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Shortcuts to non-Abelian braiding

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Topological quantum information processing relies on adiabatic braiding of non-Abelian quasiparticles. Performing the braiding operations in finite time introduces transitions out of the ground-state manifold and deviations from the non-Abelian Berry phase. We show that these errors can be eliminated by suitably designed counterdiabatic correction terms in the Hamiltonian. We implement the resulting shortcuts to adiabaticity for simple protocols of non-Abelian braiding and show that the error suppression can be substantial even for approximate realizations of the counterdiabatic terms.

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Introduction. It is envisaged that the information processing of topological quantum computers relies on adiabatic braiding of non-Abelian quasiparticles [1–5]. Exchanging two nonabelions does not leave the quantum state unchanged, possibly up to a sign (as for fermions or bosons) or phase (as for Abelian anyons) factor, but rather effects a unitary rotation in a degenerate subspace of ground states. The ground-state degeneracy grows exponentially with the number of non-Abelian quasiparticles, and quantum information processing corresponds to manipulating the system's ground state by braiding operations. Majorana bound states in topological superconducting phases constitute the simplest example of such nonabelions [6], and there has been considerable experimental effort towards realizing a possible hardware [7–13], following a series of theoretical proposals [14–22].

Topological quantum information processing is immune to local sources of decoherence when braiding is performed adiabatically [1]. Quite generally, adiabaticity is protected by the gap of the underlying topological phase. Here we want to ask the question whether it is possible to realize the *exact* adiabatic quantum dynamics of the braiding operation, albeit in a *finite* time interval. There are obvious motivations why this would be desirable: First, any topological quantum computer would operate at a finite clock speed which necessarily entails possibly small, but nonzero, errors. Second, a topological quantum computer would presumably have to operate faster than parasitic decoherence processes such as quasiparticle poisoning or deviations from perfect ground-state degeneracy originating in the finite spatial extent of the Majorana quasiparticles. In both cases, such a scheme could then be used to offset the incurred errors, enabling longer computations or higher clock speeds.

Demirplak and Rice [23] as well as Berry [24] introduced a protocol that emulates the adiabatic dynamics of any *nondegenerate* Hamiltonian $H_0(t)$ as the exact quantum dynamics in finite time. This scheme is known alternately as transitionless quantum driving or a shortcut to adiabaticity [25–29]. The prize that comes with the shortcut is that the adiabatic quantum dynamics of $H_0(t)$ is generated by a Hamiltonian $H(t)$, which differs from H_0 by counterdiabatic terms $H_1(t)$. This shortcut to adiabaticity does not apply directly to the adiabatic braiding of non-Abelian quasiparticles because of the associated ground-state degeneracy. Here, we first generalize this scheme to systems with degenerate manifolds of states where adiabatic dynamics generates non-Abelian Berry phases. Then, we apply this generalized shortcut to non-Abelian statistics, using a simple model for braiding of Majorana bound states. Within this model, the braiding of Majorana zero modes is based on judiciously chosen temporal variations of the couplings between a number of Majorana end states. We find that shortcuts to non-Abelian braiding can be implemented by introducing a small number of additional local couplings.

Shortcuts to adiabaticity for degenerate systems. The exact quantum dynamics of a Hamiltonian $H(t)$ is generated by the corresponding time-evolution operator $\mathcal{U}(t)$ which satisfies the Schrödinger equation

$$i\partial_t \mathcal{U}(t) = H(t)\mathcal{U}(t). \tag{1}$$

Thus, we can give an explicit expression for the Hamiltonian $H(t)$ generating any prescribed quantum dynamics $\mathcal{U}(t)$,

$$H(t) = i[\partial_t \mathcal{U}]\mathcal{U}^\dagger. \tag{2}$$

The shortcut to adiabaticity [23,24] follows by inserting into this expression the adiabatic time-evolution operator

$$\mathcal{U}(t) = \sum_n e^{-i\int_0^t dt' E_n(t') + i\gamma_n(t)} |\psi_n(t)\rangle\langle\psi_n(0)| \tag{3}$$

for the Hamiltonian $H_0(t)$, with instantaneous eigenvalues $E_n(t)$, instantaneous eigenstates $|\psi_n(t)\rangle$, and Berry phase $\gamma_n(t) = i\int_0^t dt' \langle\psi_n(t')|\partial_{t'}\psi_n(t')\rangle$. One finds that $H(t) = H_0(t) + H_1(t)$ with the so-called counterdiabatic terms [23,24]

$$H_1(t) = i\sum_n (|\partial_t\psi_n\rangle\langle\psi_n| - |\psi_n\rangle\langle\psi_n|\partial_t\psi_n\rangle\langle\psi_n|). \tag{4}$$

Such shortcuts to adiabaticity have recently been implemented experimentally for effective two-level systems arising in trapped Bose-Einstein condensates [30] and for the electron spin of a single nitrogen vacancy center [31].

Following Wilczek and Zee [32], we now consider a Hamiltonian $H_0(t)$ whose instantaneous spectrum, defined through

$$H_0(t)\left|\psi_\alpha^n(t)\right\rangle = E_n(t)\left|\psi_\alpha^n(t)\right\rangle, \tag{5}$$

includes one or more sets of states $|\psi_\alpha^n(t)\rangle$ which remain degenerate for all t. Here, $\alpha = 1,\ldots,d_n$ labels the states within the degenerate subspace n of multiplicity d_n.

We first define $|\eta_\alpha^n(t)\rangle$ as the adiabatic solution of the time-dependent Schrödinger equation

$$i\partial_t\big|\eta_\alpha^n(t)\big\rangle = H_0(t)\big|\eta_\alpha^n(t)\big\rangle, \tag{6}$$

with initial condition $|\eta_\alpha^n(0)\rangle = |\psi_\alpha^n(0)\rangle$. In the adiabatic limit, the time-evolved state need not remain parallel to $|\psi_\alpha^n(t)\rangle$ but will in general be a linear combination of all basis states within the degenerate subspace,

$$\big|\eta_\alpha^n(t)\big\rangle = \sum_\beta U_{\alpha\beta}^n\big|\psi_\beta^n(t)\big\rangle. \tag{7}$$

Inserting this expansion into the time-dependent Schrödinger equation yields an equation for the coefficient matrices U^n,

$$i\partial_t U^n = U^n(E_n\mathbf{1} - A^n), \tag{8}$$

where $A_{\alpha\beta}^n = i\langle\psi_\beta^n|\partial_t\psi_\alpha^n\rangle$ denotes the non-Abelian Berry connection [32]. This is solved by

$$U^n(t) = e^{-i\int_0^t dt' E_n(t')}\tilde{T}\,e^{i\int_0^t dt' A^n(t')} \tag{9}$$

in terms of time-ordered exponentials.

The adiabatic time evolution of the Hamiltonian $H_0(t)$ follows the time-evolution operator

$$\mathcal{U}(t) = \sum_{n,\alpha}\big|\eta_\alpha^n(t)\big\rangle\big\langle\eta_\alpha^n(0)\big| = \sum_{n,\alpha\beta}U_{\alpha\beta}^n\big|\psi_\beta^n(t)\big\rangle\big\langle\psi_\alpha^n(0)\big|. \tag{10}$$

Now we use Eq. (2) to derive the Hamiltonian $H(t)$ for which this is the *exact* time-evolution operator. Inserting Eq. (10) into (2), we obtain the shortcut to adiabaticity (all quantities evaluated at time t)

$$H = i\sum_n\sum_{\alpha\beta}\big\{[(U^n)^\dagger\dot{U}^n]_{\beta\alpha}\big|\psi_\alpha^n\big\rangle\big\langle\psi_\beta^n\big| + [(U^n)^\dagger U^n]_{\beta\alpha}\big|\partial_t\psi_\alpha^n\big\rangle\big\langle\psi_\beta^n\big|\big\}. \tag{11}$$

The second term in H simplifies due to unitarity of U^n, $(U^n)^\dagger U^n = \mathbf{1}$. Combining unitarity and Eq. (8), we also have $i(U^n)^\dagger\dot{U}^n = (E_n\mathbf{1} - A^n)$, which simplifies the first term. With these identities, we readily find $H(t) = H_0(t) + H_1(t)$ with

$$H_1 = i\sum_n\left[\sum_\alpha\big|\partial_t\psi_\alpha^n\big\rangle\big\langle\psi_\alpha^n\big| - \sum_{\alpha\beta}\big|\psi_\alpha^n\big\rangle\big\langle\psi_\alpha^n\big|\partial_t\psi_\beta^n\big\rangle\big\langle\psi_\beta^n\big|\right]. \tag{12}$$

These counterdiabatic terms generalize the shortcut to adiabaticity to systems with degenerate spectra and non-Abelian Berry connections.

Majorana systems. In view of topological quantum information processing, we specifically consider the counterdiabatic terms for a Bogoliubov–de Gennes Hamiltonian in Majorana representation,

$$H_0 = i\sum_{n\alpha}\epsilon_n\gamma_{n,2\alpha-1}\gamma_{n,2\alpha}. \tag{13}$$

Here, both ϵ_n and the $\gamma_{n,\alpha}$ are explicitly time dependent and associated with the instantaneous Hamiltonian. The instantaneous many-body spectrum of H_0 contains degeneracies whenever an eigenenergy ϵ_n vanishes or when one or several nonzero ϵ_n are degenerate. The Majorana eigenmodes associated with ϵ_n are denoted by $\gamma_{n,\alpha}$, where α takes on $2N$ values for an N-fold degenerate energy ϵ_n. The counterdiabatic terms H_1 guarantee that the time evolution generated by the full Hamiltonian $H_0 + H_1$ does not take the Majorana eigenmodes $\gamma_{n,\alpha}$ out of the subspace n. At the same time, H_1 should not alter the time evolution within these subspaces. In the Supplemental Material [33] we show that these conditions yield

$$H_1 = \frac{i}{4}\sum_{n\alpha}\dot{\gamma}_{n,\alpha}\gamma_{n,\alpha} - \frac{i}{8}\sum_{n,\alpha\beta}\gamma_{n,\alpha}\{\gamma_{n,\alpha},\dot{\gamma}_{n,\beta}\}\gamma_{n,\beta}. \tag{14}$$

This result complements the counterdiabatic terms in first quantization in Eq. (12).

Application to non-Abelian braiding. A minimal model for non-Abelian braiding starts from a Y junction of three one-dimensional topological superconductors, labeled wire 1, 2, and 3, as illustrated in Fig. 1(a) [34–36]. If all three arms are in the topological phase, there are four Majorana bound states in this system. Three of these are located at the outer ends of the three wires, with Bogoliubov operators labeled γ_j for wire j, and a fourth Majorana mode γ_0 is located at the junction of the three wires. As long as the three arms have a finite length, these outer Majorana bound states hybridize with the central Majorana and the system is described by the Hamiltonian

$$H_0 = i\sum_{\alpha=1}^3\Delta_\alpha\gamma_0\gamma_\alpha. \tag{15}$$

This Hamiltonian couples the central Majorana γ_0 to a linear combination of the outer three Majoranas. We can thus readily bring it to the form of Eq. (13),

$$H_0 = ih_\Delta\gamma_0\gamma_\Delta, \tag{16}$$

with $\gamma_\Delta = (1/h_\Delta)\sum_{\alpha=1}^3\Delta_\alpha\gamma_\alpha$ and $h_\Delta = [\Delta_1^2 + \Delta_2^2 + \Delta_3^2]^{1/2}$. For any choice of the couplings Δ_j, there are also two linearly independent combinations of the outer Majoranas which do not appear in the Hamiltonian and thus remain true *zero-energy* Majoranas. Due to these zero-energy modes, the two eigenvalues of H_0 are each doubly degenerate. Specifically, when just one of the couplings Δ_j is nonzero,

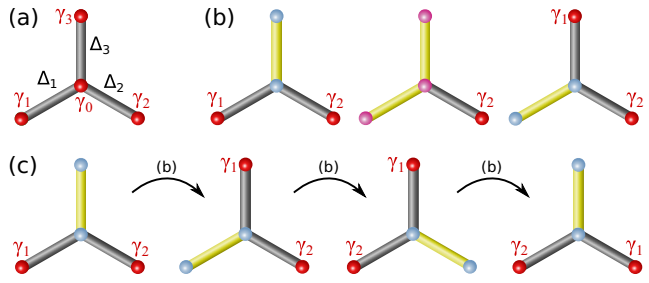

FIG. 1. (Color online) (a) Y junction with central Majorana γ_0 and three outer Majoranas γ_j ($j = 1,2,3$). The outer Majoranas are coupled to the inner Majoranas with strength Δ_j. (b) Basic step of the braiding procedure, moving a zero-energy Majorana from the end of wire 1 to the end of wire 3 by varying the Δ_j. Dark (light) wires indicate zero (nonzero) couplings Δ_j. Dark red circles correspond to zero-energy Majoranas, and light blue circles indicate Majoranas acquiring a finite energy by coupling. In the intermediate step, the zero-energy Majorana is delocalized over the three Majoranas along the light wires. (c) Three steps as described in (b) result in braiding the zero-energy Majoranas γ_1 and γ_2.

these two zero-energy Majoranas can be identified with the Majoranas located at the ends of those wires with zero coupling.

We assume that we can change the couplings Δ_j as a function of time. We can now imagine the following braiding procedure [34,36]. Initially, only Δ_3 is nonzero. Then, γ_1 and γ_2 are zero-energy Majoranas. In a first step, we move a zero-energy Majorana from the end of wire 1 to the end of wire 3, without involving the zero-energy Majorana γ_2, as shown in Fig. 1(b). To this end, first increase Δ_1 to a finite value. The zero-energy Majorana originally located at the end of wire 1 is now delocalized and a linear combination of γ_1 and γ_3. We then localize the Majorana zero mode at the end of wire 3 by reducing Δ_3 down to zero, leaving only Δ_1 nonzero. The braiding process is completed by two completely analogous moves [see Fig. 1(c)]: We first move the zero-energy Majorana from the end of wire 2 to the end of wire 1, and finally the zero-energy Majorana from wire 3 to wire 2. The combined effect of this procedure is to exchange the initial zero-energy Majoranas at the ends of wires 1 and 2. One can check easily [36] that the change of the state of the system under this adiabatic exchange is described by the familiar braiding matrix $U_{12} = \exp(\pi\gamma_1\gamma_2/4)$.

When performing this exchange operation over a finite time interval, there will be corrections to the adiabatic time evolution. We can now apply one of the non-Abelian shortcut formulas in Eqs. (12) or (14). As shown in the Supplemental Material [33], we obtain

$$H_1 = \frac{i}{2}\dot{\gamma}_\Delta\gamma_\Delta = \frac{i}{2h_\Delta^2}\sum_{\alpha<\beta}(\Delta_\beta\dot{\Delta}_\alpha - \Delta_\alpha\dot{\Delta}_\beta)\gamma_\alpha\gamma_\beta. \qquad (17)$$

Thus, the shortcut is based on additional couplings between the outer Majoranas, while the adiabatic braiding protocol only uses couplings between the central and the outer Majoranas [see Fig. 2(a)]. Specifically, during the basic step of moving a zero-energy Majorana from the end of wire i to wire j, only the couplings Δ_i and Δ_j are nonzero. According to Eq. (17), performing this step accurately in finite time merely requires the additional coupling between γ_i and γ_j.

Practical implementation. There has been considerable work on how to implement braiding based on one-dimensional superconducting phases [34–39]. The couplings of the Majoranas can, e.g., be varied by changing the length of the intervening topological section. However, this may not be easily compatible with the geometric constraints imposed by the shortcut protocol [cf. Fig. 2(a)]. A better approach may be to vary the magnitude of the topological gap. Both methods control the overlap of the Majorana end states and hence their coupling. Physically, this can be achieved, say, in quantum-wire based realizations, by changing the chemical potential by means of a gate electrode [34] or a supercurrent in the adjacent s-wave superconductor [40].

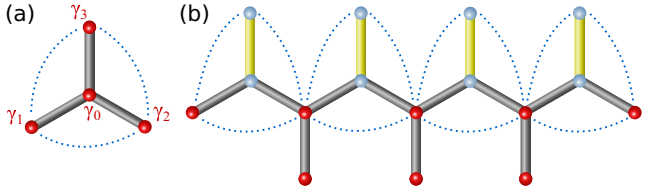

FIG. 2. (Color online) (a) Minimal implementation required for braiding with the shortcut protocol. The additional couplings needed for the shortcut protocol are shown in blue. (b) Wire network with many Majoranas allowing for pairwise exchanges of neighboring Majoranas including the shortcut protocol. Implementing the shortcut merely requires the addition of local couplings within the network.

More controlled variations of the Majorana couplings may be possible by exploiting charging effects [36] or by quantum dots [37]. For simplicity, assume that the quantum dot has a single level which is tunnel coupled to the ends of two topological wires with their Majorana end states. When the dot level is far from the Fermi energy, there is essentially no coupling between the adjacent Majoranas. Conversely, when the dot level is close to the Fermi energy, the Majoranas become strongly coupled. This approach modifies the coupling of the Majoranas by conventional gate control of a quantum-dot level and is also compatible with the geometric constraints of the shortcut protocol.

So far, we have focused on the exchange of two Majoranas within the minimal setting of a Y junction. Of course, one can readily imagine a scheme in which there is an entire keyboard of Majoranas and any two neighboring Majoranas can be readily braided. Importantly, amending this scheme to implement the counterdiabatic terms merely requires additional *local* couplings, as shown in Fig. 2(b).

Robustness. The manipulation of the quantum state is independent of the precise braiding path as long as the exchange is performed adiabatically. In contrast, the diabatic corrections are sensitive to the details of the braiding protocol. Consequently, the counterdiabatic terms (17) are not topologically protected, depend on the specifics of the braiding path, and, for full effect, have to be implemented exactly for a given $H_0(t)$.

However, we find that one can reach substantial reductions in the diabatic errors even when the shortcut protocol is implemented only with reasonable accuracy. We have computed the diabatic errors numerically, both for the bare braiding protocols and for approximate implementations of the counterdiabatic terms. Specifically, we consider the diabatic errors for

$$H_\lambda(t) = H_0(t) + \lambda H_1(t). \qquad (18)$$

For $\lambda = 1$, the counterdiabatic terms exactly compensate the diabatic corrections for any duration of the braiding protocol. As approximate implementations of the counterdiabatic terms, we consider relative errors of 10% ($\lambda = 0.9$) and 30% ($\lambda = 0.7$). We compute both the transition probability out of the degenerate subspace and the accumulated deviation from the adiabatic Berry phase.

Implementing the basic step [shown in Fig. 1(b)] of the braiding protocol in Fig. 1(c) by $\Delta_1(t) = \Delta\sin\varphi(t)$ and $\Delta_3(t) = \Delta\cos\varphi(t)$, with $\varphi(t)$ increasing from 0 to $\pi/2$, both the transition probability and the phase error exhibit a power-law dependence on the protocol duration T. The power law depends on the specific choice for $\varphi(t)$. Choosing the latter such that the first derivative vanishes at the end points yields a T^{-4} dependence. In fact, we find that for each order m of vanishing time derivatives, the T dependence of

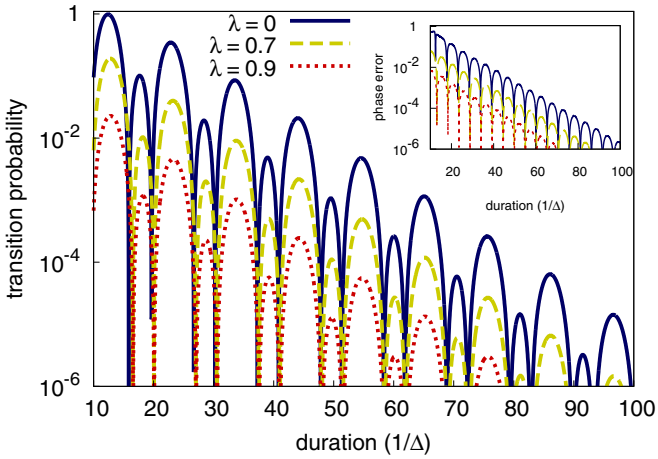

FIG. 3. (Color online) Diabatic errors vs duration of braiding protocol for the transition probability out of the degenerate subspace of the initial state. The inset shows the phase error relative to the non-Abelian Berry phase. For both quantities, curves are shown in the absence of counterdiabatic terms [$\lambda = 0$ in Eq. (18)] and with counterdiabatic terms with 10% ($\lambda = 0.9$) and 30% ($\lambda = 0.7$) relative error. There would be no diabatic error if the counterdiabatic errors were implemented exactly.

the errors scale with a power $2(m+1)$, which is a well-known effect (see, e.g., Ref. [41]; the additional factor of 2 arises since we consider expectation values rather than amplitudes). Corresponding numerical results are included with the Supplemental Material [33]. We also find similar power laws for the protocol given in Ref. [36], in which one initially increases Δ_1, leaving Δ_3 constant, and then reduces Δ_3 to zero in a second step [33].

Exponentially small transition rates can be realized by choosing $\Delta_1(t) = \Delta \sin^2 \varphi(t)$ and $\Delta_3(t) = \Delta \cos^2 \varphi(t)$. Now the gap assumes a minimum during the protocol as in the familiar Landau-Zener process. For the numerical calculation presented in Fig. 3 we have chosen $\varphi(t)$ to have a smooth derivative. The diabatic transition rate is indeed exponential in the protocol duration, which is somewhat counterintuitive as the transition rate actually *decreases* relative to the previously discussed protocols, although the gap is *smaller*. The phase error also exhibits exponential scaling, as shown in the inset of Fig. 3.

An exact implementation of the counterdiabatic terms fully corrects for these errors. As can be seen from Fig. 3, a suppression by two orders of magnitude merely requires an implementation which is accurate at the 10% level. Even a very rough implementation at the 30% level still substantially reduces the errors. More generally, we find that the relative error scales approximately as $(1-\lambda)^2$ with the accuracy of the implementation of H_1. This can be understood by treating the mismatch to the counterdiabatic Hamiltonian $(\lambda - 1)H_1(t)$ perturbatively (the power of 2 arises because the errors involve squared wave-function overlaps). It is also worth noting that the approximate counterdiabatic terms suppress the diabatic error, but do not modify its scaling with protocol duration.

Conclusions. In summary, we have generalized the concept of shortcuts to adiabaticity to non-Abelian Berry phases and showed how this can in principle be used to implement non-Abelian braiding operations exactly in a finite time. Such protocols can substantially improve the accuracy of braiding operations performed in a finite time interval. It is interesting to note that our scheme bears some resemblance with the concept of quasiadiabatic continuity for topological phases [42].

In this Rapid Communication we have focused on a simple model of non-Abelian braiding which excludes the quasiparticle continuum. The current protocols are therefore useful whenever there is a separation of scales between the finite-energy subgap states and the magnitude of the topological gap. Including the quasiparticle continuum is an interesting problem for future research. It should also be interesting to extend the current considerations for Majorana zero modes to more exotic non-Abelian quasiparticles.

Note added. Recently, we became aware of Ref. [43], which independently introduces shortcuts to non-Abelian manipulations in the context of holonomic quantum computation.

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