Unconventional Josephson Signatures of Majorana Bound States

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A junction between two topological superconductors containing a pair of Majorana fermions exhibits a "fractional" Josephson effect, 4π periodic in the superconductors' phase difference. An additional fractional Josephson effect, however, arises when the Majorana fermions are spatially separated by a superconducting barrier. This new term gives rise to a set of Shapiro steps which are essentially absent without Majorana modes and therefore provides a unique signature for these exotic states.

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Majorana fermions comprise the simplest and likely most experimentally accessible non-Abelian anyon. An unambiguous demonstration of their non-Abelian exchange statistics would be a great triumph for condensed matter physics, as this phenomenon reflects one of the most spectacular manifestations of emergence. Furthermore, non-Abelian excitations provide the foundation behind topologically protected quantum computation [1,2], with Majorana fermions playing a crucial role in prototype devices [3–7]. In the solid-state context, Majorana modes were originally perceived as zero-energy states bound to vortices in p-wave superconductors [8], and therefore are also associated with quasiparticles in the Moore-Read state [9]. More recent proposals employ topological insulators [10–13], half-metals in proximity to superconductors [14–16], as well as spin-orbit-coupled quantum wells [17,18] and nanowires [19–23] to stabilize these elusive particles. Signatures of Majorana fermions appear in tunneling spectra and noise [24–26], and more strikingly through interference effects [27,28].

Josephson effects provide yet another important experimental signature of Majorana fermions. Kitaev first predicted that a pair of Majorana fermions fused across a junction formed by two topological superconducting wires generates a Josephson current [29],

$$I = \frac{e}{\hbar} J_M \sin\left(\frac{\phi_\ell - \phi_r}{2}\right), \tag{1}$$

which exhibits a remarkable 4π periodicity in the superconducting phase difference $\phi_\ell - \phi_r$ between the left and right wires. In stark contrast to ordinary Josephson currents, this contribution reflects tunneling of *half* of a Cooper pair across the junction. Such a "fractional" Josephson effect was later established in other systems supporting Majorana modes [10,11,19,20,30,31] and in direct junctions between p-wave superconductors [32]. In this Letter we demonstrate that two topological superconductors bridged by an ordinary superconductor with phase ϕ_m generically support a second kind of unconventional Josephson effect with an associated current,

$$I' = \frac{e}{\hbar} J_Z \sin\left(\frac{\phi_\ell + \phi_r}{2} - \phi_m\right), \tag{2}$$

in the right or left superconductors, and twice that in the middle. This contribution arises solely from the fusion of spatially separated Majorana fermions across the junction and represents processes whereby a Cooper pair in the middle region splinters, with half entering the left and half entering the right topological superconductor (this process is first discussed in Ref. [33]). We will derive this emergent term in one-dimensional Majorana-supporting systems, and propose several ways of measuring its effects.

This novel Josephson coupling is derived most simply in a one-dimensional Kitaev chain. Consider a junction with Hamiltonian $H = H_\ell + H_r + \delta H$, where the left and right superconductors are described by p-wave-paired spinless fermions $c_{\alpha,x}$ ($\alpha = \ell, r$) hopping on an N-site chain [29],

$$H_\alpha = -\sum_{x=1}^{N-1} (t c^\dagger_{\alpha,x} c_{\alpha,x+1} + \Delta e^{i\phi_\alpha} c_{\alpha,x} c_{\alpha,x+1} + \text{H.c.}). \tag{3}$$

Equation (3) adiabatically connects to realistic Majorana-supporting quantum wire Hamiltonians [19,20,34] and therefore describes their universal properties as well. Following Kitaev, we express the spinless fermions in terms of two Majorana operators via $c_{\alpha,x} = \frac{1}{2} e^{-i(\phi_\alpha/2)} \times (\gamma^\alpha_{B,x} + i\gamma^\alpha_{A,x})$. When $t = \Delta$, Eq. (3) maps onto a dimerized Majorana chain: $H_\alpha = -it \sum_{x=1}^{N-1} \gamma^\alpha_{B,x} \gamma^\alpha_{A,x+1}$. The explicit absence of $\gamma^\alpha_{A,1}$ and $\gamma^\alpha_{B,N}$ in the Hamiltonians indicates the presence of zero-energy Majorana modes localized at the ends of each superconductor in the junction.

Let us now couple the two superconductors through

$$\delta H = -t_m (c^\dagger_{\ell,N} c_{r,1} + \text{H.c.}) - \Delta_m (e^{i\phi_m} c_{\ell,N} c_{r,1} + \text{H.c.}), \tag{4}$$

where the two terms describe tunneling and Cooper pairing across the junction. These couplings combine the zero-energy Majorana modes residing at the junction into a finite-energy Andreev bound state. Focusing on these zero-energy modes, one can write $c_{\ell,N} \to \frac{1}{2} e^{-i\phi_\ell/2}\gamma^\ell_{B,N}$ and $c_{r,1} \to i\frac{1}{2} e^{-i\phi_r/2}\gamma^r_{A,1}$ and define an ordinary fermion operator $f^\dagger = \frac{1}{2}(\gamma^\ell_{B,N} + i\gamma^r_{A,1})$; δH then becomes

$$\delta H \to (2f^\dagger f - 1)\{J_M \cos[(\phi_\ell - \phi_r)/2] + J_Z \cos[(\phi_\ell + \phi_r)/2 - \phi_m]\}, \quad (5)$$

with $J_M = \frac{t_m}{2}$ and $J_Z = \frac{\Delta_m}{2}$. Since the current in region s is given by $\frac{2e}{\hbar}\frac{\partial\langle\delta H\rangle}{\partial\phi_s}$, the fermion tunneling t_m gives rise to the fractional Josephson effect of Eq. (1), while pairing Δ_m across the junction produces the Josephson current in Eq. (2). Note that the sign of either current is dictated by the occupation number for the f fermion, and hence can be used as a readout method for qubit states encoded by the Majorana fermions [11,34].

A more quantitative understanding is obtained by considering more realistic models. Let us consider Majorana fermions localized on a topological insulator edge in proximity to a superconductor and subjected to a magnetic field [10]; a very similar analysis applies to quantum wires. In the Nambu spinor basis $\Psi^T = (\psi_\uparrow, \psi_\downarrow, \psi^\dagger_\downarrow, -\psi^\dagger_\uparrow)$, the Bogoliubov–de Gennes Hamiltonian for this system is

$$\mathcal{H} = v\hat{p}\sigma^z\tau^z - \mu\tau^z + \Delta(\cos\phi\tau^x - \sin\phi\tau^y) + B\sigma^x, \quad (6)$$

with v the edge-state velocity, $\hat{p}$ the momentum, B the Zeeman energy, and σ^a and τ^a Pauli matrices acting in the spin and particle-hole sectors, respectively. We allow the chemical potential μ, pairing amplitude Δ, and superconducting phase ϕ to vary spatially.

Majorana states arise at interfaces between topological (T) and trivial (S) regions of the edge [10]. With μ, Δ, and ϕ uniform the quasiparticle gap is $E_{\text{gap}} = |B - \sqrt{\Delta^2 + \mu^2}|$. When $\sqrt{\Delta^2 + \mu^2} > B$ the edge is gapped by proximity-induced superconductivity and forms a topological phase closely related to that of Kitaev's model described above [10]. In the trivial phase $\sqrt{\Delta^2 + \mu^2} < B$, and the magnetic field dominates the gap. We will study the T-S-T domain sequence of Fig. 1, which localizes Majorana fermions γ_1 at $x = 0$ and γ_2 at $x = L$. Each of the three regions, ℓ, r, m, couples to a superconductor imparting proximity strength $\Delta_{\ell/m/r}$ and phase $\phi_{\ell/m/r}$ and has a chemical potential $\mu_{\ell/m/r}$ controlled by separate gates. (The main difference in the quantum wire case is that there creating the T-S-T domain structure needed to observe the unconventional Josephson effects discussed here requires the reversed criteria: $\sqrt{\Delta^2 + \mu^2} < B$ in the outer regions and $\sqrt{\Delta^2 + \mu^2} > B$ in the middle region.)

The Majorana-related Josephson effects result from hybridization between γ_1 and γ_2. When γ_1 and γ_2 are far apart ($L \to \infty$), they constitute exact zero-energy modes

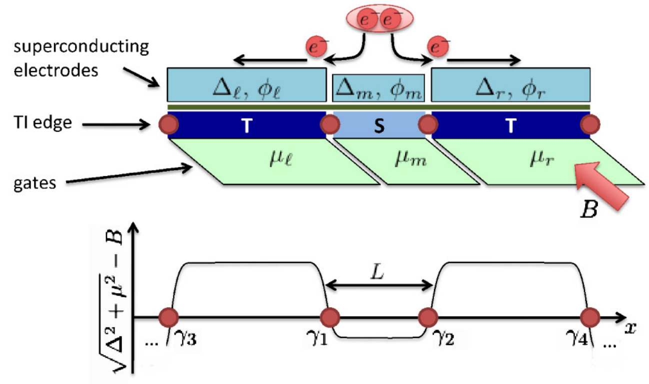

FIG. 1 (color online). Topological insulator edge subjected to a magnetic field B and sandwiched by gates and superconducting electrodes. Majorana modes (red circles) localize at domain walls where the gap $E_{\text{gap}} = |\sqrt{\mu^2 + \Delta^2} - B|$ vanishes and the argument in the absolute value changes sign. When the middle region is a trivial superconductor (S), and the sides form a topological phase (T), the novel J_Z term in Eq. (5), with current $\propto \sin(\frac{\phi_\ell+\phi_r}{2} - \phi_m)$, accompanies the usual fractional Josephson effect. This splits a Cooper pair in the middle electrode into two single electrons, injected via the two Majorana states in each topological segment. The same effect appears in spin-orbit-coupled wires in a T-S-T configuration.

and their wave functions decay exponentially in region $s = \ell, m, r$ with *two* characteristic lengths:

$$\lambda_{s\pm} = \frac{v}{|\Delta_s \pm \sqrt{B^2 - \mu_s^2}|} \quad (7)$$

(we assume $\mu_s < B$). For finite L, however, $\gamma_{1,2}$ combine into a finite-energy state with creation operator $f^\dagger = \frac{1}{2}(\gamma_1 + i\gamma_2)$. Roughly, each Majorana perceives the interface localizing the other Majorana as a perturbation, yielding a hybridization which is suppressed as a weighted sum of two decaying exponentials. This hybridization is again described by Eq. (5), with $J_{M/Z} = \frac{1}{2}(J_- e^{-L/\lambda_{m-}} \pm J_+ e^{-L/\lambda_{m+}})$. An explicit calculation for the symmetric setup, $\mu_\ell = \mu_r \equiv \mu$, $\Delta_\ell = \Delta_r \equiv \Delta > \Delta_m$, and $\mu_m = 0$, yields (for more details see [35])

$$J_+ = J_- \approx \frac{2\Delta}{\frac{\Delta(B+\Delta)+\mu^2}{\Delta^2+\mu^2-B^2} + \frac{B\Delta}{B^2-\Delta_m^2}}. \quad (8)$$

When $\Delta_m = 0$ and the middle region is normal—which is the setup typically studied [10,19]—$J_Z = 0$ and hence only the Josephson term in Eq. (1) appears. Turning on $\Delta_m \neq 0$ yields a nonzero J_Z and the second Josephson term in Eq. (2). Furthermore, since both J_Z and J_M are dominated by the slowest decay length, they will generically be of the same order. For a quantitative estimate, consider the parameters $\mu_m = 0$, $\mu_{l,r} = E$, $\Delta_m = E$, $\Delta_{l,r} = \sqrt{8}E$, $B = 2E$ with energy scale $E = 0.1$ meV. Assuming an edge velocity $v = 10^4$ m/s, for this choice we obtain $\lambda_{m+} \approx 22$ nm, $\lambda_{m-} \approx 66$ nm, and $J_\pm \approx 0.12$ meV. The effect then peaks at $L \approx 50$ nm, which yields $J_Z \approx 0.022$ meV and $I_Z = \frac{e}{\hbar} J_Z \approx 5.3$ nA.

These Josephson effects are simplest to understand conceptually when two additional Majorana fermions $\gamma_{3,4}$ straddle the T segments of the edge as shown in Fig. 1. Let us define fermion operators $f_A = \frac{1}{2}(\gamma_1 + i\gamma_3)$ and $f_B = \frac{1}{2}(\gamma_2 + i\gamma_4)$ and assume that the corresponding occupation numbers are initially $n_A = 1$ and $n_B = 0$. We will further employ a "perturbative" perspective and promote the superconducting phases to quantum operators conjugate to the Cooper pair number. One can then see that the Majorana operators in the term $J_M(2f^\dagger f - 1) \times \exp(i\frac{\phi_r - \phi_\ell}{2}) = iJ_M\gamma_1\gamma_2 \exp(i\frac{\phi_r - \phi_\ell}{2})$ hop a single fermion across the S region, changing the state of the edge from $(n_A, n_B) = (1, 0)$ to $(0, 1)$. At the same time, the exponential passes a charge e from side to side. The combination of these processes makes the term gauge invariant. The persistent superconducting current limit in this case is apparent when we consider an additional tunneling event which restores the parities of the T segments, moving a fermion back to the left but with a Cooper pair hopping to the right. A similar perspective clarifies the role of the J_Z term—the Majorana fermions in $iJ_Z\gamma_1\gamma_2 \exp[i(\frac{\phi_r + \phi_\ell}{2} - \phi_m)]$ also change the parity of the two T segments, while the exponent removes a Cooper pair from the middle region and adds charge e to each T region (see Fig. 1).

Next, we discuss the crucial issue of measuring the new Josephson term in Eq. (2). The first and most direct possibility involves independently manipulating the phase differences $\phi_\ell - \phi_m \equiv \Phi_L$ and $\phi_m - \phi_r \equiv \Phi_R$, e.g., by inserting different fluxes in the two loops in Fig. 2(a) (ignoring the voltage sources in the figure). By tuning $\Phi_L = -\Phi_R$ in a symmetric junction, one can probe the J_Z Josephson term (driving current $J_Z \sin\Phi_L$ on the middle electrode) while canceling the J_M term. Such measurements, however, are highly challenging—they require careful flux control; the Majorana-related Josephson current must be disentangled from the conventional 2π periodic contributions; and the measurement must be concluded before the parity of the two Majorana fermions changes.

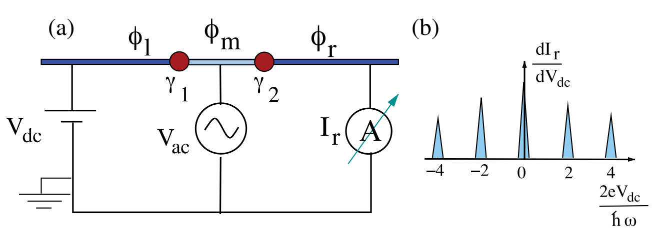

FIG. 2 (color online). Three-leg Shapiro-step measurement scheme. (a) We envision applying a dc voltage V_{dc} to the left superconducting electrode and an ac voltage with angular frequency ω in the left loop (which we model as an ac voltage applied to the middle electrode). A measurement of the dc current I_r in the right electrode will then reveal Shapiro steps stemming from the Majorana modes when the ac Josephson frequency $2eV_{dc}/\hbar$ equals an *even* harmonic of ω. (b) Sketch of dI_r/dV_{dc} indicating the predicted Shapiro steps—note the crucial absence of odd-harmonic peaks, which would appear in a conventional Shapiro-step measurement.

A potentially more promising measurement scheme relies on Shapiro steps. In a regular Josephson junction, Shapiro steps arise from a combination of a dc voltage V_{dc} and an ac voltage $V_{ac} \sin\omega t$, which together generate a current $I = I_J \sin[\phi_0 + 2eV_{dc}t/\hbar - (2eV_{ac}/\hbar\omega)\cos\omega t]$. Naively, this current averages to zero because of the constantly winding phase. This is not the case, however, when $2eV_{dc}/\hbar = n\omega$ for some integer n—here a dc current component exists, producing a step in the V vs I plot for the junction [36,37]. For the fractional Josephson term in Eq. (1), the 4π periodicity leads to Shapiro steps when $2eV_{dc}/\hbar = 2n\omega$, corresponding to even Shapiro steps of a regular Josephson junction. The halved periodicity, if established, could provide an unambiguous signature for Majorana modes. An inevitable conventional Josephson current, however, "fills in" the missing steps, making it difficult to disentangle these contributions [32].

The following three-leg Shapiro-step measurement circumvents this problem and targets the Josephson term of Eq. (2). As shown in Fig. 2(a), we envision a dc voltage applied to the left leg so that $\phi_\ell = 2eV_{dc}t/\hbar$, while an ac voltage applied to the middle leg sets $\phi_m = -(2eV_{ac}/\hbar\omega)\cos\omega t$. Since the new Josephson term induces current in all three legs, a current measurement on the right lead will find Shapiro steps emerging only when

$$2eV_{dc}/\hbar = 2n\omega \tag{9}$$

as illustrated in Fig. 2(b), without any odd-harmonic steps. This nonlocal measurement is insensitive to any parasitic two-phase Josephson terms and therefore automatically eliminates most competing processes. Furthermore, it bears the advantage of being a fast dynamic measurement (since Josephson frequencies are typically in the GHz regime), which reduces its sensitivity to temporal fluctuations of the Majorana-state occupations.

To verify the approximation methods used and to confirm the prominence of the J_Z term in the three-leg Shapiro measurement, we also numerically analyzed the Josephson effects in a topological insulator edge. Figure 3 shows that our analytical results [e.g., Eq. (7)] indeed agree very well with the exact numerical calculation. We also explored additional current contributions such as $\delta I \sin(\phi_\ell + \phi_r - 2\phi_m)$, which could obscure the Majorana signature by producing unwanted odd-harmonic Shapiro steps. This term is independent of the Majorana modes and can instead arise from conventional Bogoliubov states in the junction. In the limit of small pairing and tunneling over the middle segments, such a term reflects a high-order process. Numerically, we find that it is suppressed by at least an order of magnitude compared to the Majorana J_Z

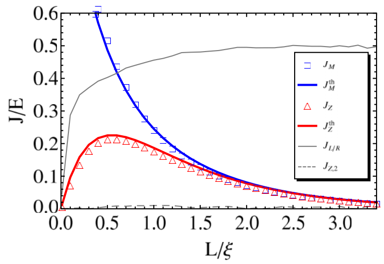

FIG. 3 (color online). Numerically determined coefficients of conventional Josephson couplings ($J_{L/R}$), Majorana-induced terms ($J_{M/Z}$) and second harmonic of the J_Z term ($J_{Z,2}$). Our analytical estimates of J_M^{th} and J_Z^{th} agree well with numerics (for more details see [35]). The energy unit is E and the length unit is $\xi = v/E$. The parameters are $\mu_{l,r} = E$, $\mu_m = 0$, $\Delta_{l,r} = \sqrt{8}E$, $\Delta_m = E$, and $B_{l,r} = B_m = 2E$. The characteristic lengths are $\lambda_{m+} = \xi/3$ and $\lambda_{m-} = \xi$. For $E = 0.1$ meV and $v = 10^4$ m/s, the length unit is $\xi = 66$ nm and the maximum current is $I_Z = \frac{e}{\hbar} J_Z \approx 5.3$ nA.

contribution in the regime where J_Z is substantial, i.e., when L is of order $\lambda_{m\pm}$.

By considering the full edge spectrum (including the Andreev bound states and continuum states exactly), we obtained the total Josephson energy of the domain configuration in Fig. 1:

$$E_{\text{tot}} \approx J_L \cos(\phi_\ell - \phi_m) + J_R \cos(\phi_r - \phi_m) + J_M \cos[(\phi_\ell - \phi_r)/2] + J_Z \cos[(\phi_\ell + \phi_r)/2 - \phi_m] + \sum_{n=2}^{\infty} J_{Z,n} \cos\{n[(\phi_\ell + \phi_r)/2 - \phi_m]\} + \cdots. \tag{10}$$

Here $J_{L/R}$ are conventional Josephson terms (to which the three-leg measurement is insensitive), $J_{M/Z}$ are the Majorana-induced contributions, and $J_{Z,n}$ denote the (unwanted) higher harmonics of the J_Z term. As Fig. 3 illustrates, J_M dominates for $L \ll \lambda_{m+}$, while for $\lambda_{m-} \gtrsim L \gtrsim \lambda_{m+}$ the J_Z term becomes comparable, enabling the three-leg Shapiro-step measurement. The higher harmonics $J_{Z,n}$ are at least an order of magnitude smaller than J_Z in this regime and can be neglected. For $L \gg \lambda_{m-}$ the Majorana signatures are strongly suppressed as expected. In contrast to systems with quadratic kinetic energy [38], the coupling energy for the linearized model [Eq. (6)] does not have oscillatory dependence on L because the wave function between the two Majorana fermions decays exponentially with no oscillations.

In this Letter, we explored a new Josephson effect that arises when a pair of Majorana fermions fuse across a junction formed by two topological superconductors separated by an ordinary superconductor. The Majorana fermions in this setup enable Cooper pairs injected into the barrier superconductor to "splinter" into the left and right legs of the junction—a process which would ordinarily be prohibited at low energies. While Majorana modes can also give rise to a novel fractional Josephson effect in T-normal-T junctions, we argued that an important advantage of our setup is that here one can more readily isolate the Majorana-mediated Josephson current through Shapiro-step measurements. The experiments we proposed could provide a relatively simple and unambiguous detection scheme for Majorana fermions and may also serve as a practical readout mechanism for qubit states encoded by these particles.

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