Disorder-induced resistive anomaly near ferromagnetic phase transitions

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We show that the resistivity $\rho(T)$ of disordered ferromagnets near the Curie temperature T_c generically exhibits a stronger anomaly than the scaling-based Fisher-Langer prediction. Treating transport beyond the Boltzmann description, we find that within mean-field theory, $d\rho/dT$ exhibits a $|T - T_c|^{-1/2}$-singularity near T_c. Our results, being solely due to impurities, are relevant to ferromagnets with low T_c, such as $SrRuO_3$ or diluted magnetic semiconductors, whose mobility near T_c is limited by disorder.

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Introduction.—It was first observed by Gerlach [1] that the resistivity of itinerant ferromagnets exhibits an anomalous temperature dependence in the vicinity of the Curie temperature T_c. This feature was later reproduced with much higher experimental accuracy by Craig *et al.* [2] as well as others [3]. Dating back to the seminal works of de Gennes and Friedel [4] as well as Fisher and Langer [5], this resistive anomaly is conventionally explained in terms of *coherent* scattering of carriers by large blocks of spins whose size is determined by the magnetic correlation length $\xi(T)$. As the temperature approaches T_c, $\xi(T)$ diverges, making the scattering more efficient.

De Gennes and Friedel [4] studied the resistive anomaly of ferromagnets within mean-field theory. They argue that due to critical slowing down, spin fluctuations can be treated as effectively static. The ensuing result for the coherent transport scattering rate from spin fluctuations above T_c is succinctly summarized by [4]

$$\frac{\tau_0}{\tau} = \frac{1}{4}\int dx\, \frac{x^3}{t + (k_F a x)^2}. \qquad (1)$$

Here, the scattering rate is normalized to the rate $1/\tau_0$ for incoherent scattering. The denominator of the integrand is the conventional mean-field (Ornstein-Zernike) correlator for the spin fluctuations, with $t = (T - T_c)/T_c$ denoting the reduced temperature. a is a microscopic length of the order of the lattice constant. The integration variable x denotes the transferred momentum in units of the Fermi wavevector k_F. The numerator of the integrand incorporates the usual factor $1 - \cos\theta$ (with θ the scattering angle) in the transport scattering rate. Performing the integral in Eq. (1) yields $\tau_0/\tau(t) = \tau_0/\tau(0) + (1/8(k_F a)^4)\, t \ln t$ which transforms into a singularity of the resistivity of the form $\rho(T) = \rho_0 - bt \ln(1/t)$ with $b > 0$ when approaching T_c from above.

Fisher and Langer [5] noticed that this singularity emerges from the lower limit of the integral in Eq. (1) while the body of the integrand is dominated by large wavevectors. Within the mean-field description, the large-wavevector behavior of the spin-spin correlator is non-singular. The central assertion of Ref. [5] is that there exists a *singular* contribution to this correlator at large wavevectors when going beyond the mean-field approximation, with the singularity governed by anomalous dimensions. Fisher and Langer conclude that while below T_c, the predictions of Ref. [4] are essentially correct, there is *no singularity within mean-field approximation* when approaching T_c from above.

This conclusion rests on their important physical observation that Eq. (1) becomes inapplicable for $t \ll (\ell/a)^2$, or equivalently, when the correlation length $\xi(T)$ exceeds the mean free path ℓ (due to phonon or impurity scattering). The reason is that the carriers can be viewed as plane waves only over distances shorter than ℓ and thus, they are no longer susceptible to the order in the spin configuration beyond ℓ.

Later, the ideas of Refs. [4, 5] were extended to include realistic features of ferromagnets [6] and to the critical behavior of other quantities such as the spin-flip scattering rate [7, 8, 9]. The effect of a finite mean free path on the resistivity was studied by a number approaches, ranging from replacing the δ-function in the Golden rule by a Lorentzian [8] to smearing the Ornstein-Zernike correlator in order to eliminate the pole [10].

All of these approaches are based on the Boltzmann-equation formalism. The prime message of this paper is that in the small-t limit, when the correlation length $\xi(T)$ exceeds the mean free path ℓ, the Boltzmann approach *fails*. The reason for this is that for $\xi(T) \gg \ell$, the smooth variations of the magnetization (on the scale $\xi(T)$) are "explored" by *diffusing* carriers. By contrast, the Boltzmann approach prescribes to treat scattering from both short-range impurities and the smooth variations of the magnetization on equal footing, i.e., to add their partial scattering rates. Thus, a theory of the resistive anomaly at small t requires one to go beyond the Boltzmann approach.

The consequences of the inapplicability of the Boltzmann description are drastic. In fact, as demonstrated below, instead of smearing the resistive anomaly, impurities cause a much stronger singularity, even within mean-field theory. More quantitatively, we find $d^2\rho/dT^2 \sim$

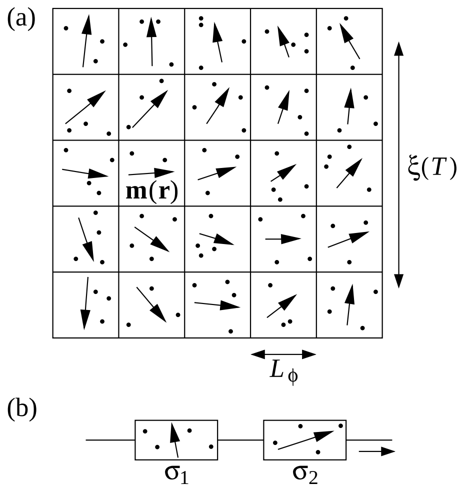

FIG. 1: (a) Schematic representation of resistor network describing disordered ferromagnets when the correlation length $\xi(T)$ exceeds the phase-coherence length L_ϕ. Each block of size L_ϕ constitutes a resistor with coarse-grained magnetization (arrows) and random impurities (dots). (b) Minimal model illustrating the importance of performing the impurity and thermal averages at the last step of the calculation.

$t^{-3/2}$ sufficiently close to T_c, as opposed to $d^2\rho/dT^2 \sim t^{-1}$ for $\xi(T) \ll \ell$ [4].

Our reasoning goes as follows. The adequate description of electric transport on scales larger than the phase-breaking length L_ϕ is a network of resistors, made up of cubes of size L_ϕ, as illustrated in Fig. 1(a). This network is inhomogeneous due to the different spin and disorder configurations in each cube. Contrary to the Boltzmann prescription, it is essential to first compute the effective resistivity of the inhomogeneous network and to perform the disorder and thermal averages only as the last step of the calculation. Then, correlations between distant spins affect the resistance through the inhomogeneous current and field distribution over the network, leading to a true singularity for $t \to 0$ even within mean-field theory.

To illustrate the importance of performing the impurity and thermal averages at the last step of the calculation, consider a minimal model of two macroscopic resistors in sequence (see Fig. 1(b)). These resistors differ in both impurity and spin configurations. Within the Boltzmann approach, both resistances become equal to $1/\overline{\sigma}$ upon configurational and thermal averaging, yielding a total resistance of $2/\overline{\sigma}$. However, for the *actual* distribution of impurities and spins, their conductivities differ, so that $\sigma_1 = \overline{\sigma} + \delta\sigma/2$ while $\sigma_2 = \overline{\sigma} - \delta\sigma/2$. Then, the effective resistance becomes $(\sigma_1 + \sigma_2)/\sigma_1\sigma_2 \simeq 2/\overline{\sigma} + \delta\sigma^2/2\overline{\sigma}^3$ (assuming $\delta\sigma \ll \overline{\sigma}$). This involves an *additional* term $\delta\sigma^2/2\overline{\sigma}^3$ with non-zero average.

Effective conductivity of an inhomogeneous medium.— Remarkably, the effective resistivity $\rho_{\rm eff} = 1/\sigma_{\rm eff}$ can be computed for an arbitrary realization $\sigma(\mathbf{r})$ of the local conductivity, provided that the relative variation in $\sigma(\mathbf{r})$ is weak [11]. To see this, we decompose the current, the conductivity, and the electric field into averages $\mathbf{j}_0$, σ_0, and $\mathbf{E}_0$ and spatially fluctuating contributions $\delta\mathbf{j}(\mathbf{r})$, $\delta\sigma(\mathbf{r})$, and $\delta\mathbf{E}(\mathbf{r})$. From Ohm's law, we have

$$\mathbf{j}_0 = \sigma_0\mathbf{E}_0 + \langle\delta\sigma(\mathbf{r})\delta\mathbf{E}(\mathbf{r})\rangle, \quad (2)$$

$$\delta\mathbf{j}(\mathbf{r}) = \delta\sigma(\mathbf{r})\mathbf{E}_0 + \sigma_0\delta\mathbf{E}(\mathbf{r}). \quad (3)$$

Here, the brackets denote a spatial average. Combining the continuity equation $\boldsymbol{\nabla}\cdot\delta\mathbf{j} = 0$ and Maxwell's equation $\boldsymbol{\nabla}\times\delta\mathbf{E} = 0$ with Eq. (3), one obtains $\delta\mathbf{E}(\mathbf{q}) = -\hat{\mathbf{q}}\,\hat{\mathbf{q}}\cdot\mathbf{E}\delta\sigma(\mathbf{q})/\sigma_0$, where $\hat{\mathbf{q}}$ denotes the unit vector in the direction of the wavevector $\mathbf{q}$. Inserting this into Eq. (2), one finds for the effective macroscopic conductivity $\sigma_{\rm eff}$ (defined by $\mathbf{j}_0 = \sigma_{\rm eff}\mathbf{E}_0$) in three dimensions

$$\sigma_{\rm eff} = \sigma_0 - \frac{\langle[\delta\sigma(\mathbf{r})]^2\rangle}{3\sigma_0}. \quad (4)$$

Remarkably, this result is independent of the geometry of the conductivity variations. To illustrate this, consider a sample consisting of a 50-50 random mixture of domains with conductivities σ_1 and σ_2, where $|\sigma_1 - \sigma_2| \ll \sigma_1, \sigma_2$. In this case, Eq. (4) yields $\sigma_{\rm eff} = \sigma_0 - (\sigma_1 - \sigma_2)^2/12\sigma_0$ (here, $\sigma_0 = (\sigma_1 + \sigma_2)/2$), which is completely independent of the arrangement of domains. It is interesting to remark that in two dimensions, one can find an exact, geometry-independent expression for such two-phase systems which is valid for arbitrarily large inhomogeneities $|\sigma_2 - \sigma_1|/(\sigma_1 + \sigma_2)$ [12].

Effective conductivity due to spin fluctuations.—The conductance of a phase-coherent sample exhibits random, sample-specific but reproducible variations as a function of external parameters such as the Fermi energy E_F. These conductance fluctuations arise due to interference between different elastic scattering paths of a carrier that diffuses through the sample [13]. Thus, the conductance $g(\mathbf{r}, E_F)$ varies from block to block because of their different impurity configurations. The fluctuations in the conductivity entering Eq. (4) can then be expressed as

$$\delta\sigma(\mathbf{r}) = \delta g(\mathbf{r}, E_F; \mathbf{m}(\mathbf{r}))/L_\phi. \quad (5)$$

and the spatial average in Eq. (4) can be replaced by an average over impurity configurations with subsequent thermal average. Note that in this way, the variance of the conductivity on the rhs of Eq. (4) reduces to a much-studied quantity describing the properties of the universal conductance fluctuations [13].

Two spin subbands.—In our case, the external parameter is a vector, namely the magnetization $\mathbf{m}(\mathbf{r})$. To proceed, we first consider the simplest case in which the dominant effect of the impurity spins on the carriers is an effective Zeeman field arising from the exchange interaction, which is proportional to the magnetization $m(\mathbf{r})$. Then, the *coarse-grained* magnetization $\mathbf{m}(\mathbf{r})$ which is

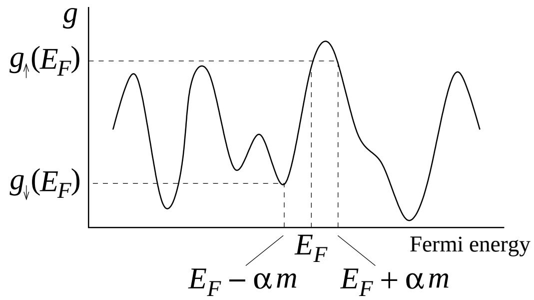

FIG. 2: Sample-specific variation of $g(E_F)$ for a block of the resistor network (universal conductance fluctuations) in the absence of spin. The conductances for each spin direction are obtained by including equal, but opposite exchange-induced Zeeman shifts of the Fermi energy. As indicated, this leads to a difference in the conductances for the two spin directions.

averaged over each cube of size L_ϕ, can be incorporated via equal, but opposite energy shifts $\pm\alpha m(\mathbf{r})$ for spin-up and spin-down carriers, and $\delta\sigma$ becomes a sum of contributions from the two spin projections,

$$\begin{aligned}\delta\sigma(\mathbf{r}) &= [\delta g_\uparrow(\mathbf{r},E_F)+\delta g_\downarrow(\mathbf{r},E_F)]/L_\phi\\ &= [\delta g(\mathbf{r},E_F^+)+\delta g(\mathbf{r},E_F^-)]/L_\phi. \end{aligned}\qquad (6)$$

Here, $g(\mathbf{r},E_F)$ is the *spinless* conductance of the cube at position $\mathbf{r}$ and we introduced the shifted Fermi energies $E_F^\pm = E_F \pm \alpha m$. We used that carriers with both spin projections are scattered from the *same* impurities, see Fig. 2. Substituting Eq. (6) into Eq. (4), we obtain

$$\rho_{\rm eff}-\rho_0 = \frac{2\rho_0^3}{3L_\phi^2}\left\{\langle[\delta g(\mathbf{r},E_F^+)]^2\rangle + \langle[\delta g(\mathbf{r},E_F^-)]^2\rangle \right. \\ \left. +2\langle\delta g(\mathbf{r},E_F+\alpha m)\delta g(\mathbf{r},E_F-\alpha m)\rangle\right\}. \qquad (7)$$

The first two terms on the rhs are magnetization-independent constants (see below). Thus, the t-dependence of $\rho_{\rm eff}$ reduces to the t-dependence of the conductance correlator on the rhs.

It is well known [13] that this correlator is a function $F(x)$ of the dimensionless ratio $x=\alpha m/E_c$, where $E_c = D/L_\phi^2$ is the inverse diffusion time through a block of size L_ϕ (D denotes the diffusion constant, related to σ by the Einstein relation). In three dimensions, the asymptotic behaviors of $F(x)$ are given by [13]

$$F(x) = F(0)\left\{\begin{array}{ll}(1-C_1x^2) & x\ll 1\\ C_2x^{-1/2} & x\gg 1\end{array}\right., \qquad (8)$$

where C_1 and C_2 are constants of order unity. The remaining step is to substitute $F(x)$ into Eq. (7) and to perform the thermodynamic average over $\mathbf{m}$, using the mean-field distribution for $t>0$,

$$\mathcal{P}[\mathbf{m}(\mathbf{r})] \propto \exp\left\{-\frac{c}{2T}\int d\mathbf{r}\left[a^2\nabla_i\mathbf{m}\cdot\nabla_i\mathbf{m}+t\mathbf{m}^2\right]\right\}. \qquad (9)$$

Here, ca^2 is a spin stiffness. Using this distribution, one readily finds the typical amplitude of magnetization fluctuations

$$\langle\mathbf{m}^2(\mathbf{r})\rangle = \int_{q<1/L_\phi}\frac{d\mathbf{q}}{(2\pi)^3}\,\frac{3T_c/c}{t+q^2a^2}. \qquad (10)$$

The restriction in the range of the $\mathbf{q}$ integration accounts for the fact that we are computing fluctuations of the coarse-grained magnetization. The appearance of t in the denominator of Eq. (10) manifests the fact that the *net* strength of fluctuations grows when approaching T_c. This is in contrast to the conventional origin of the t-dependence, namely the spin *correlations*. Performing the integration in Eq. (10), we obtain $\langle\mathbf{m}^2(\mathbf{r})\rangle = (3T_c/2\pi^2ca^2)\,[1/L_\phi - (1/\xi(T))\arctan(\xi(T)/L_\phi)]$ in terms of the mean-field correlation length $\xi(T)=a/\sqrt{t}$.

As a next step, we replace $m(\mathbf{r})$ in Eq. (7) by $\langle\mathbf{m}^2(\mathbf{r})\rangle^{1/2}$. Using the distribution (9), this procedure can be shown to be exact in the limits of small and large x. Expanding the result in the small parameter $L_\phi/\xi(T)$, we readily obtain

$$\begin{aligned}\rho_{\rm eff}-\rho_0 &= \frac{2\rho_0^3}{3L_\phi^2}\left[F(x_0)-F'(x_0)x_0\frac{\pi L_\phi}{4\xi(T)}\right]\\ &= \frac{2\rho_0^3}{3L_\phi^2}\left[F(x_0)-F'(x_0)x_0\frac{\pi L_\phi\sqrt{t}}{4a}\right],\end{aligned} \qquad (11)$$

where $x_0 = (\alpha/E_c)(3T_c/2\pi^2ca^2L_\phi)^{1/2}$. The t-dependent part of $\rho_{\rm eff}$ is given by the second term on the rhs.

So far, our model completely disregards spin-orbit (SO) coupling which is present in the vast majority of ferromagnets. Below, we incorporate SO coupling into the calculation of the effective resistivity.

Spin-orbit coupling.—In the presence of SO coupling, the variance $\langle[\delta g(\mathbf{r},E_F;\mathbf{m}(\mathbf{r}))]^2\rangle_{\rm imp}$ increases by a factor of two when applying a sufficiently strong Zeeman field [14]. In terms of the relevant dimensionless measure of the exchange-induced Zeeman field $y=\alpha m\tau_{\rm so}$ (here $\tau_{\rm so}$ is the SO time), we can then write $\langle[\delta g(\mathbf{r},E_F;\mathbf{m}(\mathbf{r}))]^2\rangle_{\rm imp} = H(y)$ with $H(\infty)/H(0)=2$ [14]. Combining Eqs. (5) and (4), substituting $H(y)$, and proceeding as in the derivation of Eq. (11), we obtain

$$\begin{aligned}\rho_{\rm eff}-\rho_0 &= \frac{2\rho_0^3}{3L_\phi^2}\left[H(y_0)-H'(y_0)y_0\frac{\pi L_\phi}{4\xi(T)}\right]\\ &= \frac{2\rho_0^3}{3L_\phi^2}\left[H(y_0)-H'(y_0)y_0\frac{\pi L_\phi\sqrt{t}}{4a}\right],\end{aligned} \qquad (12)$$

where $y_0=\alpha\tau_{\rm so}(3T_c/2\pi^2ca^2L_\phi)^{1/2}$. In deriving Eq. (12), we have assumed that the SO length $\ell_{\rm so}=(D\tau_{\rm so})^{1/2}$ is smaller than L_ϕ.

Discussion.—By going beyond the Boltzmann description in calculating the critical behavior of the resistivity, we have expressed $\rho_{\rm eff}$ through mesoscopic characteristics. Recall that our results, Eqs. (11) and (12), were

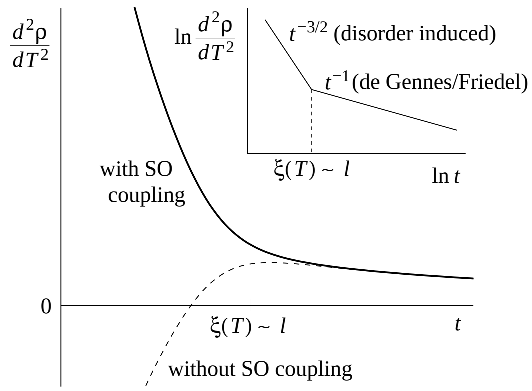

FIG. 3: Resistive anomaly within mean-field theory for $L_\phi \sim \ell$ (schematic) with (full line) and without (dashed line) SO coupling. The anomaly is described by the de Gennes-Friedel mechanism for $\xi(T) \ll \ell$, while the disorder-induced mechanism of this paper dominates closer to T_c where $\xi(T) \gg L_\phi$. When $L_\phi \gg \ell$, there is an additional intermediate regime. Inset: anomaly with SO coupling in a log-log plot.

obtained within mean-field theory so that the singularity in the t-dependence of ρ_0 on the lhs of these equations, arising within the Boltzmann formalism, is suppressed by impurities, since $\xi(T) \gg \ell$. Eqs. (11) and (12) show that in addition to this suppression, impurity scattering leads to a $\sqrt{t}$ singularity which is much stronger than the de Gennes-Friedel result $t \ln(1/t)$ and the large-wavevector Fisher-Langer contribution. This singularity, which in essence is governed by Kirchhoff's laws, constitutes our central result. To resolve the anomaly on top of a monotonous phonon contribution to ρ, it is customary to consider $d^2\rho/dT^2$. Then, our disorder-induced mean-field anomaly becomes $d^2\rho/dT^2 \sim t^{-3/2}$. In a log-log plot, the slope is -1 in the Boltzmann regime $\xi(T) \ll \ell$ and $-3/2$ for the disorder-induced anomaly, as illustrated in Fig. 3.

According to Eqs. (11) and (12), the sign of the anomaly is governed by the sign of either $F'(x_0)$ or $H'(y_0)$, depending on the strength of SO coupling. Since $F'(x_0) < 0$ while $H'(y_0) > 0$, the disorder-induced anomaly corresponds to a *decrease* of $d^2\rho/dT^2$ when approaching T_c from above in the absence of SO coupling and to an *increase* in the realistic case of strong SO coupling. The difference in signs comes about because the Zeeman field suppresses the correlator in Eq. (7), while it increases the conductance fluctuations for strong SO interactions.

The magnitude of the disorder-induced anomaly is controlled by $F(0)$ and $H(0)$ which are the variances of the conductance, $\langle(\delta g)^2\rangle_{\rm imp}$, cf. Eqs. (11) and (12). This quantity assumes different values in different regimes which are defined by the relations between the relevant lengths, namely L_ϕ, the thermal length $L_T = (D/T)^{1/2}$, and the spin-flip length ℓ_s. In the simplest case, when L_ϕ is the smallest length, $L_\phi \ll L_T, \ell_s$, we have $\langle(\delta g)^2\rangle_{\rm imp} \sim (e^2/h)^2$ [13]. If L_ϕ is larger than L_T or ℓ_s, then $\langle(\delta g)^2\rangle_{\rm imp}$ is suppressed below the universal limit, $\langle(\delta g)^2\rangle_{\rm imp} \sim (e^2/h)^2[\min\{L_T, \ell_s\}/L_\phi]$ [13].

The disorder-induced anomaly proposed in this paper is most relevant to ferromagnets with high resistance and low T_c since in such systems (i) the mean free path is dominated by impurity scattering [15] and (ii) the phase-coherence length exceeds the mean free path at T_c. Natural candidates for disordered low-T_c ferromagnets are $SrRuO_3$ [16] which belongs to the class of extremely poor metals [17] as well as diluted magnetic semiconductors (DMS) [18] which lately attracted considerable attention in view of possible spintronics applications [19]. Indeed, both types of materials show resistive anomalies which differ significantly from the predictions of Fisher-Langer theory. Metallic samples of DMS exhibit pronounced maxima near T_c even in ρ vs. T [20, 21, 22, 23, 24, 25, 26, 27, 28], which are unrelated to the $T = 0$ metal-insulator transition [22].

In closing, it is interesting to point out that our principal result, namely the enhancement of the resistive anomaly by disorder, can be viewed in perspective of the enhanced coupling of the spin fluctuations to the carriers, brought about by the diffusive carrier dynamics.

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