Half-filled Landau level as a Fermi liquid of dipolar quasiparticles

Ady Stern

Department of Condensed Matter Physics, The Weizmann Institute of Science, Rehovot 76100, Israel

Bertrand I. Halperin

Physics Department, Harvard University, Cambridge, Massachusetts 02138

Felix von Oppen*

Max-Planck-Institut für Kernphysik, Postfach 103980, 69029 Heidelberg, Germany
and Institut für Theoretische Physik, Universität zu Köln, Zülpicher Strasse 77, 50937 Köln, Germany

Steven H. Simon

Lucent Technologies Bell Laboratories, Murray Hill, New Jersey 07974
(Received 10 December 1998)

In this paper we study the relation between the conventional Fermion-Chern-Simons (FCS) theory of the half-filled Landau level ($\nu = 1/2$), and alternate descriptions that are based on the notion of neutral quasiparticles that carry electric-dipole moments. We have previously argued that these two approaches are equivalent, and that, e.g., the finite compressibility obtained in the FCS approach is also obtained from the alternate approach, provided that one properly takes into account a peculiar symmetry of the dipolar quasiparticles—the invariance of their energy to a shift of their center of mass momentum. Here, we demonstrate the equivalence of these two approaches in detail. We first study a model where the charge and flux of each fermion is smeared over a radius Q^{-1} where results can be calculated to leading order in the small parameter Q/k_F. We study two dipolar-quasiparticle descriptions of the $\nu = 1/2$ state in the small-Q model and confirm that they yield the same density-response function as in the FCS approach. We also study the single-particle Green's function and the effective mass, for one form of dipolar quasiparticles, and find the effective mass to be infrared divergent, exactly as in the FCS approach. Finally, we propose a form for a Fermi-liquid theory for the dipolar quasiparticles, which should be valid in the physical case where Q is infinite. [S0163-1829(99)05119-X]

I. INTRODUCTION

A. Overview

Many aspects of the behavior of an interacting two-dimensional electron system in the lowest Landau level have been understood using the ''composite fermion'' picture.[1,2] This description, in practice, includes a number of related concepts and computational methods. The term ''composite fermion'' was first introduced by Jain[3] in connection with the trial wave functions that he used to describe electronic ground states at the most prominent quantized Hall plateaus, where the filling fractions are of the form $\nu = p/(2p+1)$, with p an integer. A ''Fermion-Chern-Simons'' (FCS) approach was employed by Lopez and Fradkin,[4] and others[5] as an alternate way of understanding the composite fermion ground states and as a method for understanding the spectrum of collective excitations in the quantized Hall states. The FCS approach was also used by Halperin, Lee, and Read[6] (HLR) and by Kalmeyer and Zhang[7] to describe phenomena at, or near, even-denominator filling fractions, such as $\nu = 1/2$, where quantized Hall plateaus are not observed. The HLR analysis predicted that the ground state at these filling fractions should be *compressible*,[8] and that its properties could be understood by a perturbative analysis, starting from a ground state which is a filled Fermi sea of appropriately defined fermions.

The FCS approach begins with a unitary transformation, in which the electron system is converted to a system of fermions that interact with each other via a fictitious gauge field $\mathbf{a}_{CS}$ of the Chern-Simons type, as well as via the usual electromagnetic Coulomb repulsion. The Hamiltonian of the fermions is given below in Eq. (1).

A key prediction of the FCS theory is that the quasiparticles at a filling fraction such as $\nu = 1/2$ can travel in straight lines over large distances, oblivious to the effects of the strong applied magnetic field. Within the FCS formulation used by HLR,[6] this is most naturally explained by saying that the quasiparticles feel an effective magnetic field ΔB, which is the difference of the applied magnetic field and the mean value of a fictitious Chern-Simons magnetic field, and these two contributions cancel each other precisely at $\nu = 1/2$. At filling fractions slightly away from $\nu = 1/2$, the cancellation is not perfect, so $\Delta B \neq 0$. As a result, the quasiparticles should move in a circle, with a radius given by the ''effective cyclotron radius'' $R_c^* = k_F/|\Delta B|$, a prediction that has been confirmed by several experiments.[9] [Here k_F is the Fermi momentum, related to the electron density n_0 by $k_F = (4\pi n_0)^{1/2}$. We use units where $\hbar = 1$, the speed of light is 1, and the electron charge is -1.]

An alternate view of the HLR predictions, which was emphasized by Read,[10] is to say that the actual low-energy quasiparticles, obtained from the bare fermions of the HLR theory after ''screening'' by relaxation of the high-frequency magnetoplasma modes (or Kohn modes), which occur at the bare cyclotron frequency of the electron system, are actually

electrically neutral at $\nu = 1/2$. At nearby filling fractions, the low-energy quasiparticles have charge $e^* = (2\nu - 1)$, which coincides with the quasiparticle charge $-1/(2p+1)$ expected for the fractional quantized Hall states at filling fractions $\nu = p/(2p+1)$.[11] A quasiparticle with charge e^*, which sees the full magnetic field B, will have the same effective cyclotron radius R_c^* as a fermion of charge -1 that sees the effective field ΔB. Another important point that was also noted by Read, is that the low-energy quasiparticles at $\nu = 1/2$, though they are overall neutral, carry an electric dipole moment that is proportional to, and perpendicular to, the canonical momentum of the quasiparticle.[10,12]

Although the FCS theory has had many successes, and has several advantages over other formulations, it also has several disadvantages. First, the coupling to the Chern-Simons field, which is one of the parameters of the perturbation expansion, is not small in any case of physical interest. Second, the FCS analysis does not directly reveal a key feature of the system of electrons in the lowest Landau level: namely that all the intra-Landau-level excitation energies must vanish in the limit where the Coulomb repulsion is taken to zero. (An equivalent statement is that the effective masses and excitation energies remain finite if the band mass tends to zero while the strength of the electron-electron repulsion is held fixed.) Another disadvantage of the FCS approach is that it employs a perturbation theory where the fundamental entities are bare fermions with charge -1, which are quite far from the actual low-energy quasiparticles. (The true charge of the quasiparticles is revealed only after their interaction with the gauge field is taken into account, as described above.)

Various alternate formulations of the composite fermion picture, which attempt to overcome the disadvantages explained above, have been recently proposed.[13–16] Several authors have proposed formulations of composite-fermion theory which, by construction, lie entirely within the subspace of the lowest Landau level.[14–16] These formulations automatically incorporate the feature that the energy scale and effective mass are set by the electron-electron interaction. (A recent formulation by Lee[16] also incorporates the feature of exact particle-hole symmetry at $\nu = 1/2$ for electrons restricted to the lowest Landau level.)

In the present paper, however, we shall be more interested in an approach proposed recently by Murthy and Shankar[13] (MS). MS begin with the exact Hamiltonian of the FCS approach, given by Eq. (1) below, and make a unitary transformation in which the field operators for the fermions and the Chern-Simons vector potential in the FCS theory, $\psi_{CS}^\dagger, \psi_{CS}, \mathbf{a}_{CS}$, are transformed to a new set of operators $\psi^\dagger, \psi, \mathbf{a}$, which have several desirable features. First, in the transformed Hamiltonian the fermionic operators $\psi^\dagger, \psi$ are approximately decoupled from the gauge field $\mathbf{a}$. Second, the transformed fermions do indeed carry electric dipoles at $\nu = 1/2$. And third, by treating the transformed Hamiltonian in an appropriate approximation, Murthy and Shankar obtain results for the effective mass, which are tied to the strength of the electron-electron repulsion and independent of the bare mass.

If the low-energy quasiparticles are decoupled from the high-energy magnetoplasma modes at long wavelengths, it should be possible to describe the low-frequency response of the electron system at $\nu = 1/2$ directly in terms of these quasiparticles. However, an attempt to do this leads to a paradox, first noted by Shankar and Murthy:[13] if the dipolar fermions are assumed to behave like a conventional Fermi liquid of neutral particles, which carry only an electric-dipole moment, then one finds that it is costly, in terms of kinetic energy, to produce a fluctuation in the charge density at long wavelengths, and one finds that the electron system is incompressible, i.e., that the zero-frequency electron density response function at $\nu = 1/2$ vanishes $\propto q^2$ in the limit $q \to 0$, which would be contrary to the predictions of HLR. The possibility of incompressibility at $\nu = 1/2$ was also suggested in preprints by several other authors.[14,16]

The resolution of this paradox was outlined in a Comment[17] by two of the present authors (HS). As had been noted earlier by Haldane[18] for the case of composite fermions restricted to the lowest Landau level, the system of dipolar fermions has the special property that the total energy is unchanged if a constant $\mathbf{K}$ is added to the momentum of every particle. (This property, which we call "total-momentum invariance," or K invariance, does not hold, of course, for free fermions.) Consequently, as noted by HS, it costs very little energy to produce a long wavelength fluctuation in the transverse momentum density of the dipolar particles. Since the relation between the quasiparticle dipole moment and its momentum implies that the electron charge density is proportional to the gradient of the transverse momentum density, HS argued that the K invariance implies the finite compressibility of the electron system. The possibility of such a resolution of the paradox was mentioned in the original publication of Shankar and Murthy,[13] who speculated that the "drifting Fermi sea" peculiar to this system might lead to a finite compressibility. The conclusion of finite compressibility was also reached more recently by Read[15] and by Lee (in a revised version of Ref. 16), independently of HS.

It must be noted that the electronic density-density linear-response function is a measurable quantity, which should determine the outcome of experiments measuring surface-acoustic wave propagation, compressibility, magnetocapacitance, Coulomb drag, and optical response.[9] Results of all of these experiments have been interpreted in terms of the predictions of the conventional FCS theory, and would seem to support the prediction of finite compressibility.[9] Nevertheless, it is difficult to interpret them as definitively ruling out the possibility of incompressibility at $\nu = 1/2$, and MS have argued that some of the results can be fit quite well using a response function that has the feature of incompressibility. The experimental situation is complicated further by the presence of impurities, which might give rise to a finite compressibility even if the pure system were incompressible. Thus, it is comforting that the issue can be resolved on theoretical grounds, and that one finds the pure system to be compressible within all the current approaches.

The purpose of the present paper is to explore in greater detail the resolution of the compressibility paradox. We wish to see not only how the dipolar gas leads to a finite compressibility, but to examine more closely the relation between the FCS approach and several possible formulations in terms of dipolar fermions. We begin by studying a modification of the model *where it is possible to calculate the static*

compressibility and the dynamic response functions exactly, to lowest order in a small parameter, using either the FCS approach or an approach based on dipolar quasiparticles. We verify that in that calculation, one obtains exactly the same density-density response functions using the different approaches.

B. The Fermion-Chern-Simons approach

The unitary transformation, which transforms the electron system to a composite fermion description in the FCS approach, was originally employed by Leinaas and Myrheim[19] in the 1977 paper that introduced the concept of fractional statistics for two-dimensional systems. We shall follow common usage, however, and refer to the transformation as a "Chern-Simons transformation," and we shall refer to the resulting transformed fermions as "(bare) Chern-Simons fermions," or "CS fermions."

For $\nu = 1/2$, in the temporal (Weyl) gauge for the Chern-Simons gauge field ($a_0 = 0$), the Hamiltonian takes the form

$$H_{CS} = \int d\mathbf{r} \frac{1}{2m} |(-i\boldsymbol{\nabla} - a_{CS})\psi_{CS}(\mathbf{r})|^2$$

$$+ \frac{1}{2} \int d\mathbf{r}\, d\mathbf{r}' \, [|\psi_{CS}(\mathbf{r})|^2 - n_0]$$

$$\times v(\mathbf{r} - \mathbf{r}') \, [|\psi_{CS}(\mathbf{r}')|^2 - n_0]. \quad (1)$$

Here the two components of the Chern-Simons vector potential $\mathbf{a}_{CS}$ are canonically conjugate, satisfying $[a_x(\mathbf{r}), a_y(\mathbf{r}')] = i2\pi\tilde{\phi}\delta(\mathbf{r} - \mathbf{r}')$, n_0 is the average electron density, $v(\mathbf{r})$ is the electron-electron interaction, and the even integer $\tilde{\phi}$ is the number of flux quanta attached to each electron by the Chern-Simons transformation. (For $\nu = 1/2$, we choose $\tilde{\phi} = 2$.) The vector potential due to the applied magnetic field has been absorbed by shifting the Chern-Simons field $\mathbf{a}_{CS}$. Hence, in the Weyl gauge, the Chern-Simons field is dynamic and we have enlarged the Hilbert space compared to the Coulomb gauge used in HLR. The physical states of the theory are those that satisfy the Chern-Simons constraint,

$$\boldsymbol{\nabla} \times \mathbf{a}_{CS} = 2\pi\tilde{\phi}(\psi_{CS}^\dagger \psi_{CS} - n_0). \quad (2)$$

The density operator for the bare CS fermions, $\psi_{CS}^\dagger \psi_{CS}$, is the same in the Weyl and Coulomb gauges, and is identical to the density operator for the electrons.

In the FCS approach the ground state of the Hamiltonian (1) and the energies of low-lying excited states are first obtained in the Hartree approximation; then one calculates response functions and corrections to the excitation energies using the random-phase approximation[4,6] (RPA) or more sophisticated approximations based on a Feynman-diagram analysis of the perturbations arising from the Chern-Simons and Coulomb interactions.[20–23] In particular, these calculations conclude that the $\nu = 1/2$ state is "compressible;" i.e., the zero-frequency density-density response function of the electrons is found to be finite in the limit of small wave vector q if the interaction between the electrons is of short range, and to equal $q\epsilon/2\pi e^2$ if the electrons interact via an unscreened Coulomb interaction (here ϵ is the dielectric constant). Furthermore, it is concluded that the relaxation of

charge fluctuations follows a dispersion law of $\omega \propto iq^3$ for short-range electron-electron interaction (and $\omega \propto iq^2$ for unscreened Coulomb interaction). In addition, it has been argued that the effective mass of the quasiparticles should diverge close to the Fermi surface (logarithmically in the case of Coulomb interactions), and that this divergence should be manifest in the behavior of the energy gaps of the quantized Hall states close to $\nu = 1/2$.

The modified model used for our calculations employs a momentum cutoff $Q \ll k_F$ for the Chern-Simons interaction. A similar cutoff was used previously by one of the authors[24] in the context of anyon models of high-temperature superconductivity. More recently, such a cutoff was employed within the context of the quantum Hall effects by Raghav Chari *et al.*,[25] who termed the approach "fat flux quanta." The modified model, which is defined more precisely below [see Eq. 30], may be thought of as a system of composite fermions whose charge and flux tubes are spread over a finite radius, of order Q^{-1}. Barring singular phase transitions, the use of a small value of Q should make the RPA exact, to lowest order in Q, and it enables us to carry out explicitly the Murthy-Shankar unitary transformation from CS fermions to dipole fermions. The Hamiltonian we obtain for the dipole fermions is not that of free dipolar particles, but rather a more complicated one, given by Eq. (46) below. The Hamiltonian contains several terms that are additional to the ones considered explicitly by MS.

C. Alternative definitions of the quasiparticles

An important point, which will be discussed further below, is that in intermediate stages, various definitions of the low-energy dipolar quasiparticles are possible, which are not identical in the long wavelength limit, even though they all lead to the same density-density response function for the electrons. In all of the descriptions we consider here, for an infinitesimal fluctuation about $\nu = 1/2$, the electron density ρ^e is given by the relation

$$\rho^e = \frac{\boldsymbol{\nabla} \times \mathbf{g}}{2\pi\tilde{\phi}n_0}, \quad (3)$$

where $\mathbf{g}$ is the momentum density of the quasiparticles. Equation (3) relates the momentum density to the charge density. Nevertheless, one has freedom in the precise definition of the position of a quasiparticle, and hence in the relation between the electron density and the density of quasiparticles at nonzero wave vector $\mathbf{q}$.

In particular, in Sec. II we shall derive a description of the half-filled Landau level that employs dipolar quasiparticles that obey a constraint

$$\rho(\mathbf{q}) = \frac{i\mathbf{q} \times \mathbf{g}}{2\pi\tilde{\phi}n_0} \quad (4)$$

and whose density is related to the electron density by

$$\rho(\mathbf{q}) = \rho^e(\mathbf{q}). \quad (5)$$

One sees that in the long-wavelength limit the positions of the quasiparticles employed in Sec. II must coincide, on av-

erage, with the positions of the electrons in the system. We shall describe such quasiparticles as ''electron-centered'' quasiparticles.

In Sec. III below we discuss the quasiparticles used by MS, which obey a constraint

$$\rho(\mathbf{q}) = \frac{i}{2} \frac{\mathbf{q} \times \mathbf{g}}{2\pi\tilde{\phi}n_0}, \tag{6}$$

while the relation between ρ^e and the quasiparticle density ρ, for $q \neq 0$, is $\rho(\mathbf{q}) = (1/2)\rho^e(\mathbf{q})$. The positions of these quasiparticles are shifted, on average, relative to the electron positions, by an amount $\hat{z} \times \langle \mathbf{g}(\mathbf{r}) \rangle / (4\pi\tilde{\phi})$ where $\langle \mathbf{g}(\mathbf{r}) \rangle$ is the momentum density averaged over a volume of radius Q^{-1} about the position of the quasiparticle. The relations (3) and (6) would be satisfied by a collection of dipolar quasiparticles when the dipole moment of a quasiparticle of momentum $\mathbf{k}$ is $-\hat{z} \times \mathbf{k}/2\pi\tilde{\phi}n_0$ and *the position of the quasiparticle is defined to be halfway between the negative and positive charges of the electric dipole*. We describe quasiparticles of this type as ''shifted quasiparticles.''

Although the explicit calculation we carry out is to lowest order in Q, we believe that it sheds light also on the physical case, in which no upper cutoff to Q is present. In particular, it leads us to show that while the density response of a ''conventional'' Fermi liquid of dipoles to a scalar potential would be weak, leading to an incompressibility of the liquid, this is not true once the fermions satisfy K invariance and their dipole moment is proportional and perpendicular to their momentum. Under these conditions, the fermions density-density response function has the q, ω dependence one obtains from the FCS approach (for small q and ω) independent of whether Q/k_F is small or large.

D. Outline

The outline of the paper is as follows. In the next section, we show how one can derive a description in terms of electron-centered dipolar quasiparticles, in a simple fashion, from the fermion Chern-Simons Hamiltonian (1). This is done most naturally by using a Lagrangian formalism, and eliminating the high-energy modes, as well as the Chern-Simons vector potential, in an RPA-like approximation to the Lagrangian. In Sec. III, we obtain shifted quasiparticles, from the same starting Hamiltonian (1), using a Hamiltonian formalism and the unitary transformation employed by MS, for the model with small momentum-cutoff Q. In both cases, we calculate the electron-density response function, and obtain the same results as originally obtained by HLR. In Sec. IV, we make some additional comments on the physical significance of the small-Q model and the validity of the RPA in that model. In Sec. V, we discuss the effective mass and the one-fermion Green's function for the shifted quasiparticles employed in Sec. III, in the small-Q model, and compare them to results of the conventional FCS approach. The Fermi-liquid description, which we believe to be valid in the actual $\nu = 1/2$ system, where there is no small cutoff Q, is discussed in Sec. VI, for the case of electron-centered quasiparticles. Our conclusions are then summarized in Sec. VII. Some details of the response matrix used in the calculations of Sec. III are presented in the Appendix.

II. ELECTRON-CENTERED QUASIPARTICLES: A LAGRANGIAN APPROACH

In this section, we begin with a Lagrangian formulation equivalent to the Hamiltonian (1) of the FCS theory in the temporal gauge, and integrate out the variables associated with the Chern-Simons field. Using the random phase approximation, we obtain the same density-response function as was obtained by HLR, using the Coulomb gauge for the Chern-Simons field. We show, however, that the low-energy part of the fermionic theory has a natural interpretation in terms of dipolar quasiparticles whose density coincides with the density of the electrons, at long wavelengths (i.e., they are ''electron-centered'' quasiparticles). We also find an explicit form for the effective Lagrangian of the quasiparticles at low energies, using approximations that, along with the RPA, are valid in the limit where there is a wave vector cutoff Q, which is small compared to k_F.

The zero-temperature action corresponding to the Hamiltonian (1) is

$$S = \int dt\, d\mathbf{r} \left\{ \frac{1}{2} \frac{1}{2\pi\tilde{\phi}} (a_x \partial_0 a_y - a_y \partial_0 a_x) \right.$$
$$\left. + \bar{\psi} i \partial_0 \psi - \frac{1}{2m} \bar{\psi}(-i\nabla + \mathbf{A} - \mathbf{a})^2 \psi \right\} + S_{\text{Coul}}. \tag{7}$$

We drop the subscript CS in this section, because we will not consider here a canonical transformation of the fields. Using the constraint (2) inherent in the CS formulation for $\nu = 1/2$, the Coulomb interaction can be written in terms of the CS field as

$$S_{\text{Coul}} = -\frac{1}{2} \frac{1}{(2\pi\tilde{\phi})^2}$$
$$\times \int dt\, d\mathbf{r} d\mathbf{r}' [\nabla \times \mathbf{a}(\mathbf{r})] v(\mathbf{r} - \mathbf{r}') [\nabla' \times \mathbf{a}(\mathbf{r}')].$$
$$\tag{8}$$

We included in the action an external vector potential $\mathbf{A}$ as a source field for generating the electronic current-response function $K_{\alpha\beta}^e(\mathbf{q}, \omega)$. For the purposes of this section, it will be useful to consider $K_{\alpha\beta}^e$ as a 2×2 matrix, where α, β can take on the values l and t, for the longitudinal and transverse directions with respect to $\mathbf{q}$. The density-density response function $K_{\rho\rho}^e$ is related to $K_{\alpha\beta}^e$ by current conservation and gauge invariance, so that $K_{\rho\rho}^e(\mathbf{q}, \omega) = (q^2/\omega^2) K_{ll}^e(\mathbf{q}, \omega)$.

We simplify the action by the approximation $\bar{\psi}\psi \rightarrow n_0$ in the diamagnetic terms with n_0 the average electron density. This simplification is implicit in the random-phase approximation used by HLR and was also used by MS. It becomes exact in the limit where the momentum cutoff Q, discussed in the Introduction and in Sec. IV below, is taken to be very small.

As a first step towards an effective theory of the low-energy quasiparticles, we integrate out the CS field $\mathbf{a}$, which describes the magnetoplasma modes, and obtain

$$S = \bar{\psi} i \partial_0 \psi - \frac{1}{2m} |\nabla \psi|^2 - \frac{1}{2m^2} \mathbf{g} \frac{U}{1 - n_0 U/m} \mathbf{g}$$

$$- \frac{1}{m} \mathbf{A} \frac{1}{1 - n_0 U/m} \mathbf{g} - \frac{n_0}{2m} \mathbf{A} \frac{1}{1 - n_0 U/m} \mathbf{A}. \qquad (9)$$

Here, space and time integrals are left implicit, $\mathbf{g}$ denotes the canonical momentum density of the fermions, $\mathbf{g} = -(i/2)[\bar{\psi}\nabla\psi - (\nabla\bar{\psi}\psi)]$, and the operator U, in frequency and momentum representation, is

$$U(\mathbf{q}, \omega) = \begin{pmatrix} (q^2/\omega^2)v(q) & 2\pi\tilde{\phi}/i\omega \\ -2\pi\tilde{\phi}/i\omega & 0 \end{pmatrix}. \qquad (10)$$

Differentiating the effective action (9) with respect to the source field $\mathbf{A}$, we obtain for the electronic current-response function a matrix K^e, which we write in the form $K^e = K^{mp} + K^d$ with

$$K^{mp}(\mathbf{q}, \omega) = -\frac{n_0}{m}\left[1 - \frac{n_0}{m}U\right]^{-1}, \qquad (11)$$

and

$$K^d_{\alpha\beta}(\mathbf{q}, \omega) = \left\langle \left[\frac{-1/m}{1 - n_0 U/m}\mathbf{g}(\mathbf{q}, \omega)\right]_\alpha \right.$$
$$\left. \times \left[\frac{-1/m}{1 - n_0 U/m}\mathbf{g}(-\mathbf{q}, -\omega)\right]_\beta \right\rangle. \qquad (12)$$

Since we are interested in response functions, here and below expressions like $\langle A_{\mathbf{q},\omega}B_{-\mathbf{q},-\omega}\rangle$ are to be understood as the Fourier transform of the retarded correlator $i\theta(t)\langle[A(\mathbf{x},t), B(0,0)]\rangle$. In this section, angular brackets denote an expectation value in the ground state of the action (9) with $\mathbf{A} = 0$. The superscripts mp and d anticipate the fact that the two contributions to K^e will be identified with the magnetoplasma oscillators and the dipolar quasiparticles, respectively.

We first consider K^{mp} in more detail. One readily establishes that the corresponding contribution to the electronic density-density correlator is

$$K^e_{\rho\rho}(\mathbf{q}, \omega) = \frac{n_0}{m} \frac{q^2}{\omega_c^2 - \omega^2 + \frac{n_0}{m}q^2 v(q)}. \qquad (13)$$

Hence, this contribution to K^e reproduces Kohn's mode. The corresponding result for the conductivity tensor for small $\mathbf{q}$ and ω reproduces the correct Hall conductivity,

$$\sigma^{mp}(\mathbf{q}, \omega) = \frac{1}{2\pi\tilde{\phi}}\begin{pmatrix} i\omega/\omega_c & 1 \\ -1 & i\omega/\omega_c \end{pmatrix}. \qquad (14)$$

These results reflect the facts that there is no contribution to the Hall conductivity from K^d at $\nu = 1/2$ in a system without impurities, and that the contribution of the quasiparticles to the electromagnetic response is negligible compared to that of the Kohn mode, for any finite frequency, in the limit $q \rightarrow 0$.

Nevertheless, the fermions are responsible for interesting low-energy physics. Before deriving an effective low-energy action, it is useful to collect the conditions that such an approximation would need to satisfy:

(A) The first condition (condition A) arises because, due to gauge invariance, we can compute the fermionic contribution to the density-density correlator $K^e_{\rho\rho}$ in two different ways. We can either use directly that $\rho^e = \rho = \bar{\psi}\psi$, or compute K^e_{ll} from Eq. (12) and then use its relation to $K^e_{\rho\rho}$. We will see below that assuring that both approaches yield the same result is closely related to a consistent treatment of the constraint in the Hamiltonian approach.

(B) The second condition (condition B) is a consequence of the fact that even once the Weyl gauge $a_0 = 0$ is specified, we can still make the limited gauge transformations

$$a_l(\mathbf{q}, \omega = 0) \rightarrow a_l(\mathbf{q}, \omega = 0) + f(\mathbf{q})$$
$$(15)$$
$$a_t(\mathbf{q} = 0, \omega = 0) \rightarrow a_t(\mathbf{q} = 0, \omega = 0) + \text{const.}$$

These transformations must leave the action and the physical current unchanged provided that we simultaneously change the canonical momentum density of the fermions by

$$g_l(\mathbf{q}, \omega = 0) \rightarrow g_l(\mathbf{q}, \omega = 0) - n_0 f(\mathbf{q})$$
$$(16)$$
$$g_t(\mathbf{q} = 0, \omega = 0) \rightarrow g_t(\mathbf{q} = 0, \omega = 0) - n_0 \text{ const.}$$

Here, we made the replacement $\bar{\psi}\psi \rightarrow n_0$ as in the diamagnetic term of the action. For $q = 0$ the transformation (16) shifts the momentum of each fermion by a constant. We shall shortly employ an effective low-energy action where the field $\mathbf{a}$ no longer appears [see Eq. (17) below, which results from Eq. (9)]. In this case, the transformation (16) by itself, without Eq. (15), must leave the action and the physical currents unchanged. This is what we have called K-invariance. As a consequence of this invariance, the zero frequency correlator of the momentum density must diverge as $q \rightarrow 0$, being the inverse of the energy cost associated with a uniform momentum shift.

We now consider the action (9), for $\mathbf{A} = 0$, in the limit of small frequency and wave vector. In this limit, we can expand in powers of mU^{-1}/n_0 whose matrix elements are proportional to either q or ω. This gives the effective low-energy action

$$S = \bar{\psi} i \partial_0 \psi - \frac{1}{2m}|\nabla\psi|^2 + \frac{1}{2n_0 m}\mathbf{g}\,\mathbf{g} + \frac{1}{2n_0^2}\mathbf{g}\,U^{-1}\mathbf{g}. \qquad (17)$$

Applying the same expansion to K^d, one obtains

$$K^d_{\alpha\beta}(\mathbf{q}, \omega) \simeq \left\langle\left[\frac{1}{n_0}U^{-1}\mathbf{g}\right]_\alpha\left[\frac{1}{n_0}U^{-1}\mathbf{g}\right]_\beta\right\rangle. \qquad (18)$$

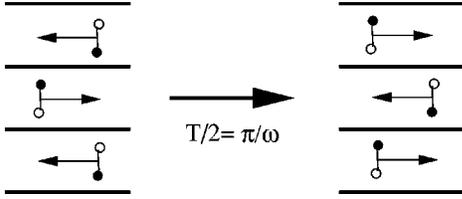This implies that a fermionic momentum density $\mathbf{g}$ is associated with a charge current

FIG. 1. Schematic drawing of a transverse quasiparticle momentum density $g_t = g_0 \cos(qy - \omega t)$ at two times separated by half a period. Full (empty) circles indicate positive (negative) charges and arrows the direction of the quasiparticle momentum. In agreement with Eqs. (19) and (20), the gradient of a transverse momentum density is associated with a charge density and its time derivative implies a longitudinal charge current.

$$\mathbf{j}^e = \frac{1}{n_0} U^{-1} \mathbf{g} = \begin{bmatrix} -\dfrac{i\omega}{2\pi\tilde{\phi} n_0} g_t \\[2ex] \dfrac{i\omega}{2\pi\tilde{\phi} n_0} g_l - \dfrac{v(q)q^2}{(2\pi\tilde{\phi})^2 n_0} g_t \end{bmatrix}. \qquad (19)$$

The (electronic) continuity equation yields for the charge density

$$\rho^e(\mathbf{q},\omega) = \frac{iq}{2\pi\tilde{\phi} n_0} g_t(\mathbf{q},\omega). \qquad (20)$$

These equations are naturally interpreted if one identifies the fermions as dipoles with dipole-moment perpendicular to their canonical momentum. This is illustrated in Fig. 1. We see that the time (space) derivative of a transverse momentum density is associated with a longitudinal charge current (charge density). Similarly, one convinces oneself that the time derivative of a longitudinal quasiparticle momentum corresponds to a transverse charge current. The additional contribution to the transverse charge current associated with the Coulomb interaction can be understood as follows. According to Eq. (20), a transverse fermion momentum density is associated with a charge density, which in turn produces an electric field. Due to the finite Hall conductivity, this longitudinal electric field induces a transverse Hall current.

The above discussion, which was restricted to the contributions of fluctuations at long wavelengths, should be directly applicable to the model where there is a wave vector cutoff Q, which is taken to zero, a model in which the RPA is presumably exact (see Sec. IV). However, there is another way of looking at the expansion leading to the low-energy action (17), which suggests that it should have wider applicability. As was discussed in the Introduction, we expect that in the limit where the band mass $m \to 0$, so that the cyclotron frequency is infinite, there remains a finite energy scale for intra-Landau-level excitations, and there should be a finite contribution to the response functions from these excitations. If one expands the effective action (9) in powers of m, keeping only terms that do not vanish in the limit of $m \to 0$, one readily recovers from Eq. (9) the low-energy action (17). While neither of these expansions is truly satisfactory, it is encouraging that both lead to the same result.

An important feature of the quasiparticle action (17) is that it contains a coupling between longitudinal and transverse momentum densities due to the off-diagonal terms in

$\mathbf{g} U^{-1} \mathbf{g}$. This additional term is crucial in order for the action (17) to satisfy condition A. To see this, one can derive the analog of the continuity equation for the dipole action (17). The continuity equation is modified in the present case because the interactions involve the momentum currents $\mathbf{g}$. Within the saddle-point approximation, we can proceed either by directly applying the Euler-Lagrange equations of motion or by deriving the Noether current associated with the invariance of the action under a global phase change of ψ. One thus obtains

$$i\omega \left\{ \rho(\mathbf{q},\omega) - \frac{iq}{2\pi\tilde{\phi} n_0} g_t(\mathbf{q},\omega) \right\} = 0. \qquad (21)$$

In deriving this equation, we have again made the replacement $\bar{\psi}\psi \to n_0$. Hence, we observe that for nonzero frequency and at the RPA level, this modified continuity equation guarantees that condition A is satisfied. By contrast, if one neglects the coupling of longitudinal and transverse momentum densities, one finds $i\omega\rho(\mathbf{q},\omega) = 0$, in violation of condition A. Indeed, we will see below that this coupling is crucial for precisely reproducing the RPA results of HLR from the present approach.

We now turn to obtaining the dipole contribution to the electronic density-density correlator by using Eq. (18). To this end, we compute the quasiparticle momentum-density correlation function $D^d_{\alpha\beta}(\mathbf{q},\omega) = \langle g_\alpha(\mathbf{q},\omega) g_\beta(-\mathbf{q},-\omega) \rangle$ in the random-phase approximation. One finds

$$[(D^d)^{-1}]_{\alpha\beta} = [(D^0)^{-1}]_{\alpha\beta} - \frac{1}{n_0 m} \delta_{\alpha\beta} - \frac{1}{n_0^2} [U^{-1}]_{\alpha\beta}, \qquad (22)$$

where D^0 denotes the correlator of free fermions. In the limit $\omega \ll v_F q$, we have

$$\frac{1}{m^2} D^0_{ll}(q,\omega) = \frac{n_0}{m} + \frac{\omega^2}{q^2} \left[\frac{m}{2\pi} + \frac{m}{2\pi} \frac{i\omega}{v_F q} \right],$$

$$\frac{1}{m^2} D^0_{tt}(q,\omega) = \frac{n_0}{m} - \frac{q^2}{24\pi m} + \frac{i2n_0\omega}{k_F q}. \qquad (23)$$

To bring out the unusual properties of the dipoles, we first focus on the limit of zero frequency. In this limit, $D^0_{ll}(\mathbf{q},\omega=0) = n_0 m$ and $D^0_{tt}(\mathbf{q},\omega=0) = n_0 m - mq^2/24\pi$. The result for $D^0_{ll}(\mathbf{q},\omega=0)$ follows directly from gauge invariance, specifically the fact that a *time-independent* longitudinal vector potential can have no physical effect, and the usual expression for the physical current operator.

Using Eqs. (22) and (23), one finds for $\omega = 0$ that D^d of the dipoles is diagonal with

$$[(D^d)^{-1}(\mathbf{q},\omega=0)]_{ll} = 0,$$

$$[(D^d)^{-1}(\mathbf{q},\omega=0)]_{tt} = \frac{q^2}{24\pi m n_0^2} + \frac{q^2 v(q)}{(2\pi\tilde{\phi})^2 n^2}. \qquad (24)$$

Hence, the longitudinal correlator of the dipoles diverges for all $\mathbf{q}$ once $\omega = 0$, while the transverse correlator diverges for $\mathbf{q} \to 0$ like $1/q$ for Coulomb interaction and like $1/q^2$ for a short-range interaction, in stark contrast to the free-fermion

results quoted above. These results are a consequence of condition B. We conclude that the dipole action (17) satisfies condition B at least at the RPA level.

How do the dipoles evade the usual theorem that the zero-frequency correlator $D_{ll}(\mathbf{q}, \omega = 0)$ be $n_0 m$? To make the dipole action (17) gauge invariant, we would need to replace the canonical momenta in the interaction terms by minimally coupled kinetic momenta. This introduces the vector potential into the interaction terms. Computing the gauge-invariant current operator by taking the derivative of this Hamiltonian with respect to the vector potential, we see that it takes a form different from the usual one. It is for this reason that the standard argument for $D_{ll}^0(\mathbf{q}, \omega = 0)$ does not apply to the dipoles.

Using Eq. (20) and the RPA Eqs. (22) and (23), one finds for the electronic density-density correlator in the limit $\omega \ll v_F q$

$$K_{\rho\rho}^e(\mathbf{q}, \omega) = \cfrac{1}{v(q) + \cfrac{2\pi}{m} + \cfrac{(2\pi\tilde{\phi})^2}{24\pi m} - i(2\pi\tilde{\phi})^2 \cfrac{2n_0\omega}{k_F q^3}}.$$

(25)

This coincides exactly with the corresponding expression obtained by HLR.[6] One easily checks that the second term in the denominator is due to the coupling between longitudinal and transverse momentum density in the above dipole action.

It is instructive to compare these calculations to those in the Coulomb gauge $\nabla \cdot \mathbf{a} = 0$ used by HLR. In that gauge, the action is

$$S = \int dt\, d\mathbf{r} \left\{ \bar{\psi} i \partial_0 \psi + a_0 \left(-\frac{\nabla \times \mathbf{a}}{2\pi\tilde{\phi}} + \bar{\psi}\psi - n_0 \right) \right.$$
$$\left. - \frac{1}{2m} \bar{\psi}(-i\nabla + \mathbf{A} + \mathbf{a})^2 \psi \right\} + S_{\text{Coul}}.$$

(26)

While the CS field $\mathbf{a}$ has only a transverse component, we keep both longitudinal and transverse component for the source field $\mathbf{A}$. Integrating out the CS field, one obtains the effective fermionic action

$$S = \bar{\psi}\partial_0 \psi - \frac{1}{2m}|\nabla\psi|^2 - \frac{1}{m}\mathbf{A}\mathbf{g} - \frac{n_0}{2m}\mathbf{A}^2$$
$$- \frac{1}{2}(\bar{\psi}\psi - n_0)\left[v(q) + \frac{(2\pi\tilde{\phi})^2 n_0}{mq^2} \right](\bar{\psi}\psi - n_0)$$
$$- (\bar{\psi}\psi - n_0)\frac{2\pi\tilde{\phi}}{iq}\left(\frac{1}{m}g_t + \frac{n_0}{m}A_t \right).$$

(27)

This effective action does not suggest a simple low-energy expansion analogous to the one found above for the Weyl gauge. Differentiating S with respect to the source field $\mathbf{A}$, we obtain for charge density and current

$$\rho^e = \bar{\psi}\psi,$$

$$j_l^e = -\frac{1}{m}g_l,$$

(28)

$$j_t^e = -\frac{1}{m}g_t - \frac{n_0}{m}a_t,$$

where $a_t = (2\pi\tilde{\phi}/iq)(\bar{\psi}\psi - n_0)$. These expressions are appropriate for composite fermions that carry charge e and that are subject to an effective magnetic field associated with deviations in density from half filling. The vector potential a_t appears explicitly in the formula for the electron current, and there is no manifest separation between the high-energy and low-energy physics at this stage.

To summarize, by analyzing the low-energy response of the Chern-Simons Lagrangian in the Weyl gauge within the RPA we identified quasiparticles of a dipolar nature, whose position coincides with that of the electrons. In the next section we carry out an analysis of the Chern-Simons Hamiltonian that leads us to describe the same low-energy dynamics in terms of quasiparticles whose position is shifted with respect to that of the electrons.

III. SHIFTED QUASIPARTICLES: A HAMILTONIAN APPROACH

The Hamiltonian (1) represents the problem in terms of two coupled sets of degrees of freedom, ψ_{CS} and $\mathbf{a}_{\text{CS}}$. The dipolar field-theoretic approach to this Hamiltonian, initiated by Murthy and Shankar,[13] is motivated by the hope that if transformed properly, the gauge-field degrees of freedom should be decoupled from the fermionic degrees of freedom at the long wavelength limit. The former would then describe a set of harmonic oscillators representing the inter-Landau-level magneto plasmons, while the latter would describe intra-Landau-level physics. An insight into this long wavelength decoupling, as well as into the K invariance, is gained when the Hamiltonian (1) is written as

$$H_{\text{CS}} = \int d\mathbf{r} \left[\frac{1}{2m}|\nabla\psi_{\text{CS}}|^2 - \mathbf{g}_{\text{CS}}\left(\frac{1}{2mn_{\text{CS}}} \right)\mathbf{g}_{\text{CS}} \right.$$
$$+ \left(\mathbf{a}_{\text{CS}} - \mathbf{g}_{\text{CS}}\frac{1}{n_{\text{CS}}} \right)\frac{n_{\text{CS}}}{2m}\left(\mathbf{a}_{\text{CS}} - \frac{1}{n_{\text{CS}}}\mathbf{g}_{\text{CS}} \right) \right]$$
$$+ \frac{1}{2}\int d\mathbf{r}\int d\mathbf{r}'[n_{\text{CS}}(\mathbf{r}) - n_0]$$
$$\times v(\mathbf{r} - \mathbf{r}')[n_{\text{CS}}(\mathbf{r}') - n_0].$$

(29)

In Eq. (29), the momentum density of the fermions is $\mathbf{g}_{\text{CS}}(\mathbf{r}) \equiv -(i/2)[\psi_{\text{CS}}^\dagger \nabla\psi_{\text{CS}} - (\nabla\psi_{\text{CS}}^\dagger)\psi_{\text{CS}}]$, their density is $n_{\text{CS}}(\mathbf{r}) \equiv \psi_{\text{CS}}^\dagger(\mathbf{r})\psi_{\text{CS}}(\mathbf{r})$, and the $\mathbf{r}$ dependence of ψ, $\mathbf{g}$, and n is suppressed for clarity.

Given the form of the Hamiltonian, one is led to approximate $n_{\text{CS}} \approx n_0$ inside the square brackets of Eq. (29), and to write the Hamiltonian in momentum space as (note that the area of the system is taken to be 1)

$$H_{CS} \approx \sum_{\mathbf{k}} \frac{k^2}{2m} \psi_{CS}(\mathbf{k}) \psi_{CS}(-\mathbf{k})$$

$$- \frac{1}{2mn_0} \sum_{\mathbf{q}} \mathbf{g}_{CS}(\mathbf{q}) \mathbf{g}_{CS}(-\mathbf{q})$$

$$+ \sum_{\mathbf{q} \neq 0} n_{CS}(\mathbf{q}) v(q) n_{CS}(-\mathbf{q})$$

$$+ \sum_{\mathbf{q}} \frac{n_0}{2m} \left| \mathbf{a}_{CS}(\mathbf{q}) - \frac{1}{n_0} \mathbf{g}_{CS}(\mathbf{q}) \right|^2. \quad (30)$$

Our small Q model is the Hamiltonian (30) with all the sums over q ranging between $0 < q < Q$, and Q treated as a small parameter. Physically, this model amounts to considering the charge and the flux tubes of the composite fermions as smeared over distance $\sim Q^{-1}$. Our analysis below focuses on the kinetic part of the Hamiltonian. Thus, at this stage we omit the electron-electron interaction part $[v(q)]$. Its inclusion in the analysis below is straightforward.

Within the small Q model, we employ the unitary transformation suggested by Murthy and Shankar,[13]

$$U = \exp i \frac{1}{2\pi\tilde{\phi}n_0} \sum_{\mathbf{q}}^{Q} \mathbf{g}_{\mathbf{q}} \times \mathbf{a}_{-\mathbf{q}} \quad (31)$$

in order to redefine the gauge-field coordinates to be $\mathbf{a}_{CS} - 1/n_0 \mathbf{g}_{CS}$, and make the coupling between the redefined gauge field and fermion coordinates vanish in the long wavelength limit. Note that in Eq. (31), as in Eq. (30), the sum over q is limited to $0 \leq q < Q$. In our notation the operators before the transformation have a subscript CS, while this subscript is missing from their transformed counterparts. For example, the transformed Hamiltonian H is defined by $H_{CS} = U^{\dagger} H U$.

Let us first consider the case $Q = 0$ in which the sums in Eqs. (30) and (31) are limited to the $q = 0$ term. The fermions and the oscillators are exactly decoupled by the transformation, and the transformed Hamiltonian can be written as

$$H_{Q=0} = \frac{1}{2m} |(-i\boldsymbol{\nabla} - \mathbf{g}_{q=0})\psi|^2 + \frac{n_0}{2m} \mathbf{a}_{\mathbf{q}=\mathbf{0}}^2. \quad (32)$$

The Hamiltonian (32) is manifestly K invariant. The source of that invariance is the invariance of the untransformed Hamiltonian (29) to the gauge transformation

$$\mathbf{a}_{CS}(\mathbf{r}) \rightarrow \mathbf{a}_{CS}(\mathbf{r}) + \mathbf{K},$$

$$\psi_{CS}(\mathbf{r}) \rightarrow e^{i\mathbf{K} \cdot \mathbf{r}} \psi_{CS}(\mathbf{r}). \quad (33)$$

This gauge transformation, which is analogous to Eqs. (15) and (16), leaves the redefined gauge field coordinate $\mathbf{a} = \mathbf{a}_{CS} - 1/n_0 \mathbf{g}_{CS}$ unchanged. Thus, it leaves unchanged the energy stored in magnetoplasmons. Therefore, for the total energy to be gauge invariant *the energy stored in the transformed fermionic degrees of freedom should be also invariant under the gauge transformation, which shifts the momentum of each fermion by* $\mathbf{K}$. As we see below, for nonzero Q the unitary transformation that redefines $\mathbf{a}_{CS}$ redefines the fermions and introduces a coupling between the transformed fermions and gauge fields. However, it is still true that in the small Q limit the invariance of the pretransformed Hamiltonian H_{CS} to the transformation (33) implies the K invariance of the transformed fermions.

A Hamiltonian similar to the first term in Eq. (32), with the bare mass replaced by a renormalized one, was suggested by Haldane as an effective Hamiltonian for low-energy excitations of the $\nu = 1/2$ state.[18]

For a small but nonzero Q the momentum current $\mathbf{g}_{\mathbf{q}}$ does not commute with other fermionic operators appearing in the Hamiltonian. Consequently, we are unable to calculate the transformed Hamiltonian exactly, but use the following small Q approximation: we neglect all terms in the transformed Hamiltonian that involve more than one integral over q, and calculate all response functions by neglecting all diagrams that contain interaction lines in which the momentum exchange is integrated. These diagrams are neglected since the integration over the momentum exchange is of the form $\int^Q d\mathbf{q}$ and yields terms of higher order in Q. Their neglect constitutes the RPA.

As is well known, the RPA can be defined either by a diagrammatic classification, as given above, or, equivalently, by approximating commutation relations. The operators whose commutation relation are relevant for applying the unitary transformation at hand are the density $\rho_{\mathbf{q}}$, the Cartesian components of the momentum density $g_{\alpha,\mathbf{q}}$, and the Cartesian components of the vector $\mathbf{C}_{\mathbf{q}}$, which is defined by

$$\mathbf{C}_{\mathbf{q}} \equiv i[H_K, \mathbf{g}_{\mathbf{q}}] = i \sum_{\mathbf{k}} \frac{\mathbf{q} \cdot \mathbf{k}}{m} \mathbf{k} \psi_{\mathbf{k}+\mathbf{q}/2}^{\dagger} \psi_{\mathbf{k}-\mathbf{q}/2}, \quad (34)$$

with $H_K \equiv \Sigma_{\mathbf{k}} \frac{k^2}{2m} \psi_{\mathbf{k}}^{\dagger} \psi_{\mathbf{k}}$.

The approximate commutation relations between these operators are:

$$[\rho_{\mathbf{q}}, \mathbf{g}_{\mathbf{q}'}] = \mathbf{q}\rho_{\mathbf{q}+\mathbf{q}'} \approx \mathbf{q}n_0 \delta_{\mathbf{q},-\mathbf{q}'}, \quad (35)$$

$$[g_{\alpha,\mathbf{q}}, g_{\beta,\mathbf{q}'}] = q_{\alpha}' g_{\beta,\mathbf{q}+\mathbf{q}'} - q_{\beta} g_{\alpha,\mathbf{q}+\mathbf{q}'} \approx 0, \quad (36)$$

$$[g_{l,\mathbf{q}}, C_{l,\mathbf{q}'}] \equiv [\hat{q} \cdot \mathbf{g}_{\mathbf{q}}, \hat{q}' \cdot \mathbf{C}_{\mathbf{q}'}] \approx i \frac{3\pi n_0^2 q^2}{m} \delta_{\mathbf{q},-\mathbf{q}'}, \quad (37)$$

$$[g_{t,\mathbf{q}}, C_{t,\mathbf{q}'}] \equiv [\hat{q} \times \mathbf{g}_{\mathbf{q}}, \hat{q}' \times \mathbf{C}_{\mathbf{q}'}] \approx i \frac{\pi n_0^2 q^2}{m} \delta_{\mathbf{q},-\mathbf{q}'}, \quad (38)$$

$$[g_{l,\mathbf{q}}, C_{t,\mathbf{q}'}] = [g_{t,\mathbf{q}}, C_{l,\mathbf{q}'}] = [C_{\alpha,\mathbf{q}}, C_{\beta,\mathbf{q}'}] = 0. \quad (39)$$

Within this approximation scheme, the following simple relations hold between the pre- and post-transformation operators:

$$\mathbf{a}_{CS}(\mathbf{q}) = \mathbf{a}_{\mathbf{q}} + \frac{1}{n_0} \mathbf{g}_{\mathbf{q}}$$

$$\rho_{CS}(\mathbf{q}) = \rho_{\mathbf{q}} + i \frac{\mathbf{q} \times \mathbf{a}_{\mathbf{q}}}{2\pi\tilde{\phi}} + \frac{i}{2} \frac{\mathbf{q} \times \mathbf{g}_{\mathbf{q}}}{2\pi\tilde{\phi}n_0}$$

$$\mathbf{g}_{CS}(\mathbf{q}) = \mathbf{g}_{\mathbf{q}}. \quad (40)$$

The transformation of $[H_K]_{CS}$ is more complicated. Carrying out the transformation to lowest order in Q requires an ex-

pansion of the exponent (31) to fourth order. The resulting transformation is the following, with each numbered line giving the contribution of one order in the expansion,

$$[H_K]_{CS} = H_K \tag{41}$$

$$+ \frac{1}{(2\pi\tilde{\phi})n_0} \sum_{\mathbf{q}}^{Q} \mathbf{C_q} \times \mathbf{a_{-q}} \tag{42}$$

$$- \frac{1}{2(2\pi\tilde{\phi})n_0^2} \sum_{\mathbf{q}}^{Q} \mathbf{C_q} \times \mathbf{g_{-q}}$$

$$+ \frac{\pi}{2(2\pi\tilde{\phi})^2 m} \sum_{\mathbf{q}}^{Q} [3|\mathbf{q}\times\mathbf{a_q}|^2 + |\mathbf{q}\cdot\mathbf{a_q}|^2] \tag{43}$$

$$- \frac{\pi}{2(2\pi\tilde{\phi})^2 n_0 m} \sum_{\mathbf{q}}^{Q} [3(\mathbf{q}\times\mathbf{a_q})\cdot(\mathbf{q}\times\mathbf{g_{-q}})$$

$$+ (\mathbf{q}\cdot\mathbf{a_q})(\mathbf{q}\cdot\mathbf{g_{-q}})] \tag{44}$$

$$+ \frac{\pi}{8(2\pi\tilde{\phi})^2 n_0^2 m} \sum_{\mathbf{q}}^{Q} [3|\mathbf{q}\times\mathbf{g_q}|^2 + |\mathbf{q}\cdot\mathbf{g_q}|^2]. \tag{45}$$

We now use Eqs. (40)–(45) to calculate the transformed form of the Hamiltonian and the constraint. The Hamiltonian is

$$H(\text{small } Q) \approx \sum_{\mathbf{k}} \frac{k^2}{2m} \psi_{\mathbf{k}}^{\dagger}\psi_{\mathbf{k}} - \frac{1}{2mn_0} \sum_{\mathbf{q}}^{Q} \mathbf{g_q}\mathbf{g_{-q}}$$

$$+ \frac{\pi}{8(2\pi\tilde{\phi})^2 n_0^2 m} \sum_{\mathbf{q}}^{Q} [3|\mathbf{q}\times\mathbf{g_q}|^2 + |\mathbf{q}\cdot\mathbf{g_q}|^2]$$

$$- \frac{1}{2(2\pi\tilde{\phi})n_0^2} \sum_{\mathbf{q}}^{Q} \mathbf{C_q} \times \mathbf{g_{-q}}$$

$$+ \frac{1}{(2\pi\tilde{\phi})n_0} \sum_{\mathbf{q}}^{Q} \mathbf{C_q} \times \mathbf{a_{-q}}$$

$$- \frac{\pi}{2(2\pi\tilde{\phi})^2 n_0 m} [3(\mathbf{q}\times\mathbf{a_q})\cdot(\mathbf{q}\times\mathbf{g_{-q}})$$

$$+ (\mathbf{q}\cdot\mathbf{a_q})(\mathbf{q}\cdot\mathbf{g_{-q}})] + \sum_{\mathbf{q}}^{Q} \frac{n_0}{2m} |(\mathbf{a_q}-\mathbf{A_q})|^2$$

$$+ \frac{\pi}{2(2\pi\tilde{\phi})^2 m} \sum_{\mathbf{q}}^{Q} [3|\mathbf{q}\times\mathbf{a_q}|^2 + |\mathbf{q}\cdot\mathbf{a_q}|^2]$$

$$+ \sum_{\mathbf{q}}^{Q} \left[\rho(\mathbf{q}) + \frac{i\mathbf{q}\times\mathbf{a}(\mathbf{q})}{2\pi\tilde{\phi}} + \frac{i}{2}\frac{\mathbf{q}\times\mathbf{g}}{2\pi\tilde{\phi}n_0} \right] V_{-\mathbf{q}}. \tag{46}$$

This Hamiltonian is composed of several terms. Purely fermionic terms include a kinetic energy, a current-current interaction energy, and an interaction term of the form $\mathbf{C}\times\mathbf{g}$. The current-current interaction has two terms, the sec-

ond and third in Eq. (46). The latter is small, by a factor of q^2/n, compared to the former. Pure gauge-field terms include the self-energy of the magnetoplasmons, which, too, has an additional term of order q^2. There are two types of fermion-gauge-field coupling terms, one coupling $\mathbf{C}$ to $\mathbf{a}$, and one, of order q^2, coupling $\mathbf{g}$ to $\mathbf{a}$. Finally, we have incorporated in this expression also a probing potential $(V_{\mathbf{q}}, \mathbf{A_q})$ (which may also be time dependent) for the future use of calculating response functions.

The Chern-Simons constraint is transformed to the constraint (6):

$$\rho(\mathbf{q}) = \frac{i}{2} \frac{\mathbf{q}\times\mathbf{g_q}}{2\pi\tilde{\phi}n_0}. \tag{47}$$

The transformed constraint commutes with the transformed Hamiltonian. This observation reflects the consistency of our approximation scheme, since the pretransformed constraint commutes with the pretransformed Hamiltonian.

Equation (6) together with Eq. (40) leads to the identification of the electronic density as

$$\rho_{CS}(\mathbf{q}) = i\frac{\mathbf{q}\times\mathbf{g_q}}{2\pi\tilde{\phi}n_0} + i\frac{\mathbf{q}\times\mathbf{a_q}}{2\pi\tilde{\phi}}. \tag{48}$$

The first term indicates that $\hat{z}\times\mathbf{g}/(2\pi\tilde{\phi}n_0)$ is a dipolar field. Thus, a transformed fermion carrying a momentum $\mathbf{k}$ carries an electronic-dipole moment $e\hat{z}\times\mathbf{k}/(2\pi\tilde{\phi}n_0)$, as one expects from a dipole in a magnetic field.[26] Since the spectrum of the oscillators is gapped, with the lowest frequency being the electronic cyclotron frequency, their response to a driving force of low-frequency ω is small by a factor ω/ω_c, compared to that of the dipoles. Thus, the contribution of the oscillators to the electronic charge density (48) is negligible, and we may approximate $\rho_{CS}(\mathbf{q}) \approx 2\rho(\mathbf{q}) = i\mathbf{q}\times\mathbf{g}/(2\pi\tilde{\phi}n_0)$. As explained in the Introduction, the factor of 2 between the electronic density ρ_{CS} and the quasiparticle density ρ indicates the shifting of the quasiparticle position from the electronic one.

The electronic physical current $\mathbf{j}^e$ may be identified by taking the derivative of the Hamiltonian (46) with respect to $\mathbf{A}$. To leading order in Q, it is,

$$\mathbf{j}^e = \frac{n_0}{m}(\mathbf{a}-\mathbf{A}). \tag{49}$$

It is expressed in terms of the oscillators alone, and has a prefactor n_0/m. An oscillators' response of order ω/ω_c results, then, in an electronic current that is independent of the mass.

We now turn to calculate response functions of the $\nu = 1/2$ state using the decoupled Hamiltonian (46), the Chern-Simons constraint (6) and the expression (48) for the electronic density. Most particularly, we are interested in the density response $\rho(\mathbf{q}, \omega)$ to a scalar potential $V(\mathbf{q}, \omega)$, so we may set $\mathbf{A}=0$. Furthermore, we are interested in the limit of low frequency $\omega \ll \omega_c$ and long wavelength $q \ll k_F$, in which the fermions' response is much stronger than the oscillators'. Thus, we may set the magnetoplasmons frozen in their ground states and replace $\mathbf{a}$ in the Hamiltonian (46) by its expectation value, zero. We are then left with the dipoles,

whose density response to V we calculate within RPA. Generally, within RPA one first calculates the response functions of free fermions, denoted by Π, and then approximates the response function of the interacting fermions $\mathcal{K}$ by

$$\mathcal{K}^{-1} = \Pi^{-1} + \mathcal{V}, \qquad (50)$$

where $\mathcal{V}$ is the fermion-fermion interaction.

In the present case Π, $\mathcal{V}$, and $\mathcal{K}$ are all 5×5 matrices. To understand that unusual dimension, we note that the interac-

tion terms in the Hamiltonian (46) couple $\mathbf{g}$ and $\mathbf{C}$, and we are interested in the density response. Thus, Π, $\mathcal{V}$, and $\mathcal{K}$ must have ρ, C_l, g_l, C_t, and g_t entries, which we label by indices i,j running from 0 to 4. The matrix Π is the response matrix of free fermions, namely fermions subject to the Hamiltonian H_K. This generates a simple relation between terms involving $\mathbf{g}$ and $\mathbf{C}$ in Π. The calculation of the matrix elements of Π is straightforward, and is given in the Appendix. It results in

$$\Pi = \begin{pmatrix} \Pi_{00} & iqn_0 + i\dfrac{m\omega^2}{q}\Pi_{00} & \dfrac{m\omega}{q}\Pi_{00} & 0 & 0 \\[2ex] -iqn_0 - i\dfrac{m\omega^2}{q}\Pi_{00} & \dfrac{3\pi(qn_0)^2}{m} + mn_0\omega^2 & -i\omega\left(mn_0 + \dfrac{(m\omega)^2}{q^2}\Pi_{00}\right) & 0 & 0 \\[2ex] \dfrac{m\omega}{q}\Pi_{00} & i\omega\left(mn_0 + \dfrac{(m\omega)^2}{q^2}\Pi_{00}\right) & mn_0 + \dfrac{(m\omega)^2}{q^2}\Pi_{00} & 0 & 0 \\[2ex] 0 & 0 & 0 & \dfrac{\pi(qn_0)^2}{m} + \omega^2\Pi_{tt} & -i\omega\Pi_{tt} \\[2ex] 0 & 0 & 0 & i\omega\Pi_{tt} & \Pi_{tt} \end{pmatrix}. \qquad (51)$$

In Eq. (51) we used the notation Π_{00} for the density-density response functions of free fermions and Π_{tt} for the transverse current-current response functions of free fermions. In the limit of small q,ω, with $\omega/q \to 0$,

$$\Pi_{00} = \frac{m}{2\pi}\left(1 + \frac{i\omega}{v_F q}\right), \qquad (52)$$

$$\Pi_{tt} = n_0 m - \frac{q^2 m}{24\pi} + \frac{i2n_0\omega m^2}{k_F q},$$

as one sees in Eq. (23).

The interaction matrix $\mathcal{V}$ is read off from the Hamiltonian (Eq. 46):

$$\mathcal{V} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\[2ex] 0 & 0 & 0 & 0 & \dfrac{1}{4\pi\tilde{\phi}n_0^2} \\[2ex] 0 & 0 & -\dfrac{1}{mn_0} + \dfrac{q^2\pi}{4m(2\pi\tilde{\phi}n_0)^2} & -\dfrac{1}{4\pi\tilde{\phi}n_0^2} & 0 \\[2ex] 0 & 0 & -\dfrac{1}{4\pi\tilde{\phi}n_0^2} & 0 & 0 \\[2ex] 0 & \dfrac{1}{4\pi\tilde{\phi}n_0^2} & 0 & 0 & -\dfrac{1}{mn_0} + \dfrac{3q^2\pi}{4m(2\pi\tilde{\phi}n_0)^2} \end{pmatrix}. \qquad (53)$$

Combining Eqs. (50), (52), (51), and (53) to calculate the matrix $\mathcal{K}$, we find that as long as $\omega \neq 0$ the matrix elements of $\mathcal{K}$ satisfy the constraint (6). Since the shifted quasiparticle density is just half of the electronic density, the electronic density-density response function is four times that of the shifted quasiparticles, i.e., it is $4\mathcal{K}_{00}$. Calculating that ele-

ment, we find it to equal Eq. (25) with $v(q)$ turned to zero. Including the electron-electron interaction in the matrix $\mathcal{V}$ reproduces exactly Eq. (25), which is the electronic density-density response function calculated either by using electron centered quasiparticles or by using the FCS approach. Thus, our Hamiltonian approach, which describes quasiparticles

that are shifted away from the electrons, yields the same response functions as our Lagrangian approach, which described electron-centered quasiparticles.

The calculation outlined in the previous paragraph is carried out for a nonzero ω, and its $\omega \rightarrow 0$ limit yields the FCS result for the static limit. The $\omega = 0$ case can be directly calculated from Eqs. (50), (52), (51), and (53), but some care is necessary, as there are two subtle points. First, at $\omega = 0$, for all q, the matrix $\mathcal{K}^{-1}$ has a zero eigenvalue. This eigenvalue corresponds to a fluctuation in g_l, coupled to the C_t. Thus, when inverting the matrix, one must work in the subspace orthogonal to this eigenmode, which is the 3×3 subspace spanned by ρ, g_t, C_l. The infinite susceptibility corresponding to the zero eigenvalue is not a problem, because there is no physical observable or force field that couples to the mode. The mode can be understood as a remnant of the gauge invariance of the original problem, where an arbitrary longitudinal momentum density could be added and compensated by a change in the longitudinal $\mathbf{a}_{CS}$, with no change in energy or any other physical property. A similar zero mode was found in Sec. II, in the simpler case of a 2×2 matrix.

The second subtle point is that the commutativity of the constraint (6) with the Hamiltonian (46) causes the constraint to be satisfied automatically only for $\omega \neq 0$. Exactly at $\omega = 0$, the constraint needs to be imposed explicitly, as by adding a term to the energy, which becomes very large if the constraint is violated. Specifically, we add to the matrix $\mathcal{V}$, defined in Eq. (53), a matrix $\mathcal{V}'_{ij} = \lambda U_i^* U_j$, with

$$U_j = \delta_{j0} - iq \, \delta_{j4} / (4 \pi \tilde{\phi} n_0) \tag{54}$$

and $\lambda \rightarrow \infty$.

The agreement we find between the density-density response functions calculated using the dipolar quasiparticle approach and the FCS approach also extends to other physical quantities, such as the quasiparticle effective mass, as will be discussed in Sec. V below. First, however, we shall discuss further the physical meaning and some mathematical consequences of the small Q model, in Sec. IV.

Before concluding this section, we note that unlike the $Q = 0$ Hamiltonian, the Hamiltonian (46) is not exactly K invariant. A boost of the fermions' momentum by $\mathbf{K}$ does affect their energy, but this effect is of high order in Q. To find out what that order is, we note that shifting the momentum of each particle by $\mathbf{K}$ amounts to the shift $\mathbf{g}(\mathbf{q}) \rightarrow \mathbf{g}(\mathbf{q}) + \mathbf{K} \rho(\mathbf{q})$. With this shift, we find that the Hamiltonian (46) acquires some new components, of which the most important one is

$$- \sum_{\mathbf{q} \neq 0}^{Q} \left[\frac{\mathbf{K} \cdot \mathbf{g_q} \rho_{-\mathbf{q}}}{mn_0} + \frac{K^2 \rho_{\mathbf{q}} \rho_{-\mathbf{q}}}{2mn_0} \right]. \tag{55}$$

Due to isotropy, the expectation value of the first term must vanish if the unshifted state is centered around zero momentum. As for the second term, its expectation value is

$$\delta E(K) = - \sum_{\mathbf{q} \neq 0}^{Q} \frac{K^2}{2m} \langle \rho_{\mathbf{q}} \rho_{-\mathbf{q}} \rangle = - \sum_{\mathbf{q} \neq 0}^{Q} \frac{K^2}{2mn_0} S(q). \tag{56}$$

For a Fermi liquid of particles interacting via short-range interaction in two dimensions, $S(q) \sim q$ so that $\delta E(K) \sim Q^3$. In the presence of interactions of longer range, the power of
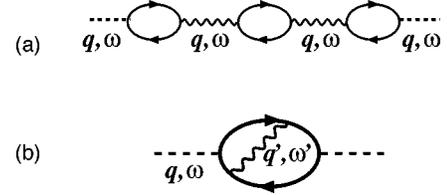

FIG. 2. (a) (top) A diagram included in the RPA for the density-density response functions. The momentum flowing in the interaction line is restricted to be the external momentum $\mathbf{q}$. (b) (bottom) A diagram neglected by RPA. The momentum in the interaction line is summed over.

Q gets even larger. The current–current interaction between the fermions at hand makes this power larger as well. Thus, the Hamiltonian (46) satisfies the K invariance, at least to order Q^2, for small Q; but it does not do so at high orders in Q, and therefore violates K invariance seriously for large values of the cutoff Q.

IV. COMMENTS ON THE SMALL Q LIMIT

In this section we make several comments regarding the small Q model we use. First, it is instructive to identify the electronic problem whose composite fermion formulation is our small Q model. In the small Q composite fermion problem each fermion carries two flux quanta antiparallel to the external magnetic field, as well as a charge -1, both smeared over a distance Q^{-1}. To transform back from composite fermions to electrons we undo the Chern-Simons transformation by attaching two unsmeared flux quanta to each composite fermion, parallel to the external field direction. The electronic problem we get is that of electrons at filling factor $\nu = 1/2$ that carry a charge smeared over a distance Q^{-1}, and a total of zero magnetic flux. This total flux, however, is made of two smeared flux quanta antiparallel to the external field and two δ-function flux quanta parallel to the field. Due to the smeared flux, a moving electron exerts a transverse electric field on all electrons within a distance Q^{-1}. This electric field is proportional to the velocity of the electron, which is of order $1/m$. Thus, the ground state of electrons experiencing this kind of interaction is not made solely of states from the lowest Landau level, and the effective mass of these electrons should not be expected to renormalize to a scale determined by Coulomb electron-electron interaction. This observation explains why the bare mass does appear in the RPA response functions such as Eq. (25).

As we mentioned in previous sections, barring phase transitions, we expect that, in the small Q limit, the random-phase approximation becomes exact for response functions. We now elaborate on that point. The RPA amounts to calculating the response functions for a particular wave vector and frequency $\mathbf{q}, \omega$ by summing all diagrams in which all the interaction lines are constrained to carry the momentum $\mathbf{q}$ and energy ω. In other words, the RPA neglects all the diagrams that contain interaction lines in which the momentum exchange is summed over. Examples of both types of diagrams are shown in Fig. 2.

Consider now one of the diagrams that are neglected by RPA. The sum over momentum exchange by the interaction is limited to $0 < q \lesssim Q$, and in the limit of a large system, this

sum may be replaced by an integral over $0 < q < Q$. If this integral is dominated by its upper cutoff then for small Q, the contribution of the diagram is at least of order Q^2. To lowest order in Q, it may therefore be neglected. (Note that this does not constitute a neglect of the interaction altogether. Interaction lines appear in the RPA diagrams, in which their momentum is not summed over.)

Unfortunately, the smallness of Q does not *always* make the contribution of a diagram negligible, even if the momentum exchange is summed over. Exceptions are diagrams in which the integrand diverges sufficiently fast at small wave vectors that the integration over momentum exchange is dominated by an infrared cutoff. In these cases the result may be independent of (or weakly dependent on) Q. An example of this is found in the computation of the composite fermion effective mass.

It is well known that, using the FCS approach, one finds a divergence in the quasiparticle effective mass at the Fermi energy,[6,20] which has a logarithmic form in the case of Coulomb interactions between electrons. This divergence also occurs if an upper momentum cutoff Q is incorporated into the FCS calculation. Moreover, the coefficient of the logarithm is independent of the value of Q. However, the logarithmic contribution only occurs for quasiparticles whose distance from the Fermi surface is smaller than order Q^2, which is a very small region of phase space when Q is small. (For the case of short-range interactions, the divergence in the effective mass is found to be stronger than in the Coulomb case, but it only occurs for quasiparticles whose wave vectors are closer to the Fermi surface than a distance of order Q^3.) We shall see in the next section that similar singularities are found in the dipolar approach as well.

If the singularity in the quasiparticle effective mass were to carry over to the density-density response function, we would expect that the RPA results would become invalid at very small wave vectors, say for $q \ll \mathcal{O}(Q^2)$, in the Coulomb case. The RPA would still be valid, and nontrivial, in the range $\mathcal{O}(Q^2) < q < \mathcal{O}(Q)$, so that the agreement between the FCS and dipolar approaches found in the previous section would at least be meaningful in that range. However, all existing analyses of these singularities at $\nu = 1/2$, by perturbative methods, renormalization group, $1/N$ expansion and bosonization find the self-energy singularities to cancel out from response functions, and the response functions to be regular at small q.[20–22] Thus, the identification of the small Q limit with RPA, for response functions, appears to be correct for arbitrarily small wave vector q.

V. FERMION GREEN'S FUNCTION AND THE EFFECTIVE MASS

In this section, we discuss the one-fermion Green's function, and the effective mass that may be derived from it, comparing results from the FCS and dipolar-quasiparticle approaches. (Here, we restrict ourselves to the case of shifted quasiparticles, as derived in Sec. III using the Hamiltonian approach.)

The imaginary part of the one-fermion Green's function $G(\mathbf{k}, \omega)$ describes the spectral resolution of the state where a composite fermion of momentum $\mathbf{k}$ is instantaneously added to or removed from the ground state. Naturally, the Green's function depends on the precise definition of the injected particle, and will clearly depend on the gauge that is used, if there are any vector potentials coupling to the fermion. However, if the Green's function contains a sharp pole that can be identified as arising from a low-energy quasiparticle excitation, then the behavior of the quasiparticle energy ϵ_k near the Fermi surface should be well defined and should be independent of the precise definition of the bare injected quasiparticle, as long as there is a reasonable degree of overlap between the bare particle and the low-energy excitation.

A. Effective mass

The effective mass m^* is defined by

$$k_F/m^* \equiv d\epsilon_k/dk \qquad (57)$$

in the limit $k \rightarrow k_F$. At least in the case of Coulomb interactions, it is expected that the decay rate of a quasiparticle should be small compared to the real part of the energy, near the Fermi surface, so that the m^* is well defined in this limit.

Although the effective mass does not appear directly in the density-density correlation function at long wavelengths and low frequencies, it is nevertheless measurable, in principle. For example, in a system precisely at $\nu = 1/2$, with no impurity scattering, the specific heat at low temperatures can be related to m^*. Similarly, the behavior of m^* near the Fermi surface at $\nu = 1/2$ determines the asymptotic form of the energy gaps in the principal quantized Hall states, $\nu = p/(2p+1)$, for $p \rightarrow \infty$.[6,20]

As was mentioned previously, in the FCS approach a perturbative calculation of the composite fermion self energy leads to the discovery that the composite fermion effective mass depends on energy, and diverges as the energy of the composite fermion gets close to the Fermi energy. This divergence is of the infrared type, resulting from the interaction of the fermion with low-energy transverse fluctuations in the Chern-Simons gauge field, which result, in turn, from long wavelength low-frequency fluctuations in the fermion density, due to the constraint $\nabla \times \mathbf{a}_{CS} \propto \rho^e$ [Eq. 2]. The divergence is found already at the RPA level, as a result of the contribution to the self-energy from the diagram shown in Fig. 4(a), but it has been argued to be valid exactly, at least in the case of Coulomb interactions.[20,23] A detailed discussion of the effective-mass divergence in the FCS approach is given in Ref. 20.

In the Murthy-Shankar calculation, by contrast, the composite fermion mass was found to be renormalized from its bare value to a constant m^*, independent of the momentum and energy of the fermion. The value of m^* was determined by the strength of the electron-electron interaction and was independent of the bare mass. However, this result depended crucially on several aspects of the approximation used by MS, including particularly the fact that the ultraviolet cutoff in their theory was chosen to be precisely equal to k_F. There were no infrared divergences in their approximation.

Motivated by these findings, we now study the effective mass within our small Q model, using the formalism of Sec. III. With respect to the infrared divergence, we may envision several possible scenarios: the effective mass divergence may be discovered as resulting from the Hamiltonian (46); it may result from terms that are of higher order in Q, and are

FIG. 3. The Hartree-Fock diagrams. The zig-zag interaction line in these diagrams represents bare $\mathbf{g} \cdot \mathbf{g}$ interaction [cf. Eq. (46)]. The fermionic lines represent bare Green's functions. As usual, the Hartree diagram (left) does not make a contribution. The Fock diagram (right) leads to Eq. (58).
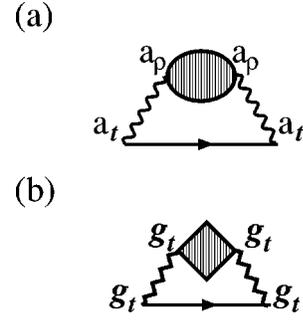

FIG. 4. (a) The fermion self-energy in the FCS approach. Here, a wiggly line is a Chern-Simons interaction, and the elliptic full bubble represents the electronic density-density correlation function. The two wiggly lines combined with the full bubble form the transverse gauge-field propagator $\langle a_t a_t \rangle$ due to the Chern-Simons constraint, (2). (b) The fermion self-energy in the shifted quasiparticle picture is the sum of this diagram and the Fock diagram in Fig. 3. A zig-zag line is a bare transverse momentum interaction, while the full diamond shaped bubble is the $\langle g_t g_t \rangle$ correlation function. At low frequencies, this correlation function is related to electronic density-density response function [the elliptic bubble of (a)] by the relation (48). At high frequencies the $g_t g_t$ interaction is dominated by the bare term in Fig. 3, as explained in the text.

therefore absent from the Hamiltonian (46); or it may be absent here altogether, a result that would raise questions about the results obtained from the FCS approach. We find the first scenario to happen. With respect to the ultraviolet renormalization, we may expect to find a smaller renormalization than the one found by MS, since we take Q to be small. We indeed find this to be the case, and clarify the different regimes at which each of the two calculations dominate.

We first note that a Hartree-Fock treatment of the Hamiltonian (46) leads, in the small Q limit, to a small renormalization of the bare mass, of order Q. To leading order in Q the Hartree-Fock contribution to the self energy, Σ_{HF} is dominated by the second term in Eq. (46). It is described by the diagram in Fig. 3, and is given by

$$\Sigma_{HF}(\mathbf{k},E) = -\frac{1}{mn} \sum_{0<|\mathbf{q}|<Q} [1 - n_F(\mathbf{k}+\mathbf{q})], \qquad (58)$$

where n_F is the Fermi occupation number. Since Σ_{HF} depends on wave vector only, it leads to a mass renormalization according to

$$\frac{1}{m^*} - \frac{1}{m} = \frac{1}{k_F} \frac{\partial \Sigma_{HF}}{\partial k} = -\frac{2Q}{m \pi k_F}. \qquad (59)$$

This renormalization has a similar source as the one found by MS.

Going beyond the Hartree-Fock approximation, we replace the bare $\mathbf{g} \cdot \mathbf{g}$ interaction by the dressed one, which means that we must include in the self-energy the diagram shown in Fig. 4(b), as well as the Hartree-Fock term of Fig. 3. The contribution of Fig. 4(b) leads to the same infrared singularities as in the FCS approach, including the same divergence of the effective mass. The role played in the FCS approach by the interaction with transverse gauge-field fluctuations is now played by the transverse part of the second term in Eq. (46), namely the transverse part of the fermion current-current interaction. The transverse fermion $\langle \mathbf{gg} \rangle$ propagator is related to the electronic density response function (25), at low frequencies, by the constraint $\nabla \times \mathbf{g} \propto \rho^e$ [Eq. (3)], which makes its q,ω dependence exactly identical to that of the transverse gauge-field propagator in the FCS approach, and leads to the same infrared singularities in perturbative calculations. (We have already seen that the electron density-density propagator is the same, at the RPA level, in both approaches.)

The divergence of m^* as a function of the energy E is a consequence of the strong dependence of the self-energy on E. This dependence results from virtual processes of energy transfers $\sim E$ and momentum transfer of order $q_0 \sim E^{1/3}$ for

short-range interaction and $q_0 \sim E^{1/2}$ for $1/r$ Coulomb interactions. If E is larger than order Q^3 (or Q^2 in the Coulomb case), the dependence of the self-energy on E weakens and the mass renormalization from Fig. 4(b) becomes small. In that regime, the biggest contribution to the mass renormalization comes from the Hartree-Fock term previously discussed, which gives an effective mass that is independent of the energy or wave vector of the quasiparticle, provided that the quasiparticle energy ϵ_k is still small compared to Q/v_F.

In the FCS approach, one can obtain a contribution to the effective mass identical to that of the Hartree-Fock diagram in the present approach by including in the self-energy of Fig. 4(a) the contribution to the transverse gauge fluctuations arising from the high-frequency Kohn mode. Since this contribution does not lead to any singularity at low energies, it has generally been ignored in the FCS literature.

We believe that the general state of affairs, which we find to hold for small Q, holds also for the physical case where no upper cutoff Q exists. For low enough energies, the effective mass is determined by singular infrared contributions, and diverges close to the Fermi surface. The leading singularity in the effective mass is independent of Q, at least in the case of Coulomb interactions. This is another manifestation of the statement made in Ref. 20: the leading singularity in m^* is captured exactly by the perturbative calculation. For high enough energies, on the other hand, the effective mass is determined by the short-distance behavior of the interaction. In the physical case, unlike the small Q limit, the high-energy effective mass is renormalized from the bare mass scale to a scale determined by electron-electron interaction.

The fact that the low-energy divergence of the effective mass depends on the range of electron-electron interaction leads us to express a word of caution regarding numerical calculations of the effective mass. The singular q,ω dependence of Eq. (25), in all its derivations, is obtained by summing infinitely many terms (the RPA geometric series), where the nth term represents the amplitude for excitation of

n particle-hole pairs. Numerical calculations based on trial wave functions often calculate the effective mass by comparing the energy of a ground state with the energy of an excited state with *one* excited particle hole pair. Although Jain-type trial wave functions used in these calculations are not the wave functions that emerge from our analysis, they are not unrelated (see the discussion by MS[13]). Moreover, the trial wave functions used in numerical calculations are constructed in a way that is independent of the range of electron-electron interaction. Thus, we believe that calculations done using these wave functions are very useful in understanding the behavior of the effective mass at high energies, but are not able to discover the infrared divergence at low energies, which depends strongly on the range of electron-electron interaction and results from many particle-hole pairs being excited.

B. Quasiparticle-weight in the fermion Green's function

As has been noted in the literature, the amplitude *z* of the single-quasiparticle contribution to the Green's function for the bare composite fermion of the FCS approach actually vanishes in the thermodynamic limit and at low energies. In fact there are two separate reasons for this.

The first point, noted by HLR,[6] is that, in the Coulomb gauge, the operator that inserts a bare composite fermion at some instant of time will excite a number n_m of long-wavelength magnetoplasma modes, whose mean value diverges logarithmically with the size of the system. This means that the probability of winding up in the ground state of the magnetoplasma modes falls off as a power of the area of the system, and the quasiparticle amplitude *z* must fall off accordingly. Although this effect is an infrared divergence in the sense that it depends on the size of the system, it is not sensitive to the wave vector or energy of the quasiparticle, since the responsible fluctuations are at very high energy. In fact, the problem occurs equally well for a fractional quantized Hall state, with an energy gap, as it does for $\nu = 1/2$. Within the HLR formalism, this diverging contribution arises from interactions of the bare composite fermion with fluctuations in the Chern-Simons scalar potential associated with the magnetoplasmon modes.

The diverging renormalization of *z* due to magnetoplasmon modes does not occur for the dipolar fermions we consider here, because the fermions are decoupled from the oscillators at long wavelengths. This makes sense, because the fermion Green's function here describes the propagation of an added composite fermion after all states with nonzero oscillator occupation have been eliminated.

The second type of divergent renormalization of the quasiparticle weight *z*, encountered in the FCS approach, is an infrared divergence, resulting from interactions with fluctuations in the transverse Chern-Simons field due to the low-frequency density mode. This renormalization of *z* arises from the same frequency-dependent contribution to the self-energy as is responsible for the infrared singularity in the effective mass. This renormalization is present equally well for the dipolar fermions considered in the present section. The quasiparticle weight is thus predicted to vanish as a power of the quasiparticle energy *E*, in the case of short-range interactions, and to vanish as $1/\log E$ in the case of Coulomb interactions.

The fact that the quasiparticle weight in the one-fermion Green's function vanishes at low energies does not lead to any major effects in the density-density response, as has been previously noted, because all the diverging contributions are cancelled in this case.[21] The diverging contribution of the long-wavelength magnetoplasmons, which led to a vanishing of the quasiparticle weight in the one-fermion Green's function of the FCS theory is absent in the density-response function because of the opposite signs of the interaction for the particle and the hole. (This has been discussed explicitly in Ref. 27.)

Finally, we must note that in calculating the one-fermion Green's function for our dipolar quasiparticles, we have implicitly assumed that the initial state has zero total momentum, as well as vanishing amplitude for all the unphysical modes of the longitudinal momentum density, which occur at zero-frequency and nonzero **q**. (This is similar to fixing the gauge when calculating the Green's function for a charged particle.) Clearly, if we considered the physically equivalent state obtained by adding a constant momentum **K** to every fermion, we would find a Green's function where the Fermi surface was itself displaced by the constant momentum. If one averaged the Green's function over a set of physically equivalent initial states with different values of **K** one would lose all useful information, as would also happen if one averaged over initial states with nonzero amplitudes in unphysical zero frequency and nonzero **q** modes.

VI. LARGE Q: FERMI LIQUID PICTURE

A. General conjectures

In the previous sections we discussed in detail a model in which the random-phase approximation is exact. In this section, we conjecture the form of a Fermi liquid theory for the dipolar quasiparticles, which describes the electronic response functions for the physical case, in which there is no momentum cutoff *Q*, and discuss what features of the RPA are expected to remain valid in the physical case.

As we saw in previous sections, there are various different formulations of the small *Q* limit, such as the electron-centered and shifted quasiparticles. Naturally, this ambiguity carries over to the physical case. Below, we first enumerate four assumptions, which we believe to be common to all Fermi-liquid descriptions of the $\nu = 1/2$ state in terms of dipolar quasiparticles. We then consider the static limit and show that these assumptions are already sufficient to establish that the electronic system is compressible. In a second step, we specify to electron-centered quasiparticles and discuss the dynamic density-density response function.

We conjecture that any Fermi-liquid theory of the $\nu = 1/2$ state in terms of dipolar quasiparticles will satisfy the following four assumptions:

(1) The electronic density is to be described in terms of low-energy quasiparticles and high-energy magnetoplasmons. The low-energy quasiparticles are neutral, and carry an electric-dipole moment proportional and perpendicular to their momentum. Thus, in the limit of small **q** and ω, the electronic density ρ^e and the quasiparticle momentum density **g** are related by

$$\rho^e = \frac{i\mathbf{q} \times \mathbf{g}}{2\pi\tilde{\phi}n}. \tag{60}$$

This relation was found to hold in the RPA approach for both electron-centered and shifted quasiparticles.

The relation (60) is valid precisely at $\nu = 1/2$. If an infinitesimal driving force is applied to the system by means of a vector potential $\mathbf{A}_{\mathbf{q},\omega}$ then Eq. (60) has to be modified to

$$\rho^e = \frac{i\mathbf{q} \times \mathbf{g}}{2\pi\tilde{\phi}n} + \frac{i\mathbf{q} \times \mathbf{A}}{2\pi\tilde{\phi}}. \tag{61}$$

This means that if the electron-density ρ^e tracks the magnetic field, so that the system is everywhere *locally* at $\nu = 1/2$, we still have $\mathbf{q} \times \mathbf{g} = 0$.

(2) The low-energy quasiparticles form a Fermi liquid characterized by response functions that are regular in the small $\mathbf{q}, \omega$ limit. Moreover, we assume that this liquid is compressible.

(3) The Fermi-liquid theory satisfies K invariance, i.e., the quasiparticles' energy is invariant to a shift of the Fermi surface by a constant $\mathbf{K}$. Moreover, the invariance holds even for a position dependent boost such as $\mathbf{K} = \mathbf{K_0}\cos(\mathbf{q} \cdot \mathbf{r})$, with $\mathbf{K_0} \| \mathbf{q}$. Put in different words, this Fermi liquid possesses zero-energy excitation modes at small nonzero values of q, associated with static longitudinal fluctuations in the momentum density $\mathbf{g}$. This assumption carries over from the RPA results. The zero-energy modes do not appear in any physical observables.

(4) The quasiparticle density ρ is related to the electronic density ρ^e [or, alternatively, to the quasiparticle momentum density $\mathbf{g}$ due to Eq. (60)] in a nonsingular manner in the limit $q \to 0$. For small Q, we found $\rho(\mathbf{q}) \propto \rho^e(\mathbf{q})$ with the proportionality constant varying between different descriptions. We assume that the proportionality relation continues to hold for large Q, though the constant of proportionality may be modified in some of the descriptions.

In our discussions we ignore the possibility of infrared divergences in the quasiparticle effective mass m^*, and we implicitly assume that the decay rate of a quasiparticle is small compared to its energy. These assumptions should be literally correct if the electron-electron interaction is longer range than $1/r$. The arguments go through with relatively minor modifications in the case of Coulomb interactions, where there is a weak logarithmic divergence of the effective mass, and the quasiparticle decay rate is asymptotically small compared to the energy.[20] It is not clear how much of the Fermi-liquid description can be retained for short-ranged interactions, however, where the quasiparticle decay rate is predicted to be proportional to the energy, as one approaches the Fermi surface.

B. Static response

Making the assumptions above, we now show that the $\nu = 1/2$ state is compressible. An electronic system is compressible if the energy cost involved in producing a (static) modulation $\delta\rho^e(\mathbf{q})$ of the electronic density is finite for $q \to 0$. (Here, we exclude the direct Coulomb energy of the density fluctuation, which, of course, diverges for $q \to 0$.) In the present case, due to assumptions (1) and (4), a modula-

tion $\delta\rho^e$ in the charge density is associated with both a quasiparticle momentum density $\delta\mathbf{g}$ (of order q^{-1}) and a quasiparticle density modulation $\delta\rho$ (of order q^0). The energy cost associated with a small $\delta\rho^e$ can then be written as

$$\delta E(\delta\rho^e) = \frac{1}{2}\mathcal{A}(\delta\mathbf{g})^2 + \mathcal{B}\delta g\,\delta\rho + \frac{1}{2}\mathcal{C}(\delta\rho)^2. \tag{62}$$

In this expression, $\mathcal{A}$ should vanish at least as fast as q^2 for small q, as a consequence of K invariance and the assumption (2). The energy cost associated with the first term of Eq. (62) is then finite. In the second term, $\mathcal{B}$ must be at least of order q as a consequence of assumption (2), and thus the energy cost associated with this term is finite. Finally, the assumption of a compressible quasiparticle Fermi liquid requires $\mathcal{C}$ to be a constant in the limit $q \to 0$. Combining all terms in Eq. (62) we find $\delta E(\delta\rho^e)$ to be finite.

C. Dynamical response

Generally, dynamical response in Fermi-liquid theory is analyzed by means of a Boltzmann equation, which describes the nonequilibrium state of the liquid in terms of the function $\delta n(\mathbf{k}, \mathbf{r}, t)$, the deviation of the quasiparticle distribution function from the equilibrium Fermi-Dirac function. At zero temperature one may write $\delta n(\mathbf{k}, \mathbf{r}, t) = (2\pi/m^*)\delta(\epsilon_k - \mu)\nu(\theta, \mathbf{r}, t)$ (where θ is the angle of the vector $\mathbf{k}$ with the x axis), and expand $\nu(\theta, \mathbf{r}, t) = \Sigma_l\nu_l\exp(il\theta)$. The quasiparticle density is then $\rho(\mathbf{r}, t) = \nu_0(\mathbf{r}, t)$, while the two components of the kinetic momentum density are

$$g_x(\mathbf{r}, t) + n_0 A_x(\mathbf{r}, t) \equiv \sum_{\mathbf{k}} \mathbf{k}_x\,\delta n(\mathbf{k}, \mathbf{r}, t)$$

$$= \frac{1}{2}k_F[\nu_1(\mathbf{r}, t) + \nu_{-1}(\mathbf{r}, t)],$$

$$g_y(\mathbf{r}, t) + n_0 A_y(\mathbf{r}, t) \equiv \sum_{\mathbf{k}} \mathbf{k}_y\,\delta n(\mathbf{k}, \mathbf{r}, t) \tag{63}$$

$$= \frac{i}{2}k_F[\nu_1(\mathbf{r}, t) - \nu_{-1}(\mathbf{r}, t)],$$

where $\mathbf{A}(\mathbf{r}, t)$ is an infinitesimal externally applied driving vector potential. Note that in the presence of $\mathbf{A}(\mathbf{r}, t)$ the vector $\mathbf{k}$ is the kinetic momentum, $\mathbf{k} = \mathbf{p} + \mathbf{A}$, rather than the canonical momentum $\mathbf{p}$. The energy ϵ_k is thus unaffected by the vector potential and the equilibrium Fermi sphere remains centered at $k = 0$.

The kinetic momentum density is distinguished in Fermi-liquid theory from the quasiparticle current, which is expressed in terms of $\nu_{\pm 1}$ and the (dimensionless) Landau parameters $F_{\pm 1}$ as

$$j_x = \frac{1}{2}v_F^*[(1 + F_1)\nu_1 + (1 + F_{-1})\nu_{-1}],$$

$$\tag{64}$$

$$j_y = \frac{i}{2}v_F^*[(1 + F_1)\nu_1 - (1 + F_{-1})\nu_{-1}].$$

It is convenient to work in momentum-frequency representation, so from now on, $\nu(\theta)$ and ν_l will be implicitly assumed to depend on $\mathbf{q}$ and ω. The Boltzmann equation for $\nu(\theta)$ in the presence of an electric field $\mathbf{E} \propto e^{i\mathbf{q}\cdot\mathbf{r}-i\omega t}$ is,

$$-i\omega\nu(\theta)+iv_F^* q\cos\theta[\nu(\theta)+\delta\nu(\theta)]=-\frac{k_F}{2\pi}\mathbf{E}\cdot\hat{\mathbf{n}}(\theta),$$
(65)

with $\hat{\mathbf{n}}(\theta)=(\cos\theta,\sin\theta)$ and $\delta\nu(\theta)=\int(d\theta'/2\pi)F(\theta-\theta')\nu(\theta')$. The angular Fourier components of the quasiparticle interaction function $F(\theta-\theta')$ are the Landau parameters. The field $\mathbf{E}$ is the self-consistent electric field, including both the external probing field and the field produced by the long-range Coulomb potential arising from the induced inhomogeneities in the electron density.

Equation (65) easily yields a continuity equation $\omega\rho=\mathbf{q}\cdot\mathbf{j}$. In our case, however, the quasiparticle density ρ and current j may be related to the electronic density ρ^e and current j^e in a nontrivial way. In fact, this relation depends on the particular description we choose, and varies between Sec. II and Sec. III. To be specific, we now assume that we may construct a Fermi liquid description in which the quasiparticles are electron centered, as in Sec. II, i.e.,

$$\rho^e(\mathbf{q})=\rho(\mathbf{q}).$$
(66)

By the continuity equation, Eq. (66) fixes the longitudinal component of the electronic current to be the projection of Eq. (64) onto the direction of $\mathbf{q}$. Furthermore, Eq. (61), when combined with Eqs. (64) and (66), allows us to identify

$$F_{\pm1}=-1\pm\frac{m^*}{n_0}\frac{\omega}{2\pi\tilde{\phi}}.$$
(67)

This expression for $F_{\pm1}$ is reminiscent of the results of Sec. II. In the static limit $\omega=0$ we have $F_{\pm1}=-1$. Generally, the cost in the total energy ($E-\mu N$) associated with a spatially slowly varying infinitesimal static deformation $\nu(\theta)$ of the Fermi sea is proportional to $\Sigma_l(1+F_l)|\nu_l|^2$. A small uniform boost of the Fermi sea by momentum $\mathbf{K}$ is described by $\nu_{\pm1}\sim\frac{1}{2}(K_x\pm iK_y)$. *Thus, $F_{\pm1}=-1$ eliminates any energy cost associated with this uniform boost of the Fermi sea in the zero-frequency limit.* This is precisely the statement of the K invariance. Note that in this description K invariance is not an independent assumption but rather results from Eqs. (60) and (66). At nonzero ω Eq. (67) has the unusual feature of being dependent on the frequency. While the frequency dependence of F_1 is hard to interpret in the context of an energy functional, it can be quite naturally included in a Boltzmann equation.

We now turn to use the Boltzmann equation for studying the current $\mathbf{j}$ induced, in linear response, by an electric field $\mathbf{E}$. Before doing so, we make three comments of caution. First, we note that the considerations above, which identified j_l^e with j_l, do not identify j_t^e with j_t. In fact, such an identification does not hold even in the small Q limit [cf. Eq. (19)]. Consequently, we are able to draw conclusions only with regard to the longitudinal electronic current response. Second, we note that since we are not able to calculate the Landau parameters F_l for $l\neq\pm1$, the information we may obtain from the Boltzmann equation is limited. And third, as

usual, the semiclassical Boltzmann equation does not properly account for the quasiparticle Landau diamagnetism, which, as we saw in Secs. II and III, affects the electrons' compressibility. Its effect has to be included in the equation by hand.

As explained in detail in Refs. 12 and 28, generally the effect of $F_{\pm1}$ on response functions can be understood by the following procedure: we write the Boltzmann equation as

$$-i\omega\nu(\theta)+iv_F^* q\cos\theta\left[\nu(\theta)+\int(d\theta'/2\pi)\tilde{F}(\theta-\theta')\nu(\theta')\right]$$
$$=-\frac{k_F}{2\pi}(\mathbf{E}+\mathbf{E}^{\text{eff}})\cdot\hat{\mathbf{n}}(\theta),$$
(68)

where

$$\mathbf{E}^{\text{eff}}=-\frac{2\pi i\omega}{k_F^2}[(F_1+F_{-1})\mathbf{g}+i(F_1-F_{-1})\hat{\mathbf{z}}\times\mathbf{g}],$$
(69)

and $\tilde{F}(\theta)=F(\theta)-F_1e^{i\theta}-F_{-1}e^{-i\theta}$. [The Landau parameters $\tilde{F}_l$ corresponding to $\tilde{F}(\theta)$ are those corresponding to $F(\theta)$, except for the case $l=\pm1$, in which $\tilde{F}_{\pm1}=0$.] For our particular values of $F_{\pm1}$ [Eq. (67)] and given Eqs. (63) and (64), we get

$$\mathbf{E}^{\text{eff}}=-2\pi\tilde{\phi}\hat{\mathbf{z}}\times\mathbf{j}-\frac{i\omega}{n_0/m^*}\mathbf{j}.$$
(70)

The second term in Eq. (70) is $\mathcal{O}(\omega)$ smaller than the first, and can, therefore, be neglected.

If we define a conductivity tensor $\sigma(\mathbf{q},\omega)$ by $\mathbf{j}=\sigma\mathbf{E}$, and a "quasiparticle conductivity tensor" $\sigma^*(\mathbf{q},\omega)$ by $\mathbf{j}=\sigma^*(\mathbf{E}+\mathbf{E}^{\text{eff}})$, then the two quantities are related by

$$\sigma^{-1}=(\sigma^*)^{-1}+2\pi\tilde{\phi}\begin{pmatrix}0&-1\\1&0\end{pmatrix}.$$
(71)

The quantity $\sigma^*(\mathbf{q},\omega)$, in turn, is the linear-response function, which gives the relation between induced current and the electric field for a conventional Fermi liquid, with effective mass m^* and Landau parameters $\tilde{F}_l$.

Equation (71) also clarifies the relation between the Fermi liquid picture we develop here from the dipolar approach and the Fermi liquid picture developed in the FCS approach. In the latter one gets Eq. (71) with one more term on the right-hand side, given by $i\omega/n_0(m^*-m)$. This term does not affect the response at low frequency, but is essential for getting the correct electronic response at $\omega\approx\omega_c$, particularly Kohn's mode. Thus, our dipolar picture, which we confined to low frequency, coincides with the FCS approach in that limit.[12,28]

As we emphasized before, only one component of $\sigma(\mathbf{q},\omega)$, namely $\sigma_{ll}(q,\omega)$, can be ascribed a physical meaning. The electronic density-density response function is related to that term by

$$K_{\rho\rho}^e(\mathbf{q},\omega)=\frac{1}{v(q)-i\omega/q^2\sigma_{ll}}.$$
(72)

In the limit $\omega,q\rightarrow0$ and $\omega/q\rightarrow0$, this takes the form

$$K_{\rho\rho}^e(\mathbf{q},\omega) = \frac{1}{v(q) + \widetilde{C} - i(2\pi\widetilde{\phi})^2 \dfrac{2n_0\omega}{k_F q^3}}, \qquad (73)$$

where $\widetilde{C}$ is the inverse compressibility of the Fermi liquid. Within the Boltzmann theory approach of Eq. (68), the inverse compressibility is given by

$$\widetilde{C} = 2\pi(1+F_0)/m^*, \qquad (74)$$

where $F_0 = 1/2\pi\int_0^{2p}d\theta\, F(\theta)$ is the 0th Fourier mode of the Landau interaction function.

The functional dependence of Eq. (73) on $\mathbf{q},\psi$ is the same as that of the small Q result, Eq. (25). The constant $\widetilde{C}$, however, is modified. We are not able to calculate the value of F_0 in Eq. (74) (and thus we can not calculate $\widetilde{C}$) in terms of hte original parameters of the electronic problem.

Whether or not Eq. (74) is correct, or even well defined, is problematic for the actual system of interest with large Q. There may not be a unique way to define $\widetilde{F}$ except to impose Eq. (74) as a definition. The ambiguities here arise due to the constraint Eq. (61), that for a longitudinal $\mathbf{A}$ a quasiparticle density excitation must involve an excitation of a transverse $\mathbf{g}$. In the absence of the constraint, the dynamics of a density excitation involves F_0 and is independent of the diamagnetic susceptibility χ, while the dynamics of a g_t excitation involves χ and is independent of F_0. Thus, F_0 and χ will be two independent quantities. Here, with the two excitations correlated, it is not clear whether F_0 and χ are uniquely and independently defined.

Even if the function $\widetilde{F}$ is well defined, a further complication arises due to the singular nature of the dipolar system, that $1+F_{\pm 1}=0$. Our analysis suggests[29] that if one generalizes Eq. (68) to include corrections to $\widetilde{F}$ that are proportional to q, this can lead to a correction to Eq. (74) and can effect the value of $\widetilde{C}$ even in the limit $q\to 0$.

It should be noted that the value of $\widetilde{C}$ does not affect the leading behavior of the density-response function at long wavelengths, for any frequency, in the case of Coulomb interactions, because the constant $\widetilde{C}$ is small compared to the interaction term $v(q)$. Even in the case of short-range interactions, the value of $\widetilde{C}$ is unimportant for the response function as long as $\omega \gg v_F q^3/k_F^2$.

The Fermi liquid picture we obtain shares some similarities with the Fermi liquid obtained recently by Read[15] for bosons at $\nu=1$ constrained to the lowest Landau level. In particular, the identification of $F_1 = -1$ (K invariance) and the linear response function (73) are obtained also in Read's approach. This similarity indicates that some of the features obtained in a calculation constrained to the lowest Landau level are, in fact, unchanged when this constraint is removed. In particular, our study concludes that for Coulomb interactions the long wavelength limit of Eq. (73) (where $1/v(q) \gg \widetilde{C}$) is valid independent of whether the electrons are confined to the lowest Landau level. Moreover, the diverging effective mass and the energy gaps at filling factors $p/(2p+1)$ with $p\to\infty$ are fully determined by the long wavelength limit of Eq. (73), and are found to depend only on the scale

of electron-electron interaction.[6,20] Consequently, we conclude that, in the case of Coulomb interaction, for any ratio of the interaction energy to the cyclotron energy at which electrons form fractional quantized Hall states at $\nu=p/(2p+1)$ the energy gaps correponding to these states are solely determined by the interaction energy in the limit $p\to\infty$. In practice, since this statement results from the weak, logarithmic, divergence of the effective mass, the limit $p\to\infty$ may be realized only for extremely large values of p.

VII. CONCLUSIONS

When analyzing the role of electron-electron interaction in the physics of electrons in zero magnetic field, the common strategy treats the interaction as a perturbation. The interaction may then be handled by means of a Hartree-Fock approximation, an RPA, or a Fermi-liquid approach. These schemes attempt to find (approximately) what are the low-energy excitations (quasiparticles and quasiholes) of the problem, and using these excitations, what is the electronic response of the system to driving forces.

This strategy cannot be directly applied to the case of a half-filled Landau level, since in the absence of the interaction the ground state is vastly degenerate. The Leinaas-Myrheim-Chern-Simons transformation, which attaches $\widetilde{\phi}=2$ flux quanta to each electron, opens the way to a perturbative calculation in which the unperturbed ground state is nondegenerate. However, this transformation, which transforms electrons into CS fermions, generates a new interaction, the Chern-Simons interaction, whose coupling constant $\widetilde{\phi}$ is not small.

An important feature of the transformed problem is that the singular nature of the interaction for $q\to 0$ leads to a strong coupling between the CS fermions and the high-energy collective modes (the magnetoplasmons) at long wavelengths. Due to this strong interaction, the low-energy excitations of the transformed problem, the dressed composite fermions or quasiparticles, are very different from the bare Chern-Simons fermions.

There are some strong similarities between the fermion-Chern-Simons system and the ordinary three-dimensional (3D) electron system, where the Coulomb interaction gives rise to plasma modes whose frequency remains finite at $q=0$, and where coupling to the plasma modes gives rise to complete screening of the charge of the quasiparticle excitations at low energies. As in the case of the 3D electron system, there are several ways of approaching the FCS system. The conventional FCS approach treats the CS interaction and the Coulomb interactions as though they were weak perturbations, taking care of the most singular aspects of the CS interaction by using (at least) the RPA to calculate the long-wavelength density and current response functions of the electron system. An important result is the density-density response function in the limit of small $q,\omega,\omega/q$. This function has a unique q,ω dependence, which is calculated explicitly for small $\widetilde{\phi}$ [giving Eq. (25)] and is argued to be correct for the physical value of $\widetilde{\phi}$. In particular, it predicts the $\nu=1/2$ state to be compressible.

The conventional FCS approach can also be used to extract properties of the low-energy quasiparticles from the

Green's function of the bare CS fermion. Although the overlap with the propagator for the low-energy quasiparticles is vanishingly small in the limit of an infinite system, one nevertheless gains insight into the quasiparticle effective mass m^* from the structure of the CS fermion self-energy, learning for example that there is a logarithmic infrared divergence in case of Coulombic electron-electron interactions.[6,20–23]

The conventional FCS approach is analogous to the perturbative approach most commonly employed in the study of three-dimensional electron systems. An alternate approach, suggested by MS (Ref. 13) and motivated by an earlier paper of Read,[10] is analogous to the Bohm-Pines[30] treatment of the 3D electron system. MS fix $\tilde{\phi}$ at its physical value, and apply a unitary transformation in order to produce low-energy quasiparticles, which are decoupled, at long wavelengths, from the high-energy magnetoplasmon modes. This approach may seem intuitively more appealing than the conventional FCS approach, since it directly brings to light the dipolar nature of the quasiparticles, and thus naturally explains why they move in straight lines in a strong magnetic field. Beyond offering a different intuition, however, the MS approach suggested also the possibility that the density-density response function calculated by the FCS approach might be incorrect. Particularly, it suggested the possibility that the $\nu = 1/2$ state is incompressible, as a result of the weak coupling of dipolar particles to electromagnetic fields.

Our original goal in the present paper was to settle the seeming contradiction between the two approaches, and to show that Eq. (25) is indeed the correct density-density response function for electrons at $\nu = 1/2$. Our starting point was the Hamiltonian (1). We followed MS in using the temporal (Weyl) gauge, in which the Hamiltonian contains two sets of dynamical degrees of freedom, the CS fermions and the CS gauge field. Our first step was to formulate a simplified model, which can be analyzed systematically by means of an expansion in a small parameter, and apply both approaches to study that model. In this model, which we termed the small Q model, the charge and flux quanta carried by the Chern-Simons fermion are smeared over an area $\sim Q^{-2}$ around the fermion, and Q is the small parameter. For the FCS perturbative analysis, a small Q limit is equivalent to a small $\tilde{\phi}$ limit, and can be treated by the RPA. Having observed that, we carried out two different analyses of the small Q model in terms of dipolar quasiparticles, attempting to understand how, despite their weak coupling to electromagnetic fields, the dipolar quasiparticles form a compressible state.

In the first analysis, described in Sec. II, we used a Lagrangian formulation, and integrated out the high-energy magnetoplasmon modes, obtaining an effective low-energy action (17) for the Chern-Simons fermions. This approach, while close to the FCS calculation, brought to light the dipolar nature of the CS fermions in the temporal gauge, as was seen by the relation (19) between the quasiparticle momentum and the electronic current. Moreover, the action (17) also revealed the important difference between a gas of free fermions carrying a dipole moment and the dipoles we have at hand. This difference is the K invariance of the present problem, the invariance of the energy of the fermionic system to a shift of the Fermi sphere in momentum space. In the language of Sec. II, K invariance is guaranteed by the current-current interaction in Eq. (17), which compensates the kinetic energy cost involved in a momentum space displacement of the quasiparticle's Fermi sphere and makes the associated total energy cost vanish. As we saw first in Sec. II, it is the combination of K invariance and the way the dipole moment is related to the quasiparticle's momentum that makes the $\nu = 1/2$ state compressible, and leads to the density-density response function (25), which is precisely the one obtained in the FCS approach.

In the second analysis of the small-Q limit, carried out in Sec. III, we performed the unitary transformation suggested by MS to decouple the low-energy quasiparticles (the fermions) from the high-energy magnetoplasmons (the gauge field), working systematically to lowest order in Q. We found the transformed Hamiltonian (46), in which the two degrees of freedom decouple in the long-wavelength limit. We then identified the way the electronic density is expressed in terms of the transformed variables, and calculated the electronic density-density response function, obtaining again Eq. (25). Although the low-energy quasiparticles do not interact with the transformed Chern-Simons gauge field at long wavelengths, they do retain strong momentum-dependent interactions, which must be properly taken into account in order to preserve the K invariance and to recover the correct response functions.

In both the Hamiltonian and Lagrangian descriptions, K invariance leads to the existence of zero-energy excitation modes for the longitudinal momentum density at finite wave vectors. However, these modes do not couple to any physical observables.

We have also investigated the single fermion Green's function and the fermion effective mass, in the context of our Hamiltonian dipolar formulation, in the small-Q limit, finding results identical to those of the conventional FCS calculation for the infrared singularities of the effective mass and the single particle Green's function. These issues were discussed in Secs. IV and V A, where we also discussed the difference between the small Q model and the physical model with regard to the ultraviolet renormalization of the effective mass. In this discussion we argued that present numerical calculations based on trial wave functions are well suited to study the ultraviolet renormalization but are inadequate for revealing the infrared one. We also showed that the dipolar approach is free from the inconvenient feature of the conventional FCS approach that the overlap between the Green's function for the bare CS fermion and the low-energy quasiparticle sector vanishes in the limit of an infinite system due to coupling to the high-energy magnetoplasmons.

The action (17) and the Hamiltonian (46) predict the same electronic response, despite their very different appearances. As we saw in Secs. II and III, these differences originate from a different identification of the position of the quasiparticle. In Sec. II the position of quasiparticles is (after averaging over a scale $\sim Q^{-2}$) identical to the position of the electrons, while in Sec. III the quasiparticles are shifted halfway from the electrons towards the correlation holes. This assignment of a position to a quasiparticle is arbitrary, particularly since the quasiparticles are dipolar, and different assignments should not lead to different predictions regarding the physical electronic response. The two assignments

we used may be argued to be the two natural ones, but are obviously not the only possible ones.

An additional difference between the two dipolar descriptions employed in Secs. II and III arises due the time derivative in the last term of the effective low-energy action Eq. (17). When the fields $\psi^\dagger, \psi$ entering the low-energy action of Eq. (17) are transformed into quantum mechanical operators, their commutation relations are not canonical. Consequently, the quasiparticle density operators at different points (or different wave vectors) do not commute. Preliminary study of these commutation relations indicate that in the limit of small wave vectors, the commutation relations between the density operators at different wave vectors is the commutation relation characterizing the electronic density operator after projection to the lowest Landau level.[29,13]

Our attempt to proceed from the small Q model to the physical model of unlimited Q is based on the conjecture that K invariance is not a property only of the small Q model, but rather a consequence of gauge invariance, which should hold also in the physical case. This belief is strongly supported by the analyses of Haldane,[18] who worked with trial wave functions confined to the lowest Landau level. However, the approximate unitary transformations (31) suggested by MS and used in our Sec. III do not preserve K invariance when Q is taken to be large, even if the driving force wave vector q remains very small. Consequently, we cannot use this transformation for studying the physical case. Rather, we showed, by constructing a Fermi-liquid theory, that even without a small Q cutoff, the combination of K invariance and the dipolar relation (60), together with conventional Fermi-liquid assumptions, is bound to lead to an electronic density-density response function of the form (73), which is identical in q, ω dependence to the small Q result, Eq. (25). In particular, the static limit of this response function predicts the $\nu = 1/2$ state to be compressible.

We remark that in the case of the three-dimensional electron gas, the low-energy quasiparticles in the Bohm-Pines formulation[30] form a more-or-less normal Fermi liquid, with nothing similar to the K invariance and the vanishing of $1 + F_1$, which we find in the CS system. In the three-dimensional electron system, the fact that the low-energy quasiparticles are neutral thus leads to incompressibility, in the sense that the electron density-response function, reducible with respect to the Coulomb interaction, vanishes $\propto q^2$.

To conclude, we believe that the dipolar approach to the $\nu = 1/2$ problem leads to a better understanding of the low-energy excitations of the $\nu = 1/2$ state, and predicts the same electronic density response as the FCS approach. Whether the dipolar approach can be used to improve our understanding of topics that were not discussed here, such as the electronic tunneling density of states, filling factors away from $\nu = 1/2$ and the effect of disorder, is an open question under extensive study.[16,13,29]

APPENDIX A: THE 5×5 RESPONSE MATRIX Π

In this appendix we outline the calculation of the response matrix $\Pi_{\mu\nu}$ for free fermions in zero magnetic field [See Eq. (51)]. Here $\Pi_{\mu\nu}(\mathbf{q}, \omega)$ is the 5×5 matrix defined as the Fourier transform of the retarded correlator $i\theta(t)\langle[\hat{O}_\mu(t,\mathbf{r}), \hat{O}_\nu(0,\mathbf{r}')]\rangle$ where the operator $\hat{O}_\mu$ takes the values ρ, C_l, g_l, C_t, g_t, respectively, for the five different values of the indices μ and ν. For example, Π_{ρ, C_l} is the Fourier transform of the retarded correlator $i\theta(t)\langle[\rho(t,\mathbf{r}), C_l(0,\mathbf{r}')]\rangle$. Note also that in Eqs. (51) and (53), the rows μ of the response matrix are arranged in the order ρ, C_l, g_l, C_t, g_t. We can then write the elements of this correlation function in terms of the matrix elements of time independent (Schroedinger representation) operators as

$$\Pi_{\mu\nu}(\omega, \mathbf{q}) = \int \frac{d\mathbf{k}}{(2\pi)^2} [f(\omega_{\mathbf{k}+\mathbf{q}}) - f(\omega_{\mathbf{k}})]$$

$$\times \frac{\langle \mathbf{k}+\mathbf{q}|\hat{O}_\mu(\mathbf{q})|\mathbf{k}\rangle \langle \mathbf{k}|\hat{O}_\nu^*(\mathbf{q})|\mathbf{k}+\mathbf{q}\rangle}{\omega - \omega_{\mathbf{k}+\mathbf{q}} + \omega_{\mathbf{k}} + i0^+}, \tag{A1}$$

where f is the Fermi function and $\omega_k = k^2/(2m)$. The matrix elements can be calculated directly to give

$$\left\langle \mathbf{k}+\frac{\mathbf{q}}{2}\Big|\rho(\mathbf{q})\Big|\mathbf{k}-\frac{\mathbf{q}}{2}\right\rangle = 1, \tag{A2}$$

$$\left\langle \mathbf{k}+\frac{\mathbf{q}}{2}\Big|\mathbf{g}(\mathbf{q})\Big|\mathbf{k}-\frac{\mathbf{q}}{2}\right\rangle = \mathbf{k}, \tag{A3}$$

$$\left\langle \mathbf{k}+\frac{\mathbf{q}}{2}\Big|\mathbf{C}(\mathbf{q})\Big|\mathbf{k}-\frac{\mathbf{q}}{2}\right\rangle$$

$$= i(\omega_{\mathbf{k}+\mathbf{q}/2} - \omega_{\mathbf{k}-\mathbf{q}/2}) \times \left\langle \mathbf{k}+\frac{\mathbf{q}}{2}\Big|\mathbf{g}(\mathbf{q})\Big|\mathbf{k}-\frac{\mathbf{q}}{2}\right\rangle \tag{A4}$$

$$\approx \frac{i\mathbf{q}\cdot\mathbf{k}}{m}\mathbf{k}. \tag{A5}$$

With some difficulty we can now perform the integrals in Eq. (A1) to obtain all of the elements of the matrix Π. However, one can save a great deal of effort by relating some of these elements to response functions that are well known in the literature, by noting certain relations between the different elements of the matrix Π, and by noting that some elements are zero by symmetry.

We begin by examining the symmetry of Eq. (A1). In particular, we examine the θ dependence of the integrand where θ is the angle that $\mathbf{k}$ makes with the $\hat{\mathbf{x}}$ axis. We note that the integrand can be written as matrix elements times some rational function of $\cos\theta$ only (not a function of $\sin\theta$). The matrix elements of ρ, g_l, and C_l are also analytic functions of $\cos\theta$ whereas the matrix elements of g_y and C_y can be written as $\sin\theta$ times an analytic function of $\cos\theta$. The integral over θ in Eq. (A1) is nonzero only if there is an even number of powers of $\sin\theta$. Thus, we have $\Pi_{\nu,\mu} = \Pi_{\mu,\nu} = 0$ for $\nu = g_l, \rho$, or C_l and $\mu = g_t$ or C_t which gives the 12 zero matrix elements shown in Eq. (51). We note that these are zero by time reversal symmetry for a system in zero magnetic field, just as the Hall coefficient is zero.

We now focus on elements of the response matrix $\Pi_{\mu\nu}$ where neither μ nor ν is either C_l or C_t. These elements of Π (a 3×3 submatrix) are very closely related to the electromagnetic response of a free two-dimensional electron gas to an external perturbing (scalar or vector) potential. The response matrix K^{free} is defined as $j_\mu = K^{\text{free}}_{\mu\nu} A^{ext}_\nu$ where μ here takes the values 0, l, and t (j_0 is the density and A_0 is the scalar potential, j_l and j_t are the respective components of the charge current, and A_l and A_t are the components of the vector potential). The response matrix K^{free} has been calculated many times before (see, for example, Ref. 6. Note that in this reference K^{free}_{tt} is referred to as K^0_{11}). Using Kubo formula, or simple linear response, the matrix K^{free} can be related to the retarded correlation function of paramagnetic currents plus a diamagnetic term. These correlation functions are precisely the correlation functions we need for calculation of the 3×3 submatrix of Π. Also noting that the definition of $\mathbf{g}$ differs from the definition of the physical paramagnetic current by a factor of m (the charge current is $\mathbf{j} = (\mathbf{g} - e\rho\mathbf{A})/m$), we then can obtain the relation

$$\Pi_{\mu\nu} = m^{\alpha(\mu)+\alpha(\nu)} K^{\text{free}}_{\mu\nu} + mn_0\alpha(\mu)\delta_{\mu\nu}, \quad (A6)$$

where $\alpha(\mu) = 0$ for $\mu = \rho$ and $\alpha(\mu) = 1$ for $\mu = g_l$ and for $\mu = g_t$. Here, the addition of mn_0 is the contribution of the diamagnetic term.

Current conservation and gauge invariance restrict the elements of the response matrix to have the symmetry $K^{\text{free}}_{\mu l} = (\omega/q)K^{\text{free}}_{\mu 0}$ and similarly $K^{\text{free}}_{l\mu} = (\omega/q)K^{\text{free}}_{0\mu}$, such that there are only three independent elements of the 3×3 matrix K^{free}, of which one is zero by time-reversal symmetry. Thus, we are left with two independent nonzero elements of K^{free}, which are $K^{\text{free}}_{00} = \Pi_{\rho\rho}$ and $K^{\text{free}}_{tt} = \Pi_{g_t g_t}/(m^2) - n_0/m$ [note in Eqs. (52) we refer to $\Pi_{\rho\rho}$ and $\Pi_{g_t g_t}$ as Π_{00} and Π_{tt}, respectively].

To calculate the remaining elements of Π, we first focus on zero frequency. Assuming q is small, we can replace the difference of Fermi functions in Eq. (A1) by $(\omega_{k+q} - \omega_k)df/d\omega$, which then leads to the simplification

$$\Pi_{\mu\nu}(\omega=0,\mathbf{q}) = \int \frac{d\mathbf{k}}{(2\pi)^2} \langle \mathbf{k}+\mathbf{q}|\hat{O}_\mu(\mathbf{q})|\mathbf{k}\rangle$$
$$\times \langle \mathbf{k}|\hat{O}^*_\nu(\mathbf{q})|\mathbf{k}+\mathbf{q}\rangle\delta(\omega_k - E_f), \quad (A7)$$

with E_f the Fermi energy. Plugging in the above matrix elements and performing this simple integration then yields the zero frequency results

$$\Pi_{\rho,C_l}(\omega=0,\mathbf{q}) = \Pi^*_{C_l,\rho}(\omega=0,\mathbf{q}) = iqn_0 + \cdots, \quad (A8)$$

$$\Pi_{C_l,C_l}(\omega=0,\mathbf{q}) = 3\pi(qn_0)^2/m + \cdots, \quad (A9)$$

$$\Pi_{C_t,C_t}(\omega=0,\mathbf{q}) = \pi(qn_0)^2/m + \cdots, \quad (A10)$$

where the $\cdots$ indicates higher orders in q. Note that all other nonzero elements of the matrix $\Pi_{\mu,\nu}$ which have either μ or ν equal to C_l or C_t are higher order in q at zero frequency.

Finally, we need to find the nonzero frequency results for all matrix elements $\Pi_{\mu\nu}$ with either μ or ν equal to C_l or C_t. To do this, we note that we can use Eq. (A4) to relate the $\mathbf{C}$ matrix elements to $\mathbf{g}$ matrix elements and insert these into Eq. (A1) such that we have, for example,

$$\Pi_{C_j,\nu}(\omega,\mathbf{q})$$
$$= \int \frac{d\mathbf{k}}{(2\pi)^2}[f(\omega_{\mathbf{k}+\mathbf{q}}) - f(\omega_{\mathbf{k}})]$$
$$\times \frac{i(\omega_{\mathbf{k}+\mathbf{q}} - \omega_{\mathbf{k}})\langle \mathbf{k}+\mathbf{q}|g_j(\mathbf{q})|\mathbf{k}\rangle \langle \mathbf{k}|\hat{O}^*_\nu(\mathbf{q})|\mathbf{k}+\mathbf{q}\rangle}{\omega - \omega_{\mathbf{k}+\mathbf{q}} + \omega_{\mathbf{k}} + i0^+},$$
$$(A11)$$

where j takes the value l or t. We then rewrite the frequency factors as

$$\frac{\omega_{\mathbf{k}+\mathbf{q}} - \omega_{\mathbf{k}}}{\omega - \omega_{\mathbf{k}+\mathbf{q}} + \omega_{\mathbf{k}} + i0^+} = -1 + \frac{\omega}{\omega - \omega_{\mathbf{k}+\mathbf{q}} + \omega_{\mathbf{k}} + i0^+},$$
$$(A12)$$

at which point we see that we have a frequency-independent piece (from the ''1'') and a frequency-dependent piece, which is precisely $i\omega$ times $\Pi_{g_j,\nu}$. Thus we have

$$\Pi_{C_j,\nu}(\omega,\mathbf{q}) = \Pi_{C_j,\nu}(\omega=0,\mathbf{q}) + i\omega\Pi_{g_j,\nu}(\omega,\mathbf{q}) \quad (A13)$$

and we can similarly derive

$$\Pi_{\nu,C_j}(\omega,\mathbf{q}) = \Pi_{\nu,C_j} - i\omega\Pi_{\nu,g_j}(\omega,\mathbf{q}) \quad (A14)$$

with $j = l, t$. Using these laws we can then completely calculate the rest of the matrix Π with a minimum of effort.

---

*Present address: Institut für Theoretische Physik, Universität zu Köln, Zülpicher Str. 77, 50937 Köln, Germany.

[1] For a general review of composite fermion physics, see *Composite Fermions*, edited by O. Heinonen (World Scientific, Singapore, 1998).

[2] See also, *Perspectives in Quantum Hall Effects*, edited by S. Das Sarma and A. Pinczuk (Wiley, New York, 1997).

[3] J. K. Jain, Phys. Rev. Lett. **63**, 199 (1989); for a more recent discussion of Jain's approach see the chapter by J. K. Jain and

R. Kamilla, in *Composite Fermions* (Ref. 1) as well as J. K. Jain, Adv. Phys. **41**, 105 (1992).

[4] A. Lopez and E. Fradkin, Phys. Rev. B **44**, 5246 (1991); **47**, 7080 (1993); see also the chapter by A. Lopez and E. Fradkin in *Composite Fermions* (Ref. 1).

[5] G. Moore and N. Read, Nucl. Phys. B **360**, 362 (1991); M. Greiter and F. Wilczek, Mod. Phys. Lett. B **4**, 1063 (1990); Nucl. Phys. B **370**, 577 (1992); M. Greiter, X.-G. Wen, and F. Wilczek, Phys. Rev. Lett. **66**, 3205 (1991); Nucl. Phys. B **374**, 567 (1992).

[6] B. I. Halperin, P. A. Lee, and N. Read, Phys. Rev. B **47**, 7312 (1993).

[7] V. Kalmeyer and S.-C. Zhang, Phys. Rev. B **46**, 9889 (1992).

[8] By finite compressibility, we mean that the static density-density linear response function $\chi_{\rho\rho}(q)$ approaches a finite constant for wave vector $q \to 0$, in the case of short-range interactions, or varies like $1/v(q)$ for long-range interactions $v(q)$. The linear response function is defined for infinitesimal density fluctuations $\delta\rho^e$, at finite q, in the limit $T=0$, for a system without impurities. Its applicability to finite density fluctuations is restricted to values of $\delta\rho^e$ that are small compared to $q k_F/4\pi$. (This assures that the effective cyclotron radius R_c^* in the Chern-Simons field produced by $\delta\rho^e$ is large compared to q^{-1}.) The order of limits for $\delta\rho^e \to 0$ and $q \to 0$ must be specified because, at $q=0$, the energy is a singular function of $\delta\rho^e$, due to de Haas–van Alphen-type oscillations, as discussed in Ref. 6 .

[9] A review of experimental results on composite fermion physics is given by R. L. Willett, in *Composite Fermions* (Ref. 1). See also, R. L. Willett, Adv. Phys. **46**, 447 (1997); and H. L. Stormer and D. C. Tsui, in *Perspectives in Quantum Hall Effects* (Ref. 2), Chap. 10; see also M. P. Lilly *et al.*, Phys. Rev. Lett. **80**, 1714 (1998) for Coulomb-drag experiments; see J. P. Eisenstein, L. N. Pfieffer, and K. W. West, Phys. Rev. B **50**, 1760 (1994) for compressibility measurements; see J. S. Moon *et al.*, Phys. Rev. Lett. **79**, 4457 (1997) for magnetocapacitance measurements.

[10] N. Read, Semicond. Sci. Technol. **9**, 1859 (1994); Surf. Sci. **361/362**, 7 (1996).

[11] In addition to having fractional charge, quasiparticles in fractional quantized Hall states obey fractional statistics, as noted in B. I. Halperin, Phys. Rev. Lett. **52**, 1583 (1984). However, for quasiparticles in the principal FQHE states, at filling fractions $\nu = p/(2p+1)$, the statistical angle approaches that of fermions, in the limit $\nu \to 1/2$. Quasiparticles obeying fractional statistics can, of course, be represented by fermions interacting with a residual Chern-Simons gauge field. A derivation of the fractional charge and fractional statistics of FQHE quasiparticles, starting from the FCS formulation, is outlined in E. Fradkin, *Field Theories of Condensed Matter Systems* (Addison-Wesley, Reading, MA, 1991), p. 337.

[12] A recent discussion of the relation between the wave function approaches and the dipole picture is given by S. H. Simon, in *Composite Fermions* (Ref. 1).

[13] R. Shankar and G. Murthy, Phys. Rev. Lett. **79**, 4437 (1997). See also the chapter by G. Murthy and R. Shankar, in *Composite Fermions* (Ref. 1).

[14] V. Pasquier and F. D. M. Haldane, Nucl. Phys. B **516**, 719 (1998).

[15] N. Read, Phys. Rev. B **58**, 16 262 (1998).

[16] D.-H. Lee, Phys. Rev. Lett. **80**, 4745 (1998).

[17] B. I. Halperin and A. Stern, Phys. Rev. Lett. **80**, 5457 (1998).

[18] F. D. M. Haldane (unpublished).

[19] J. M. Leinaas and J. Myrheim, Nuovo Cimento B **37**, 1 (1977). For a review of fractional statistics and related topics see F. Wilczek, *Fractional Statistics and Anyon Superconductivity* (World Scientific, Singapore, 1990).

[20] A. Stern and B. I. Halperin, Phys. Rev. B **52**, 5890 (1995); Surf. Sci. **361**, 42 (1996).

[21] Y. B. Kim, A. Furusaki, X.-G. Wen, and P. A. Lee, Phys. Rev. B **50**, 17 917 (1994); Y. B. Kim, P. A. Lee, X.-G. Wen, and P. C. E. Stamp, *ibid.* **51**, 10 779 (1995).

[22] H. J. Kwon, A. Houghton, and J. B. Marston, Phys. Rev. Lett. **73**, 284 (1994); Phys. Rev. B **52**, 8002 (1995); C. Nayak and F. Wilczek, Nucl. Phys. B **417**, 359 (1994); J. Gan and E. Wong, Phys. Rev. Lett. **71**, 4226 (1993); J. Polchinski, Nucl. Phys. B **422**, 617 (1994).

[23] B. L. Altshuler, L. Ioffe, and A. J. Millis, Phys. Rev. B **50**, 14 048 (1994).

[24] B. I. Halperin, Phys. Rev. B **45**, 5504 (1992).

[25] A. Raghav Chari, F. D. M. Haldane, and Kun Yang, cond-mat/9707055 (unpublished).

[26] C. Kallin and B. I. Halperin, Phys. Rev. B **30**, 5655 (1984).

[27] D.-H. Lee, Y. Krotov, J. Gan, and S. A. Kivelson, Phys. Rev. B **55**, 15 552 (1997); **57**, 9349(E) (1998).

[28] S. H. Simon and B. I. Halperin, Phys. Rev. B **48**, 17 368 (1993); See also Ref. 12 and S. H. Simon, J. Phys.: Condens. Matter **8**, 10 127 (1996).

[29] A. Stern, F. von Oppen, B. I. Halperin, and S. H. Simon (unpublished).

[30] D. Bohm and D. Pines, Phys. Rev. **92**, 609 (1953).