K}_{RR}^{\alpha\beta}(\mu)), \quad (\text{C14})$$

$$(\mathcal{Y}_1^{\text{neq}})_{\alpha\beta} = \frac{\Delta\mu}{4\pi} (\partial_\varepsilon^a \tilde{K}_{RL}^{\alpha\beta}(\mu) - \partial_\varepsilon^a \tilde{K}_{LR}^{\alpha\beta}(\mu)), \quad (\text{C15})$$

$$\left(\frac{\mathbf{D}}{\Delta\mu}\right)_{\alpha\beta} = \frac{1}{4\pi} (\tilde{K}_{LR}^{\alpha\beta}(\mu) + \tilde{K}_{RL}^{\alpha\beta}(\mu)) \quad (\text{C16})$$

for two leads under symmetric summation with respect to the indices α and β in linear response at zero temperature [cf. Eqs. (40) and (45)–(47)]. We defined $\tilde{K}_{kn}^{\alpha\beta}(\varepsilon) = \tilde{K}_{kn}^{\alpha\beta}(\varepsilon, \varepsilon)$.

We begin with the identification of the friction tensor. We take the expression of the friction tensor in terms of the adiabatic Green's functions from Refs. [35,40]:

$$\gamma_{\alpha\beta} = \int \frac{d\varepsilon}{2\pi} \text{tr}\{\Lambda_\alpha G^>(\varepsilon) \Lambda_\beta \partial_\varepsilon G^<(\varepsilon)\}_s. \quad (\text{C17})$$

We immediately conclude for the friction tensor in equilibrium that

$$(\mathcal{Y}_0^{\text{eq}})_{\alpha\beta} = \int \frac{d\varepsilon}{4\pi} \sum_{kn=L,R} \text{tr}\{\Lambda_\alpha G^R(\varepsilon) W^\dagger(\mu) \Pi_k(\mu) W(\mu) G^A(\mu) \Lambda_\beta G^R(\mu) W^\dagger(\mu) \Pi_n(\mu) W(\mu) G^A(\mu)\}_s \quad (\text{C18})$$

since $f_k(\varepsilon)[1 - f_k(\varepsilon)] = -T \partial_\varepsilon f_k(\varepsilon) = 0$ for $T \rightarrow 0$ as well as $f(\mu) = \frac{1}{2}$ and $-\partial_\varepsilon f(\varepsilon) = \delta(\varepsilon - \mu)$. A comparison to the definition in Eq. (C12) readily results in Eq. (C13).

Next, we address Eqs. (C14) and (C15). Hereto we write the Fermi functions of the left and right leads as $f_{L/R}(\varepsilon) = f(\varepsilon) \mp \frac{\Delta\mu}{2} \partial_\varepsilon f(\varepsilon)$. Keeping only terms linear in $\Delta\mu$ we conclude after performing an integration by parts

$$\begin{aligned} (\mathcal{Y}_1^{\text{eq}} + \mathcal{Y}_1^{\text{neq}})_{\alpha\beta} &= \frac{\Delta\mu}{8\pi} \partial_\varepsilon (\tilde{K}_{LL}^{\alpha\beta}(\mu, \varepsilon) + \tilde{K}_{LR}^{\alpha\beta}(\varepsilon, \mu))_{\varepsilon=\mu} - \frac{\Delta\mu}{8\pi} \partial_\varepsilon (\tilde{K}_{RL}^{\alpha\beta}(\mu, \varepsilon) + \tilde{K}_{RR}^{\alpha\beta}(\varepsilon, \mu))_{\varepsilon=\mu} \\ &+ \frac{\Delta\mu}{8\pi} \partial_\varepsilon (\tilde{K}_{LL}^{\alpha\beta}(\mu, \varepsilon) + \tilde{K}_{RL}^{\alpha\beta}(\varepsilon, \mu))_{\varepsilon=\mu} - \frac{\Delta\mu}{8\pi} \partial_\varepsilon (\tilde{K}_{LR}^{\alpha\beta}(\mu, \varepsilon) + \tilde{K}_{RR}^{\alpha\beta}(\varepsilon, \mu))_{\varepsilon=\mu} \\ &+ \int \frac{d\varepsilon}{2\pi} [\partial_\varepsilon f(\varepsilon)]^2 [\tilde{K}_{LL}^{\alpha\beta}(\varepsilon) + \tilde{K}_{LR}^{\alpha\beta}(\varepsilon) - \tilde{K}_{RL}^{\alpha\beta}(\varepsilon) - \tilde{K}_{RR}^{\alpha\beta}(\varepsilon) - \tilde{K}_{LL}^{\alpha\beta}(\varepsilon) - \tilde{K}_{RL}^{\alpha\beta}(\varepsilon) + \tilde{K}_{LR}^{\alpha\beta}(\varepsilon) + \tilde{K}_{RR}^{\alpha\beta}(\varepsilon)]. \end{aligned} \quad (\text{C19})$$

The last terms vanish due to the symmetric summation over α and β and since $\tilde{K}_{kn}^{\alpha\beta}(\varepsilon) = \tilde{K}_{nk}^{\alpha\beta}(\varepsilon)$. This yields Eqs. (C14) and (C15).

Finally, we identify the delta-correlated noise and prove Eq. (C16) in linear response. We rely on the expression

$$D_{\alpha\beta}(\omega) = \int \frac{d\varepsilon}{2\pi} \text{tr}\{\Lambda_\alpha G^>(\varepsilon) \Lambda_\beta G^<(\varepsilon)\}_s \quad (\text{C20})$$

obtained in the literature [35,40] in terms of the Green's functions of the dot. Using Eqs. (C7) and (C8) we immediately identify

$$D_{\alpha\beta}(\omega) = \int \frac{d\varepsilon}{2\pi} \sum_{kn=L,R} f_k(\varepsilon)[1 - f_n(\varepsilon)] \tilde{K}_{LR}^{\alpha\beta}(\varepsilon). \quad (\text{C21})$$

To linear response we find Eq. (C16) by using $\tilde{K}_{kn}^{\alpha\beta}(\varepsilon) = \tilde{K}_{nk}^{\alpha\beta}(\varepsilon)$ and the above-stated relations for the Fermi functions.

We end this appendix by showing the direct equivalence between the function $\tilde{K}_{kn}^{\alpha\beta}(\varepsilon, \varepsilon')$ defined in Eq. (C12) and $K_{kn}^{\alpha\beta}(\varepsilon, \varepsilon')$ defined in Eq. (36), that is,

$$K_{kn}^{\alpha\beta}(\varepsilon, \varepsilon') = \{ \langle \psi_k^{\text{X}^+}(\varepsilon) | \partial_\alpha H_{\text{X}} | \psi_n^{\text{X}^+}(\varepsilon') \rangle \langle \psi_n^{\text{X}^+}(\varepsilon') | \partial_\beta H_{\text{X}} | \psi_k^{\text{X}^+}(\varepsilon) \rangle \}_s \quad (\text{C22})$$

since $\partial_\alpha H_{\text{X}} = \partial_\alpha V_{\text{X}}$. We note that we can relate the Green's function of the dot and the scattering states via the Lippmann-Schwinger equation [36]

$$\Pi_D | \psi_\eta^{\text{X}^+}(\varepsilon) \rangle = G^R(\varepsilon) W^\dagger | \phi_\eta(\varepsilon) \rangle, \quad (\text{C23})$$

where Π_D denotes a projector onto the space of the dot. With the aid of $\partial_\alpha H_{\text{X}} = \Pi_D \partial_\alpha h_0(\text{X}) \Pi_D$ and the Lippmann-Schwinger equation projected onto the dot's space in Eq. (C23) we have

$$K_{kn}^{\alpha\beta}(\varepsilon, \varepsilon') = \{ \langle \phi_k(\varepsilon) | W(\varepsilon) G^A(\varepsilon) \partial_\alpha H_{\text{X}} G^R(\varepsilon') W^\dagger(\varepsilon') | \phi_n(\varepsilon') \rangle \langle \phi_n(\varepsilon') | W(\varepsilon') G^A(\varepsilon') \partial_\beta H_{\text{X}} G^R(\varepsilon) W^\dagger(\varepsilon) | \phi_k(\varepsilon) \rangle \}_s. \quad (\text{C24})$$

Due to symmetric summation under exchanging the indices α and β and using that $\Pi_n(\varepsilon') = | \phi_n(\varepsilon') \rangle \langle \phi_n(\varepsilon') |$, we conclude that

$$\begin{aligned} K_{kn}^{\alpha\beta}(\varepsilon, \varepsilon') &= \text{tr} \{ \partial_\alpha H_{\text{X}} G^R(\varepsilon') W^\dagger(\varepsilon') \Pi_n(\varepsilon') W(\varepsilon') G^A(\varepsilon') \partial_\beta H_{\text{X}} G^R(\varepsilon) W^\dagger(\varepsilon) \Pi_k(\varepsilon) W(\varepsilon) G^A(\varepsilon) \}_s \\ &= \tilde{K}_{kn}^{\alpha\beta}(\varepsilon, \varepsilon'). \end{aligned} \quad (\text{C25})$$

This shows explicitly the equivalence to the scattering states approach.

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