Fractional Shot Noise in the Kondo Regime

Eran Sela,[1] Yuval Oreg,[1] Felix von Oppen,[2] and Jens Koch[2]
[1] *Department of Condensed Matter Physics, Weizmann Institute of Science, Rehovot, 76100, Israel*
[2] *Institut für Theoretische Physik, Freie Universität Berlin, Arnimallee 14, 14195 Berlin, Germany*
(Dated: March 16, 2006)

Low temperature transport through a quantum dot in the Kondo regime proceeds by a universal combination of elastic and inelastic processes, as dictated by the low-energy Fermi-liquid fixed point. We show that as a result of inelastic processes, the charge detected by a shot-noise experiment is enhanced relative to the noninteracting situation to a universal fractional value, $e^* = 5/3e$. Thus, shot noise reveals that the Kondo effect involves many-body features even at low energies, despite its Fermi-liquid nature. We discuss the influence of symmetry breaking perturbations.

PACS numbers: 72.15.Qm, 72.70.+m,73.63.Kv

Introduction.—Shot-noise measurements in mesoscopic devices provide a direct measurement of the effective charge e^* of the current-carrying particles. Prominent examples in which this charge differs from the electron charge e include the observation of the fractional charge $e^* = e/3$ in the fractional quantum Hall regime [1], as well as the detection of the Cooper-pair charge $e^* = 2e$ in normal metal-superconductor junctions [2]. In this paper we study shot noise in quantum dots in the Kondo limit. Despite the Fermi liquid (FL) nature of the low-energy fixed point of the Kondo effect, we find that the effective backscattering charge is a universal quantity satisfying $e^* > e$. Unlike in quantum Hall systems and superconductors, this enhancement relative to the noninteracting value is *not* related to the fundamental quasiparticle charge. Instead, it is a direct consequence of interactions between quasiparticles of charge e, which lead to simultaneous backscattering of two quasiparticles.

The Kondo effect occurs in quantum dots [3] when the dot carries an effective spin, and charge fluctuations are frozen out by the strong Coulomb repulsion. Virtual tunneling of electrons into and out of the dot induces an antiferromagnetic coupling of the dot spin with the electrons in the leads. In this paper, we focus on the regime of temperatures T well below the Kondo temperature T_K, where the dot spin is locked into a singlet state with the lead electrons. Then, for two leads coupled symmetrically to the dot, the linear-response conductance is enhanced to the maximal unitary value $g_0 = 2e^2/h$ [4], corresponding to the conductance of a fully transparent channel with transmission probability $T(\epsilon) = 1$.

Shot noise in the Kondo effect was recently addressed theoretically for a wide range of temperatures and voltages V [5, 6]. For energies well above T_K, shot noise exhibits the typical enhancement $\propto \log^{-2}(eV/T_K)$, then it develops a peak around T_K, and is finally suppressed at low energies. The low-temperature suppression can be understood from the expression for the shot noise S in noninteracting systems [7]

$$S = 2g_0 \int d\epsilon T(\epsilon)[1 - T(\epsilon)][f_s(\epsilon) - f_d(\epsilon)], \qquad (1)$$

which vanishes in the unitary limit $T(\epsilon) \to 1$. Here f_s and f_d denote the Fermi distribution functions of the source and drain, respectively. Intuitively, while the incident fermionic carrier flow is fluctuationless at zero temperature, the transmitted and reflected carrier flows generally exhibit probabilistically generated noise. However, if $T(\epsilon) = 1$ or 0, no noise is generated by the scatterer.

The starting point of the present paper is the observation that close to the limit of perfect transmission, it is natural to extract the charge of the *backscattered* particles from the ratio

$$e^* = S/2I_b, \qquad (2)$$

where I_b denotes the backscattering current of reflected carriers. Indeed, this definition was used to extract the quasiparticle charge in the fractional quantum Hall effect [1]. In the noninteracting case, $I_b = 2\frac{e}{h}\int d\epsilon[1 - T(\epsilon)][f_s(\epsilon) - f_d(\epsilon)]$, and using Eqs. (1) and (2) we have $e^* = e$ when $T(\epsilon) \to 1$ for energies ϵ close to the Fermi energy. In contrast, it is the central result of this paper that $e^* = \frac{5}{3}e$ in the Kondo regime. We show below that this is a universal property of the Kondo effect, which is independent of the Kondo temperature T_K.

Near the unitary limit, it is most convenient to describe the system in the language of right movers (R-movers) propagating from source (with chemical potential μ_s) to drain (with chemical potential μ_d) and left movers (L-movers) propagating from drain to source. Deviations from the unitary limit will allow R-movers to backscatter into L-movers and vice versa, as indicated schematically by a wavy line in Fig. 1(a). We now turn to a discussion of the various relevant backscattering processes which are dictated by the low-energy fixed point of the Kondo effect and summarized pictorially in Figs. 1(b), (c), and (d).

In a naive picture, the Kondo effect is thought of as the formation of a single-quasiparticle resonance of width T_K, centered exactly at the Fermi energy E_F. Then, individual quasiparticles are backscattered as energies ϵ away from the Fermi energy become relevant due to finite temperature or voltage [see Fig. 1(b)]. The rate for this process grows quadratically in $\max\{T/T_K, V/T_K\}$.

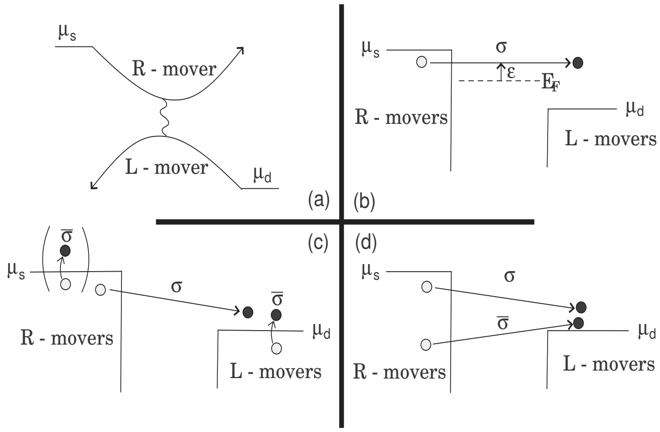

FIG. 1: Transport processes near the unitary limit. (a) In the unitary limit an electron incident from the source is almost totally transmitted into the drain, a process describing the free motion of a R-mover. Weak backscattering between R-movers and L-movers is denoted by the wavy line. (b) Elastic backscattering of one R-mover with energy ϵ relative to $E_F = (\mu_s + \mu_d)/2$. The energy dependence of this amplitude $\propto \alpha\epsilon/T_K$ corresponds to scattering off a resonant level of width $\propto T_K/\alpha$, centered at E_F. (c) Inelastic backscattering of one R-mover accompanied by creation of a particle hole pair within one reservoir. (d) Inelastic backscattering of two R-movers with opposite spins σ and $\bar{\sigma}$. The amplitudes of processes (c) and (d) are proportional to β, see Eq. (3).

However, as T or V increase, an additional inelastic channel opens for scattering between the R-movers and L-movers, in which the backscattering event is accompanied by the simultaneous creation of a particle-hole pair. In this case, the corresponding rates are again quadratic in $\max\{T/T_K, V/T_K\}$ due to phase-space restrictions for particle-hole-pair creation. If the particle-hole pair is created within the drain or within the source [see Fig. 1(c)], the process effectively backscatters a single mover. However, when particle and hole are created in drain and source, respectively [see Fig. 1(d)], we encounter an event in which *two* R-movers backscatter simultaneously [8]. These are the processes that lead to an effective backscattering charge $e < e^* < 2e$ as measured by shot noise.

The universality of e^* is a consequence of the fact that the Kondo resonance is tied to the Fermi level. This fixes the ratio between the amplitudes α for elastic scattering and β for the interactions which generate inelastic scattering. It is a central result in Nozières' FL theory of the Kondo effect [9] that $\alpha = \beta$. Thus, the fixed-point Hamiltonian Eq. (3) describes the low-energy properties by a single parameter T_K. It is interesting to note that the Wilson ratio, i.e., the ratio between the relative changes in the susceptibility and the specific heat due to the local spin, $W = (\delta\chi/\chi)/(\delta C_v/C_v) = 1 + \beta/\alpha = 2$, is another quantity which acquires a universal value due to the same reason. However, the universality of W is actually restricted to situations where the g-factors of localized spin and conduction electrons are equal [10]. We emphasize that the universality of e^* is *not* subject to this restriction.

The large value of the effective charge is surprising since there are six possible processes in which one mover is backscattered [two elastic and four inelastic processes, see Figs. 1(b) and 1(c)] compared to only a single process of two-particle backscattering [Fig. 1(d)]. However, the phase space for the two-particle process is significantly enhanced by the fact that the applied voltage acts on *both* particles scattered from source to drain. Indeed, we find for $eV \gg T$ that 2/3 of the backscattering current is carried by two-particle processes.

Calculation.—We describe a quantum dot with symmetric dot-lead couplings near the unitary limit in the basis of L and R-movers with energy $\xi_k = v_F k$, spin σ, as well as creation operators $L^\dagger_{k\sigma}$ and $R^\dagger_{k\sigma}$, respectively. The distribution of incoming R-movers (L-movers) is dictated by the chemical potential of the source (drain). Due to the LR-symmetry, the low-energy Hamiltonian $H[\psi]$ can be written entirely in terms of the symmetric combination $\psi_{k\sigma} = \frac{1}{\sqrt{2}}(L_{k\sigma} + R_{k\sigma})$.

In view of the Fermi-liquid nature of the Kondo fixed point, the low-energy physics can be completely described by the scattering phase shift suffered by an incoming quasiparticle ($\psi_{k\sigma}$), combined with the quasiparticle distribution n_σ [9]. Following Nozières, the low-energy expansion of this phase shift is $\delta_\sigma = \frac{\alpha\epsilon}{T_K} - \frac{\beta n_{\bar{\sigma}}}{\nu T_K}$ where $\bar{\sigma} = -\sigma$ and ν is the density of states. Notice that the phase shift of the electrons differs by $\pi/2$ from that of the $\psi_{k\sigma}$ particles [11]. Combining this expansion with the floating of the Kondo resonance, i.e., $\delta(\delta\epsilon, n = \nu\delta\epsilon) = \delta(\epsilon = 0, n = 0)$, one obtains the important FL relation $\alpha = \beta$ mentioned above.

Equivalently, the low-temperature physics can be described in terms of the Hamiltonian [3, 11]

$$H = \sum_{k\sigma} \xi_k \psi^\dagger_{k\sigma}\psi_{k\sigma} - \frac{\alpha}{2\pi\nu T_K}\sum_{k,k'\sigma}(\xi_k + \xi_{k'})\psi^\dagger_{k\sigma}\psi_{k'\sigma} + \frac{\beta}{\pi\nu^2 T_K}\sum_{k_1,k_2,k_3,k_4}\psi^\dagger_{k_1\uparrow}\psi_{k_2\uparrow}\psi^\dagger_{k_3\downarrow}\psi_{k_4\downarrow}, \quad (3)$$

whose t-matrix $t_\sigma = \frac{1}{2\pi i\nu}(1 - e^{2i\delta_\sigma}) \simeq -\delta_\sigma/\pi\nu$ reproduces the desired phase shift.

The term $\propto \alpha$ in Eq. (3) which yields the energy dependence of the phase shift, is consistent with the picture of a resonant level of width T_K centered at E_F. Since the phase shift grows by π across the resonance, this picture implies $\alpha \sim 1$. The term $\propto \beta$ describes the quasiparticle interactions. While the corresponding contribution to the phase shift follows from this interaction at the Hartree level, a treatment beyond Hartree involves inelastic processes in which quasiparticle scattering is accompanied by the creation of particle-hole pairs.

The current I transmitted from source to drain contains a dominant (maximal) unitary contribution $I_u =$

$2\frac{e^2}{h}V$ as well as the backscattering current, $I = I_u - I_b$. The backscattering contribution I_b, describing deviations from perfect transmission, follows from the Hamiltonian Eq. (3) by evaluating the increase in the numbers of L-movers relative to R-movers, $I_b = \frac{e}{2}\frac{d}{dt}(N_L - N_R)$, where $N_{a=L,R} = \sum_{k\sigma} a^\dagger_{k\sigma} a_{k\sigma}$. The zero-frequency current fluctuations (noise power) are defined as $S = \int dt \langle \{\delta I(0), \delta I(t)\}\rangle$, where $\delta I = I - \langle I \rangle$. At zero temperature, only I_b contributes to shot noise.

We calculate current and noise through the biased quantum dot by the Keldysh technique, which allows one to couple the L- and R-movers perturbatively in $1/T_K$. To leading order, we obtain after lengthy but straightforward calculations [12]

$$I = I_u - I_b \;=\; \frac{2e^2}{h}V\left[1 - \frac{\alpha^2+5\beta^2}{12}\left(\frac{V}{T_K}\right)^2\right], \quad (4)$$

$$S \;=\; \frac{4e^3}{h}V\frac{\alpha^2+9\beta^2}{12}\left(\frac{V}{T_K}\right)^2. \quad (5)$$

Substituting these expressions into Eq. (2), and using $\alpha = \beta$, we indeed find $e^* = \frac{5}{3}e$.

Interpretation.—We can gain physical understanding of this result by rewriting the Hamiltonian Eq. (3) in terms of the operators of left and right movers. This allows us to unravel the nature of the backscattering of R-movers into L-movers and to obtain the rates of the various backscattering processes from the relevant amplitudes combined with the voltage-dependent phase space.

Substituting $\psi_{k\sigma} = \frac{1}{\sqrt{2}}(L_{k\sigma} + R_{k\sigma})$, the inelastic term $\propto \beta$ in the Hamiltonian Eq. (3) takes the form $H_\beta = \frac{\beta}{4\pi\nu^2 T_K}\sum_{k_1,k_2,k_3,k_4}^{a,b,c,d=L,R} a^\dagger_{k_1\uparrow} b_{k_2\uparrow} c^\dagger_{k_3\downarrow} d_{k_4\downarrow}$. Clearly, it contains processes in which $0, 1$, or 2 particles are backscattered. An interesting process without net backscattering is $L^\dagger_{k_1\uparrow} R_{k_2\uparrow} R^\dagger_{k_3\downarrow} L_{k_4\downarrow}$, which contributes to the spin current [13] but not to the charge current considered here.

Backscattering of *two* R-movers arises from the terms $\propto \sum L^\dagger_{k_1\uparrow} R_{k_2\uparrow} L^\dagger_{k_3\downarrow} R_{k_4\downarrow}$ in H_β [see Fig. 1(d)]. Their contribution $I_{\beta 2} = \Gamma_{\beta 2}\cdot 2e$ to the backscattering current I_b is determined by the rate

$$\Gamma_{\beta 2} = \frac{2\pi}{\hbar}\sum_{k_1,k_2,k_3,k_4} |\langle L_{k_1\uparrow} R^\dagger_{k_2\uparrow} L_{k_3\downarrow} R^\dagger_{k_4\downarrow} H_\beta\rangle|^2 \delta(\xi_{k_1} + \xi_{k_3} - \xi_{k_2} - \xi_{k_4}) = \frac{2\pi}{\hbar}\frac{\beta^2}{16\pi^2 T_K^2}\int_{-eV}^{eV} d\epsilon (V-\epsilon)(V+\epsilon). \quad (6)$$

Here we used $\sum_k = \nu\int d\xi_k$, $\langle L_{k\sigma} L^\dagger_{k'\sigma'}\rangle = \delta_{kk'}\delta_{\sigma\sigma'}[1 - f_d(\xi_k)]$, $\langle R^\dagger_{k\sigma} R_{k'\sigma'}\rangle = \delta_{kk'}\delta_{\sigma\sigma'} f_s(\xi_k)$, and set $T = 0$. ϵ denotes the energy transfer from the spin-up to the spin-down particle, and the factors $V - \epsilon$ and $V + \epsilon$ originate from the integrations over the initial energies of the spin-up and spin-down R-movers, respectively. Performing the ϵ-integration, we obtain $I_{\beta 2} = \frac{e^2}{h}\frac{2}{3}\left(\frac{V}{T_K}\right)^2 V\beta^2$.

In a similar manner, one finds that inelastic backscattering processes of a single R-mover give a contribution to I_b which is of the form $I_{\beta 1} = 4\Gamma_{\beta 1} e = \frac{e^2}{h}\frac{1}{6}\left(\frac{V}{T_K}\right)^2 V\beta^2$. The factor of 4 reflects spin as well as the fact that the particle-hole pair can be created either in the source or in the drain [see Fig. 1(c)]. $\Gamma_{\beta 1}$ is obtained by replacing $R^\dagger_{k_4\downarrow} \to L^\dagger_{k_4\downarrow}$ in Eq. (6) for $\Gamma_{\beta 2}$. Note that the ratio of the phase-space factors for inelastic backscattering of two movers vs. a single mover is $\Gamma_{\beta 2}/\Gamma_{\beta 1} = 8$.

The backscattering current due to elastic processes [see Fig. 1(b)] follows from the elastic term in the Hamiltonian Eq. (3), which takes the form $H_\alpha = -\frac{\alpha}{4\pi\nu T_K}\sum_{k,k'\sigma}^{a,b=L,R}(\xi_k + \xi_{k'}) a^\dagger_{k\sigma} b_{k'\sigma}$ in terms of $L_{k\sigma}$ and $R_{k\sigma}$. This contains processes in which at most one mover is backscattered. The corresponding elastic contribution to I_b is given by $I_\alpha = 2\Gamma_\alpha e = \frac{e^2}{h}\frac{1}{6}\left(\frac{V}{T_K}\right)^2 V\alpha^2$, where the factor of two originates from spin.

Since the scattering events have rates $\propto \left(\frac{V}{T_K}\right)^2 \ll 1$ and are thus rare, they are uncorrelated. For this reason, the total shot noise $S = 2e(I_\alpha + I_{\beta 1} + 2I_{\beta 2})$ contains independent contributions from each process. Using Eq. (2) and $\alpha = \beta$, we recover the effective charge

$$\frac{e^*}{e} = \frac{\frac{\alpha^2}{6} + \frac{\beta^2}{6} + 2\frac{2\beta^2}{3}}{\frac{\alpha^2}{6} + \frac{\beta^2}{6} + \frac{2\beta^2}{3}} = \frac{5}{3}. \quad (7)$$

So far, we have derived this universal value of e^* for systems which reach the maximal unitary limit as $T \to 0$. A necessary condition for this to happen is that the system respects the following symmetries: (i) $SU(2)$ spin symmetry, requiring zero magnetic field $\delta_h = g\mu_B H/T_K = 0$; (ii) particle-hole symmetry leading to the absence of potential scattering, $\delta_r = 0$; (iii) LR symmetry, requiring dot-lead tunneling $t_{L,R}$ and capacitive couplings which are equal for left and right lead. Deviations from $t_L = t_R$ imply $\delta_\theta \neq 0$, where $\delta_\theta = 2\theta - \pi/2$ with $\theta \equiv \arctan|t_R/t_L|$, $0 \leq \theta \leq \pi/2$. Asymmetric capacitances may shift the position of the resonance level further. We quantify this shift by a parameter γ satisfying $E_F = \frac{\mu_s + \mu_d}{2} + V\gamma$.

Some theoretical approaches artificially break these symmetries in order to arrive at solvable models. E.g., the Schiller-Hershfield version of the Toulouse solution [5] breaks $SU(2)$ spin symmetry as well as the LR-symmetry and indeed, we find that it would predict $e^* = 2e$. Slave-boson mean field theory neglects two-particle scattering and breaks particle-hole symmetry [14]. Since it leads to a self-consistent single-electron description in terms of a resonance-level model, one necessarily has $e^*/e = 1$.

Realistic quantum dots.—The maximal unitary limit is also not easily accessible in experiment due to residual symmetry-breaking perturbations [4]. Such perturbations lead to a backscattering current linear in V, which dominates at low voltages and implies $e^* = e$. However, the previously discussed (α and β) processes grow as V^3 and will thus dominate at sufficiently high voltages, leading to a crossover of e^* to a value close to the universal value $\frac{5}{3}e$. To quantify this scenario, we note that the backscattering current $\propto V$, $I_b = 2\frac{e^2}{h}V[1 - \sin^2(2\theta)\frac{1}{2}\sum_{\sigma=\pm}\sin^2\delta_\sigma^{\rm el}]$ [3], is determined by the LR-asymmetry δ_θ and by the electronic phase shift $\delta_\sigma^{\rm el} = \pi/2 - \delta_r - \sigma\delta_h$. The latter differs by $\pi/2$ from the ψ-particles phase shift, $-\delta_r - \sigma\delta_h$. This phase shift can be included by adding to the Hamiltonian a local term, $H_{\rm loc} = \sum_{kk'\sigma}\frac{\delta_r+\delta_h\sigma}{\pi\nu}\psi_{k\sigma}^\dagger\psi_{k'\sigma}$. (A global magnetic field has a similar contribution to the phase shift through a Hartree treatment of the interaction.) If the dot is close to unitarity, $\delta_\theta, \delta_h, \delta_r \ll 1$, we have the expansion $I_b = 2\frac{e^2}{h}V\delta^2$, where $\delta^2 = \delta_\theta^2 + \delta_h^2 + \delta_r^2$. Thus, this contribution to backscattering becomes negligible once

$$V^* = T_K \max\{\delta_\theta, \delta_h, \delta_r\} \ll V \ll T_K, \tag{8}$$

and the detailed crossover of e^* takes the form

$$\frac{e^*}{e} = \frac{\delta^2 V + \frac{5}{3}(V^3/2T_K^2)}{\delta^2 V + (V^3/2T_K^2)}, \tag{9}$$

(with $\alpha = \beta = 1$), as shown in Fig. 2.

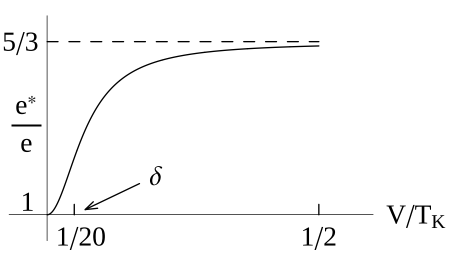

FIG. 2: Effective charge e^* vs. voltage for broken LR tunneling symmetry $\delta = \delta_\theta = 1/20$. The crossover from $e^*/e = 1$ to $e^*/e \approx 5/3$ occurs when V/T_K exceeds δ.

It may be useful to note for experimental tests of our predictions that for voltages $V \gg V^*$, it should be possible to subtract explicitly the *identical* contributions $\propto V$ in the noise $S/2e$ and in the backscattering current, cf. Eq. (10) below. In this way, one isolates the terms $\propto V^3$ and recovers the universal value 5/3 even in the presence of symmetry-breaking perturbations.

We remark that strictly speaking, symmetry-breaking perturbations also affect the terms $\propto V^3$. These corrections appear at yet higher order [e.g. $\mathcal{O}(\beta^2\delta_r^2)$]. Evaluating these corrections within the Keldysh approach for weak symmetry breaking, we obtain

$$\frac{S/2e - \delta^2 g_0 V}{I_b - \delta^2 g_0 V} - \frac{5}{3} \approx \sum_{i=r,h,\theta} c_i\delta_i^2 + c_\gamma\gamma^2 + \mathcal{O}(\delta^3), \tag{10}$$

where c_r, c_h, c_θ and c_γ are numbers of $\mathcal{O}(1)$ [15].

Summary.—We discussed the effective backscattering charge measured by shot noise in Kondo quantum dots near perfect transmission. The result $e^* = \frac{5}{3}e$ reflects the fact that transport is mediated by a combination of one and two particle scattering processes. We argued that even for real quantum dots where most of the symmetries which are often assumed in theoretical models are broken, the universal behavior can be seen at voltages larger than a voltage scale V^* reflecting the strength of symmetry breaking, but smaller than T_K.

We acknowledge useful discussions with A. M. Finkel'stein, L. Glazman, M. Heiblum, and O. Zarchin. Special thanks to A. Golub and D. Meidan. This research was supported by DIP (YO and FvO), ISF, BSF (YO), Sfb 658 (FvO), and the Studienstiftung d. dt. Volkes (JK).

[1] R. de-Picciotto et al. Nature **389**, 162 (1997); and L. Saminadayar et al. Phys. Rev. Lett. **79**, 2526 (1997).
[2] F. Lefloch *et al.* Phys. Rev. Lett. **90**, 067002 (2003).
[3] M. Pustilnik and L. I. Glazman cond-mat/0501007.
[4] W. G. van der Wiel et al., Science **289**, 2105 (2000).
[5] A. Schiller and S. Hershfield, Phys. Rev. B **58**, 14978 (1998).
[6] Y. Meir and A. Golub, Phys. Rev. Lett. **88**, 116802 (2002).
[7] G. B. Lesovik, Pis'ma Zh. Eksp. Teor. Fiz. **49**, 513 (1989), [JETP Lett **49** 592 (1989)].
[8] Two-particle backscattering in Luttinger liquids is discussed in D. Meidan and Y. Oreg, Phys. Rev. B **72**, 121312(R) (2005).
[9] P. Nozières, J. Low Temp. Phys. **17**, 31 (1974).
[10] P. B. Wiegmann and A. M. Finkel'stein, Pis'ma Zh. Eksp. Teor. Fiz. **75**, 204 (1978), [Sov. Phys. JETP **48**, 102 (1978)].
[11] I. Affleck, Nucl. Phys. B **336**, 517 (1990).
[12] The result for I agrees with Eq. (5.53) of Ref. [3].
[13] M. Kindermann, Phys. Rev. B **71**, 165332 (2005).
[14] D. Sánchez and R. López, Phys. Rev. B **71**, 035315 (2005).
[15] Assuming that the fixed point Hamiltonian of Eq. (3) is defined for band width $D \ll T_K$ and neglecting terms of $\mathcal{O}(D/T_K)$ we find $c_\theta = c_\gamma = -4/3$, $c_h = c_r = 223/144$.