Energy Relaxation and Thermalization of Hot Electrons in Quantum Wires

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We develop a theory of energy relaxation and thermalization of hot carriers in clean quantum wires. Our theory is based on a controlled perturbative approach for large excitation energies and emphasizes the important roles of the electron spin and finite temperature. Unlike in higher dimensions, relaxation in one-dimensional electron liquids requires three-body collisions and is much faster for particles than holes which relax at nonzero temperatures only. Moreover, comoving carriers thermalize more rapidly than counterpropagating carriers. Our results are quantitatively consistent with a recent experiment.

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Introduction.—The behavior of electrons confined to move in one spatial dimension is frequently described within the Tomonaga-Luttinger model which assumes a linear dispersion relation for the electrons. In this model, all excitations move at the same velocity so that electron-electron interactions become particularly significant. Consequently, the electron system can no longer be described as a Fermi liquid but instead is expected to form a Luttinger liquid. In recent years, much effort has been expended on elucidating the consequences of Luttinger-liquid physics in quantum wires [1].

A peculiar consequence of the Tomonaga-Luttinger model is the absence of inelastic processes for hot particles or holes. As emphasized by a recent experiment [2], the physics of energy relaxation is much richer in real quantum wires with a nonlinear dispersion. In this experiment, hot carriers of well-defined energy and momentum are injected into a quantum wire and their energy relaxation is probed in cleverly designed transport measurements. The experiment shows not only that hot carriers relax but also that energy relaxation is much more efficient for hot particles than for hot holes, in stark contrast to electron liquids in higher dimensions. Moreover, a simple model [2] reproducing the observations assumed that thermalization occurs much faster among comoving electrons than between right- and left-moving carriers.

Foci of recent theoretical work on 1D electron systems were nonequilibrium effects in ideal [3–5] and disordered Luttinger liquids [6] as well as deviations from Luttinger-liquid theory due to the nonlinear dispersion [7–12]. Nonequilibrium physics of systems with nonlinear dispersions has been accessible within a perturbative approach for weak interactions [13–15]. The latter is peculiar because pair collisions are ineffective for a quadratic dispersion. Indeed, by momentum and energy conservation, pair collisions result either in zero-momentum transfer or exchange of the momenta of the colliding particles. Both processes do not change the electronic distribution function. The kinetics of real 1D electron systems therefore involves three-body collisions [13].

Consider an electron injected into the quantum wire with an excitation energy ϵ which is measured from the Fermi energy ϵ_F and large compared to temperature T. Because of the quadratic dispersion $\epsilon_k = \hbar^2 k^2/2m$, its velocity differs from that of the electrons in the Fermi sea by at least $\Delta v = \epsilon/m v_F$. (Here, v_F is the Fermi velocity.) According to the condition for the validity of the Born approximation [16], we thus expect a perturbative approach to energy relaxation to be appropriate when $\epsilon \gg m v_F \tilde{U}(0)/\hbar$, where $\tilde{U}(q)$ is the Fourier transform of the electron-electron interaction. In Luttinger liquids, the difference between the spin and charge velocities is $v_c - v_s \simeq \tilde{U}(0)/\pi\hbar$, so that the condition can also be recast as $v_c - v_s \ll \Delta v$. Hence, spin and charge do not separate appreciably during the collision process.

Here, we develop a theory of energy relaxation and thermalization in clean quantum wires in this perturbative regime. While some of the basic physics—such as the asymmetry between hot particles and holes—was understood [7,9,17], no systematic and quantitative theory of this fundamental property of clean 1D electron systems exists to date. Specifically, we emphasize the important roles of finite temperature, Coulomb interaction, and spin which are not appreciated in the existing literature but are important for understanding the experiment [2].

Basic processes and results.—The asymmetry in energy relaxation between hot particles and holes can be readily understood from the basic three-body collisions as sketched in Fig. 1. Suppose a hot particle 1 on, say, the right-moving branch transfers momentum q_1 to a right mover in the Fermi sea. Because of the positive curvature of the dispersion, the energy loss $\Delta\epsilon$ of the hot particle exceeds the energy of the created particle-hole (p-h) pair. This mismatch can be fixed by simultaneously exciting a left-moving p-h pair [Fig. 1(a)]. In line with the energy mismatch, the energy transfer to the left-moving p-h pair is

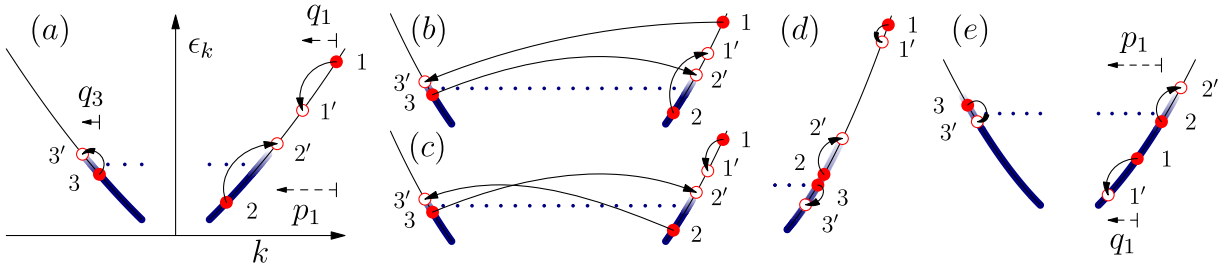

FIG. 1 (color online). Basic three-body relaxation processes of hot particles (a)–(d) and hot holes (e). (a) Small-q process $T^{123}_{1'2'3'}$ and $2k_F$ processes (b) $T^{123}_{3'1'2'}$ and (c) $T^{123}_{1'3'2'}$. The remaining processes follow by exchanging $1' \leftrightarrow 2'$. (d) Competing relaxation process involving only comoving electrons. (e) A small-q relaxation process of hot holes. Dotted lines: ϵ_F.

of order $(\epsilon/\epsilon_F)\Delta\epsilon$. The typical energy loss $\Delta\epsilon$ of hot particles in a single three-body collision is of order ϵ.

Compare this with the relaxation of hot holes sketched in Fig. 1(e) where, for a given momentum transfer, the energy gain due to filling the hole by a higher-energy electron is smaller than the energy cost of exciting a comoving p-h pair. Fixing this energy mismatch therefore requires a counterpropagating electron to give up energy. Clearly, at $T = 0$ this is forbidden by the Pauli principle and the hole is unable to relax. Indeed, this conclusion remains true for arbitrary n-body processes [2].

Hot holes do relax, however, at finite T. Because of thermal smearing, the counterpropagating electron can give up an energy of order T. Thus, the hot hole can relax, with a maximal energy loss of $\Delta\epsilon \sim \epsilon_F T/\epsilon$. This implies that hot holes float towards the Fermi energy in many small steps as long as $\epsilon \gg \epsilon_T$. Here, $\epsilon_T = \sqrt{\epsilon_F T}$ is a characteristic energy scale introduced by finite temperature. Conversely, for $\epsilon \ll \epsilon_T$, the energy loss of the hole per three-body collision is comparable to its energy.

Although hot particles always relax in a few three-body collisions, the energy scale ϵ_T is also relevant in this case. Indeed, for $\epsilon \ll \epsilon_T$, the energy transfer to the counterpropagating p-h pair is small compared to temperature. This has two important consequences. First, the phase space of the left-moving p-h pair is no longer controlled by the typical energy transfer ϵ^2/ϵ_F but by temperature T so that the energy-relaxation rate of hot particles becomes temperature dependent. Second, it is no longer relevant whether the counterpropagating electron gains or loses energy so that for $\epsilon \ll \epsilon_T$ energy relaxation becomes equally fast for hot particles and holes.

Although these three-body collisions generate both a right- and a left-moving p-h pair, these are fundamentally asymmetric in their energies. Because of the parametrically larger energy transfer to the comoving p-h pair, thermalization will happen more rapidly between electrons of the same chirality than between electrons of opposite chiralities. Injection of, say, right-moving hot particles can thus lead to different temperatures of right- and left-moving electrons over significant distances as appears to be the case in experiment [2]. We also note that the relevant thermalization rate among electrons of the same chirality is controlled by the hole relaxation rate even when injecting hot particles, since the relaxation of hot particles necessarily involves the excitation of deep holes.

We now summarize our quantitative results for energy relaxation in a spinful electron liquid with Coulomb interactions [18]. (Their derivation is sketched further below.) For the energy-relaxation rate of hot particles, we find

$$1/\tau_p = (9\epsilon_F/32\pi^3\hbar)(e^2/\kappa\hbar v_F)^4[\lambda(\epsilon)]^2(\epsilon/\epsilon_F)^2 \quad (1)$$

at high energies $\epsilon \gg \epsilon_T$, and

$$1/\tau_p = (3c_1\epsilon_F/4\pi^3\hbar)(e^2/\kappa\hbar v_F)^4[\lambda(\epsilon)]^2(T/\epsilon_F) \quad (2)$$

at low energies $\epsilon \ll \epsilon_T$. Here, $\lambda(\epsilon) = \ln|1/2k_F a|\ln|\epsilon/4\epsilon_F|$ in terms of the wire width a and the Fermi wave vector k_F with $k_F a \ll 1$, κ is the dielectric constant of the host material, and $c_1 = 4\ln(2) - 1$.

For $\epsilon \gg \epsilon_T$, the hole relaxation rate

$$1/\tau_h = (2\epsilon_F/\pi\hbar)(e^2/\kappa\hbar v_F)^4[\lambda(\epsilon)]^2(T/\epsilon)^2 \quad (3)$$

is smaller than $1/\tau_p$ (at the same ϵ) by a factor $(\epsilon_T/\epsilon)^4$. Conversely, for $\epsilon \ll \epsilon_T$, Eq. (2) also applies to hole relaxation. Finally, a temperature difference between right- and left-moving carriers equilibrates at the rate

$$1/\tau_{\text{inter}} = (9c_2\epsilon_F/2^8\pi^5\hbar)(e^2/\kappa\hbar v_F)^4[\lambda(T)]^2(T/\epsilon_F)^3, \quad (4)$$

with $c_2 \approx 103.9$ [20]. The results in Eqs. (1)–(4), based on the processes shown in Figs. 1(a)–1(c) and 1(e), dominate over the competing class of processes of Fig. 1(d) if at least one of two conditions is met: temperature is not too high, $T \ll \epsilon_F/(k_F d)^4$, or energy is not too low, $\epsilon \gg \epsilon_T$. (Here, d is the distance to a nearby metallic gate.)

Derivation.—We start with the Boltzmann equation for the electronic distribution function $n(k, t) = n^0(k) + \delta n(k, t)$, linearized about the Fermi-Dirac function $n^0(k)$,

$$\partial_t n_1 = -\sum_{231'2'3'} W^{123}_{1'2'3'} n^0_1 n^0_2 n^0_3 \bar{n}^0_{1'} \bar{n}^0_{2'} \bar{n}^0_{3'} \sum_{i=1}^{3} [y_i - y_{i'}]. \quad (5)$$

Here, $y_i = \delta n_i/[n^0_i \bar{n}^0_i]$, $n_i = n(k_i)$, and $\bar{n}_i = 1 - n_i$. The initial (final) states of the three-body collisions are labeled by i (i'), and the corresponding collision integral involves the generalized Fermi golden rule

$$W^{123}_{1'2'3'} = \frac{2\pi}{\hbar}|\langle 1'2'3'|VG_0V|123\rangle_c|^2\delta(E-E'). \quad (6)$$

Here, G_0 denotes the free Green's function and the subscript c stands for "connected" scattering events, in which all three particles participate [13]. The (unscreened) Coulomb interaction takes the form $V = (1/2L)\sum_{k_1k_2q\sigma_1\sigma_2} V_q a^\dagger_{k_1+q\sigma_1} a^\dagger_{k_2-q\sigma_2} a_{k_2\sigma_2} a_{k_1\sigma_1}$ with $V_q \simeq (2e^2/\kappa)\ln(1/qa)$. Our results are dominated by processes with large momentum transfers of order $2k_F$, where screening due to nearby gates can be neglected.

Setting $\delta n_1 = \delta_{k_1,k_F+\epsilon/\hbar v_F}$ and neglecting secondary collisions, we obtain, via $(\partial_t + \tau^{-1})\delta n_1 = 0$, the total scattering rate of hot particles,

$$\frac{1}{\tau} = \sum_{231'2'3'} W^{123}_{1'2'3'} n^0_2 n^0_3 \bar{n}^0_{1'} \bar{n}^0_{2'} \bar{n}^0_{3'}. \quad (7)$$

The transition amplitude for three-body scattering from $|123\rangle$ to $|1'2'3'\rangle$ is the sum of two processes with small momentum transfers $q \ll k_F$ and four processes involving momentum transfers of order $2k_F$; cf. Figs. 1(a)–1(c).

The matrix element in Eq. (6) can be decomposed as $\langle 1'2'3'|VG_0V|123\rangle_c = \sum_{P(1'2'3')}(-1)^p \delta_{\sigma_1\sigma_2\sigma_3,P(\sigma_{1'}\sigma_{2'}\sigma_{3'})} T^{123}_{P(1'2'3')}$, where $(-1)^p$ denotes the parity of the permutation P. Following Ref. [13], we find for the two small-q processes [cf. Fig. 1(a)], to leading order in ϵ/ϵ_F,

$$T^{123}_{1'2'3'} = (V_{q_1}/8\epsilon_F L^2)(V_{q_1} - V_{q_3}), \quad (8)$$

and the corresponding amplitude with $1' \leftrightarrow 2'$ (and thus $q_1 \leftrightarrow p_1 = k_1 - k_{2'}$, where $q_i = k_{i'} - k_i$). Similarly, we find for the four $2k_F$ processes [cf. Figs. 1(b) and 1(c)]

$$T^{123}_{3'1'2'} = -T^{123}_{2'3'1'} = (V_{2k_F}/2\hbar v_F q_1 L^2)(V_{2k_F} - V_{p_1}), \quad (9)$$

and the corresponding amplitudes with $1' \leftrightarrow 2'$.

Remarkably, the electron spin plays an important role in the transition matrix element for three-body scattering. Indeed, the amplitudes of the individual $2k_F$ processes in Eq. (9) are larger by a factor $\epsilon_F/\hbar v_F q_1$ than those of the small-q processes in Eq. (8). When the total spin of the three colliding particles is maximal, or when fermions are spinless, the leading contribution to the individual $2k_F$ processes cancels from their sum, resulting in contributions of the same order from the small-q and the $2k_F$ processes (in line with the classification of interactions for spinless fermions). In contrast, when the total spin of the three colliding particles is $1/2$, there is no cancellation in the sum over the $2k_F$ processes. The latter are thus parametrically larger and dominate in the three-body scattering rates. [Note that for Coulomb interactions, the enhancement of order (ϵ_F/ϵ) is larger than the concurrent logarithmic reduction stemming from the replacement of V_{q_1} by the smaller V_{2k_F}.]

The importance of the electron spin follows from the symmetry of the wave function. When the total spin of the three colliding particles is maximal, or when fermions are spinless, their orbital wave function must be odd, and the relevant amplitudes are suppressed by the exchange effect. In contrast, no such suppression occurs when the total spin of the three colliding particles is $1/2$, since the interacting particles can be in the same orbital state.

Integrating over the momenta of the left-moving p-h pair and doing the spin sums in Eq. (7), we can now extract the partial scattering rate $\mathcal{P}dq_1dp_1$ that the hot particle generates comoving particles near $k_{1'}$ and $k_{2'}$,

$$\mathcal{P}(q_1,p_1) = \frac{3\epsilon_F}{8\pi^3\hbar}\frac{L^4}{(\hbar v_F)^2}[(T^{123}_{3'1'2'})^2 + (T^{123}_{3'2'1'})^2] \times \frac{q_3/k_F}{(e^{\hbar v_F q_3/T}-1)} n^0_{k_1+p_1+q_1}\bar{n}^0_{k_1+p_1}\bar{n}^0_{k_1+q_1}. \quad (10)$$

By exchange symmetry, q_1 and p_1 enter symmetrically into $\mathcal{P}(q_1,p_1)$. Here, the factor involving $q_3 = -p_1q_1/2k_F$ quantifies the phase space $\sim \max\{\hbar v_F|q_3|, T\}$ of the left-moving p-h pair. The colliding right movers contribute the familiar phase space $\sim\epsilon^2$, resulting in a total phase space $\sim\epsilon^2\max\{T, \epsilon^2/\epsilon_F\}$ for the three-body collision.

We can now employ Eq. (10) to obtain the energy-relaxation rate $1/\tau_p$ of hot particles. Equation (10) shows that the typical energy loss per three-body collision is of order ϵ so that $1/\tau_p$ follows directly from integrating Eq. (10) over q_1 and p_1. At $T = 0$, this integration yields Eq. (1) above. The dominant $\sim\epsilon^2$ dependence emerges from the total phase space $\sim\epsilon^4$ combined with the singular dependence of the amplitude in Eq. (9) at small q_1, p_1, yielding a factor $\sim 1/\epsilon^2$ in the partial scattering rate.

Once $\epsilon \ll \epsilon_T$ at higher T, the phase space of the left movers is controlled by temperature. In this limit, the total scattering rate diverges logarithmically in the infrared, as $1/\tau \sim \int_{-\epsilon/\hbar v_F}^0 dq_1/q_1$. This singularity is regularized in the energy-relaxation rate which can be estimated from $1/\tau_p = \int dq_1dp_1(\Delta\epsilon/\epsilon)\mathcal{P}(q_1,p_1)$ in terms of the energy loss $\Delta\epsilon = \hbar v_F \min\{|q_1|, |p_1|\}$. This yields Eq. (2). The basic dependences on ϵ and T can again be understood from the phase-space and amplitude factors. We note that the singularity also does not carry over into a solution of the Boltzmann equation (5) as long as the injected electron distribution has a finite spectral width.

The derivation of the hole relaxation rate proceeds in close analogy. A hole injected at $k_{1'}$ generates two holes at $k_1 = k_{1'} - q_1$ and $k_2 = k_{1'} - p_1$ in a three-body collision. The corresponding partial rate $\mathcal{P}(q_1,p_1)$ is given by Eq. (10) with the replacement $k_1 \to k_{1'}$, changes of sign of q_1, p_1, and q_3, and the exchange $n^0 \leftrightarrow \bar{n}^0$. A crucial modification is the sign change of q_3, which limits the small momentum transfer process to $|q_3| \lesssim T/\hbar v_F$ and thus the energy loss $\Delta\epsilon$ of holes to $\Delta\epsilon \sim \epsilon_T^2/\epsilon$ when $\epsilon \gg \epsilon_T$. Hence, complete energy relaxation proceeds by multiple collisions and the energy-relaxation rate $1/\tau_h$ can be obtained from $d\epsilon/dt = \int dq_1dp_1\Delta\epsilon\mathcal{P}(q_1,p_1)$ through $\tau_h = \int_0^\epsilon d\epsilon'(d\epsilon'/dt)^{-1}$. Performing the remaining

integrals gives Eq. (3). This is slower than the relaxation of hot particles by $(\epsilon_T/\epsilon)^4$ since holes not only relax in $(\epsilon/\epsilon_T)^2$ steps, but phase space is also smaller by $T/(\epsilon^2/\epsilon_F)$. As discussed above, the energy-relaxation rate becomes equal for particles and holes when $\epsilon \ll \epsilon_T$.

We briefly comment on the competing process shown in Fig. 1(d), which involves comoving electrons only. In this process, two electrons near the Fermi energy are scattered in opposite directions in energy, allowing a high-energy particle to relax slightly. Assuming that the interaction is screened for these small momentum transfers, a similar calculation yields their contribution $1/\tau_p \sim (\epsilon_F/\hbar) \times (e^2/\kappa\hbar v_F)^4 (T/\epsilon_F)(\epsilon_T^6/\epsilon^2\epsilon_0^4)\ln^4|d/a|$ to the energy-relaxation rate. (Here, $\epsilon_0 = \hbar v_F/d$.) By comparing with Eqs. (1) and (2), we conclude that these processes are subdominant when $\epsilon \gg \epsilon_T$ or $T \ll \epsilon_F/(k_F d)^4$ [21].

Finally, we note that the thermalization rate for a given temperature difference ΔT between right- and left-moving electrons can be obtained in a standard manner by expanding the Boltzmann equation to linear order in ΔT. This yields the result quoted in Eq. (4) above.

Comparison with experiment.—The experiment [2] did not measure the ϵ dependence of the energy-relaxation rate, but provides bounds. The energy-relaxation rate was inferred to be larger (smaller) than 10^{11} s^{-1} for particles (holes) with $\epsilon \sim \epsilon_F/3$. This is consistent with our results which predict [22] $1/\tau_p \approx 10^{11}$ s^{-1} and $1/\tau_h \approx 5 \times 10^9$ s^{-1} for $\epsilon \approx \epsilon_F/3$. We also find $1/\tau_{\text{inter}} \approx 10^6$ s^{-1}, implying that temperature differences between left and right movers are sustained over long distances.

Conclusion.—One-dimensional electron systems out of equilibrium were brought into focus by recent experiments in various contexts such as carbon nanotubes [23], quantum Hall edge states [24,25], or quantum wires [2,26]. Here, we have discussed equilibration and thermalization of hot carriers in real quantum wires, emphasizing the important roles of spin and finite temperature within a perturbative approach for large excitation energies. Our work was motivated by and provides a quantitative framework for a recent experiment [2]. It elucidates a fundamental property of 1D electron liquids and thus has ramifications for 1D electron systems beyond quantum wires. Finally, our work also points to open issues. First and foremost, it would be interesting to understand the fate of energy relaxation for small excitation energies, where one must simultaneously cope with nonequilibrium and nonperturbative effects of the interactions.

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[21] Processes as in Fig. 1(d) may dominate energy relaxation in chiral quantum Hall edges.
[22] We use $v_F = 2 \times 10^5$ m/s, $k_F = 10^8$ m^{-1}, $a \sim 20$ nm, $\kappa = 12.4$, and $\epsilon_F = 6.6$ meV. Note that for $T = 0.25$ K, $\epsilon_T \simeq 0.38$ meV, so that $\epsilon \gg \epsilon_T$. Our estimates use a more accurate V_q than the one after Eq. (6), valid for $k_F a \sim 1$.
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