# Experiment Ma8: Superconductivity

## 1 Overview

Superconductivity is a phenomenon occurring at low temperatures.

H.K. Onnes observed in year 1911 that the electrical resistivity of some metals sank abruptly below a certain temperature (called transition temperature $T_c$). Also the specific heat, the heat conduction and the magnetic properties of superconductors differ themselves from the ones of normal conductors. The transition from the normal conducting state (NCS) to the superconducting state (SCS) is reversible and for **type 1** superconductors occurs typically below 10 K. Superconductors of **type 2** (typically alloys, $T_c<25K$) are mostly exploited for technical applications, for example for constructing magnets for high magnetic fields. The transition temperature is characteristic for each material and it is related to the Debye temperature. Not all the metals become superconductors (for example ferromagnetism excludes superconductivity).

Purpose of this experiment is the measure of the transition temperature of two metals (Tin and Indium) and to investigate magnetic fields effects. Moreover some confidence with a low temperature setup should be gained.

## 2 Theory basics

In order to understand superconductivity it is important to keep in mind the theory of the **free electron gas** [1]. The **electron density of states** in a normal conductor and the **Fermi function** define the behavior at the Fermi surface. At the superconducting transition the behavior of the electrons changes in the proximity of the Fermi surface. Why and how this happens is of crucial importance in understanding superconductivity.

This phenomenon is explained through a model, the BCS theory [4], that has been developed half a century after the discovery of superconductivity. This theory explains how two electrons can form a **Cooper pair** as well as the **isotopic effect** and the occurrence of an **energy gap** around the Fermi surface and other superconductors' properties [1],[2],[3],[4]. A pictorial way

of describing how a metal can undergo the transition to the superconducting state and transport without any resistance can be found in Ref. [1].

The **Meißner-Ochsenfeld effect** will be exploited in order to proof the transition from the normal to the superconducting state. This phenomenon occurs when a superconductor is located in an external magnetic field; the knowledge of the behavior of a superconductor under intense fields is fundamental for the experiment (in particular when the field is intense enough to destroy the superconducting state itself) [2]. The value of the **critical magnetic field** $B_c$ at which the superconducting phase is destroyed depends upon temperature [1]. For $B < B_c$ the susceptibility of the superconducting phase is $\chi = -1$.

For temperatures below $T_c$, without any external magnetic field, the difference in entropy between the normal and the superconductive phase is positive, implying that the electrons in the superconductive phase possess a higher degree of order than in the normal one. A discussion of this thermodynamical property of superconductors can be found in [1] as well as the microscopic explanation in [4].

## 3  Experimental details

### 3.1 Low temperature setup

The cryogenic system consists of two cryostats: one for the liquid nitrogen and, within this one, a second one for liquid Helium. Between the two cryostats there is an additional vacuum chamber for ensuring better isolation of the He-cryostat. The pressure of the $N_2$-cryostat is the atmospheric one while the pressure of the He cryostat can be changed by means of a pump. Both cryostats are made of glass and silvered; the inner coating has been made leaving a transparent window in order to ensure optical access to the chamber. In this way it is possible to see the liquid nitrogen ($LN_2$) in the outer Dewar and the liquid Helium in the inner Dewar. At 2.17 K the liquid He undergoes the superfluid transition and you can observe the different behavior.

**ATTENTION – PRECAUTIONS YOU NEED TO TAKE**

**A.**    All mechanical connections with the glass parts have to be applied carefully. **Risk of breaking!**

**B.**     Liquid Nitrogen and liquid Helium must be filled in the cryostats just under the control of the supervisor.

**C.**     Before filling the liquid nitrogen please be sure that there is no water left within the cryostat.  **Risk of breaking!**

**D.**     Before filling the Liquid Helium, please be sure the He cryostat and the connectors to the pump are sealed.

## 3.2 Measurement

The change of susceptibility, measured through the induction system, is used to proof the change from normal to superconducting state. The sample, placed within the measuring inductance, is located within the He bath-cryostat. Through variation of the pressure of the He chamber, the temperature can be tuned between 1.5 and 4.2 K. The He cryostat is placed within a coil; the intensity of the external applied magnetic field can be changed by tuning the current flowing through the coil. The relation between current and magnetic field is given by the calibration relation:

$$B \text{ [Gauss]} = (45 \pm 1) \, I \text{ [Ampere]}$$

The principle of measurement is based on the transformer (see Fig.1). The induced secondary voltage depends upon the coupling of the inductances. As this depends upon the magnetic properties of the probe material inserted between the inductances, a change of the secondary inductance is due to changing of the permeability or susceptibility of the sample.
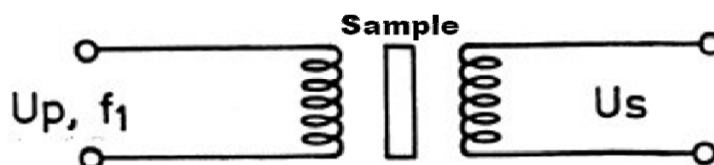


**Figure 1:** Transformer circuit for the determination of changes of the susceptibility of a sample; $U_p$ is the primary voltage at frequency $f_1$ while $U_s$ is the secondary voltage.

The secondary voltage will be registered as a function of the temperature or as a function of the external magnetic field. To increase the sensitivity of your measurement you will use two similar coupled inductances connected to each other (differential transformer). One of the two transformers will have an empty sample compartment. The change of the secondary voltage in the cryo-system will be analyzed by the lock-in amplifier in a frequency- and phase- sensitive way (see Fig. 2).
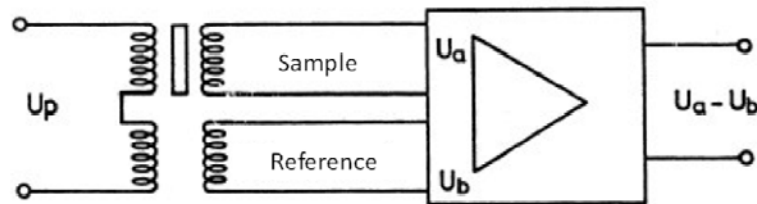


**Figure 2:** Sketch of the setup for the determination of changes in the sample voltage through the lock-in technique. For $T > T_c$ is $|U_a| \approx |U_b|$, while for $T < T_c$ is $|U_a| < |U_b|$.

For an accurate determination of the $T_c$ and the $B_c$ at different temperatures, the transition from normal to superconductor will be measured pointwise. It is thus possible to obtain $T_c$ and $B_c$ from the behavior of the $\Delta U_a$ or $\Delta U_b$ as shown from the scheme in Fig. 3. The range $2\delta T_C$ characterizes the range on which transition between the two phases takes place.



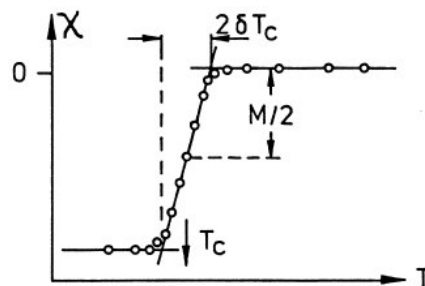**Figure 3:** Behavior of $\chi$ during the transition from NCS to SCS.

### 3.3 Instrument list

Cryostats; two stages vacuum pump; valves; gas flow meter; absolute pressure meter; Lock-in amplifier; oscilloscope; coil with power supply; multimeters.

## 3.4 Pressure to temperature conversion

The values of the He vapor pressure can be converted into the temperature ones through the following table:

T, K

| ▶ | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 | 0.10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| .50 | .00 | .00 | .00 | .00 | .00 | .00 | .00 | .00 | .00 | .00 | .00 |
| .60 | .00 | .00 | .00 | .00 | .00 | .00 | .00 | .00 | .00 | .00 | .00 |
| .70 | .00 | .00 | .00 | .01 | .01 | .01 | .01 | .01 | .01 | .01 | .02 |
| .80 | .02 | .02 | .02 | .02 | .03 | .03 | .03 | .04 | .04 | .05 | .06 |
| .90 | .06 | .06 | .07 | .08 | .09 | .10 | .11 | .12 | .13 | .15 | .16 |
| 1.00 | .16 | .18 | .19 | .21 | .23 | .25 | .28 | .30 | .33 | .36 | .39 |
| 1.10 | .39 | .42 | .46 | .50 | .54 | .58 | .62 | .67 | .72 | .78 | .83 |
| 1.20 | .83 | .89 | .96 | 1.03 | 1.10 | 1.17 | 1.25 | 1.33 | 1.42 | 1.51 | 1.61 |
| 1.30 | 1.61 | 1.71 | 1.82 | 1.93 | 2.05 | 2.17 | 2.30 | 2.43 | 2.57 | 2.72 | 2.87 |
| 1.40 | 2.87 | 3.03 | 3.20 | 3.37 | 3.55 | 3.74 | 3.94 | 4.14 | 4.35 | 4.57 | 4.80 |
| 1.50 | 4.80 | 5.03 | 5.28 | 5.53 | 5.80 | 6.07 | 6.35 | 6.65 | 6.95 | 7.26 | 7.59 |
| 1.60 | 7.59 | 7.92 | 8.27 | 8.62 | 8.99 | 9.37 | 9.76 | 10.17 | 10.58 | 11.01 | 11.45 |
| 1.70 | 11.45 | 11.91 | 12.38 | 12.86 | 13.35 | 13.85 | 14.38 | 14.92 | 15.47 | 16.04 | 16.62 |
| 1.80 | 16.62 | 17.22 | 17.83 | 18.46 | 19.10 | 19.76 | 20.43 | 21.13 | 21.84 | 22.56 | 23.30 |
| 1.90 | 23.30 | 24.06 | 24.84 | 25.63 | 26.44 | 27.27 | 28.12 | 28.98 | 29.87 | 30.77 | 31.69 |
| 2.00 | 31.69 | 32.63 | 33.58 | 34.56 | 35.55 | 36.56 | 37.59 | 38.64 | 39.71 | 40.80 | 41.90 |
| 2.10 | 41.90 | 43.02 | 44.17 | 45.33 | 46.51 | 47.70 | 48.92 | 50.15 | 51.40 | 52.66 | 53.95 |
| 2.20 | 53.95 | 55.26 | 56.59 | 57.94 | 59.31 | 60.70 | 62.12 | 63.56 | 65.02 | 66.50 | 68.01 |
| 2.30 | 68.01 | 69.54 | 71.10 | 72.67 | 74.27 | 75.90 | 77.55 | 79.23 | 80.93 | 82.65 | 84.40 |
| 2.40 | 84.40 | 86.17 | 87.97 | 89.80 | 91.65 | 93.53 | 95.43 | 97.36 | 99.32 | 101.30 | 103.32 |
| 2.50 | 103.32 | 105.35 | 107.42 | 109.51 | 111.64 | 113.79 | 115.97 | 118.17 | 120.41 | 122.67 | 124.97 |
| 2.60 | 124.97 | 127.29 | 129.64 | 132.03 | 134.44 | 136.88 | 139.36 | 141.86 | 144.39 | 146.96 | 149.55 |
| 2.70 | 149.55 | 152.18 | 154.84 | 157.53 | 160.25 | 163.00 | 165.79 | 168.61 | 171.46 | 174.34 | 177.25 |
| 2.80 | 177.25 | 180.20 | 183.19 | 185.20 | 189.25 | 192.33 | 195.45 | 198.60 | 201.78 | 205.00 | 208.25 |
| 2.90 | 208.25 | 211.54 | 214.87 | 218.23 | 221.62 | 225.05 | 228.52 | 232.02 | 235.56 | 239.13 | 242.74 |
| 3.00 | 242.74 | 246.39 | 250.08 | 253.80 | 257.56 | 261.36 | 265.20 | 269.07 | 272.98 | 276.94 | 280.92 |
| 3.10 | 280.92 | 284.95 | 289.02 | 293.13 | 297.28 | 301.46 | 305.69 | 309.95 | 314.26 | 318.61 | 322.99 |
| 3.20 | 322.99 | 327.42 | 331.89 | 336.40 | 340.95 | 345.55 | 350.18 | 354.86 | 359.58 | 364.34 | 369.14 |
| 3.30 | 369.14 | 373.99 | 378.88 | 383.81 | 388.79 | 393.81 | 398.87 | 403.98 | 409.13 | 414.32 | 419.56 |
| 3.40 | 419.56 | 424.84 | 430.17 | 435.54 | 440.96 | 446.42 | 451.93 | 457.48 | 463.08 | 468.73 | 474.42 |
| 3.50 | 474.42 | 480.16 | 485.94 | 491.77 | 497.65 | 503.58 | 509.55 | 515.57 | 521.64 | 527.76 | 533.92 |
| 3.60 | 533.92 | 540.13 | 546.39 | 552.70 | 559.05 | 565.46 | 571.91 | 578.41 | 584.97 | 591.57 | 598.22 |
| 3.70 | 598.22 | 604.92 | 611.65 | 618.48 | 625.33 | 632.24 | 639.19 | 645.20 | 653.26 | 660.37 | 667.53 |
| 3.80 | 667.53 | 674.74 | 682.01 | 689.32 | 696.69 | 704.12 | 711.59 | 719.12 | 726.71 | 734.35 | 742.04 |
| 3.90 | 742.04 | 749.78 | 757.58 | 765.44 | 773.35 | 781.31 | 789.34 | 797.41 | 805.55 | 813.74 | 821.98 |
| 4.00 | 821.98 | 830.28 | 838.64 | 847.06 | 855.53 | 864.06 | 872.65 | 881.30 | 890.00 | 898.76 | 907.58 |
| 4.10 | 907.58 | 916.46 | 925.39 | 934.39 | 943.45 | 952.56 | 961.73 | 970.97 | 980.26 | 989.61 | 999.02 |
| 4.20 | 999.02 | 1008.49 | 1018.02 | 1027.61 | 1037.26 | 1046.97 | 1056.74 | 1066.58 | 1076.47 | 1086.43 | 1096.46 |
| 4.30 | 1096.46 | 1106.55 | 1116.70 | 1126.91 | 1137.19 | 1147.54 | 1157.95 | 1168.42 | 1178.96 | 1189.57 | 1200.25 |
| 4.40 | 1200.25 | 1210.99 | 1221.79 | 1232.66 | 1243.60 | 1254.61 | 1265.68 | 1276.82 | 1288.03 | 1299.30 | 1310.65 |
| 4.50 | 1310.65 | 1322.06 | 1333.54 | 1345.10 | 1356.72 | 1368.41 | 1380.17 | 1392.00 | 1403.90 | 1415.88 | 1427.92 |
| 4.60 | 1427.92 | 1440.03 | 1452.22 | 1464.48 | 1476.81 | 1489.21 | 1501.69 | 1514.24 | 1526.86 | 1539.55 | 1552.32 |
| 4.70 | 1552.32 | 1565.17 | 1578.09 | 1591.08 | 1604.15 | 1617.29 | 1630.51 | 1643.80 | 1657.17 | 1670.62 | 1684.14 |
| 4.80 | 1684.14 | 1697.75 | 1711.42 | 1725.18 | 1739.01 | 1752.93 | 1766.92 | 1780.99 | 1795.14 | 1809.37 | 1823.68 |
| 4.90 | 1823.68 | 1838.06 | 1852.54 | 1867.09 | 1881.72 | 1896.43 | 1911.22 | 1926.09 | 1941.06 | 1956.09 | 1971.22 |
| 5.00 | 1971.22 | 1986.42 | 2001.71 | 2017.09 | 2032.55 | 2048.09 | 2063.71 | 2079.43 | 2095.22 | 2111.10 | 2127.07 |
| 5.10 | 2127.07 | 2143.13 | 2159.27 | 2175.50 | 2191.82 | 2208.23 | 2224.71 | 2241.29 | 2257.96 | 2274.72 | 2291.57 |
| 5.20 | 2291.57 | 2306.50 | 2325.53 | | | | | | | | |

## 4 Assignments

### 4.1 Measurement and evaluation

**a)** Determine the transition temperature $T_c(B=0)$ for two metals, Tin (Sn) and Indium (In). Please, choose your sampling frequency in a proper way in order to be able to follow the transition. What are the reasons for the finite transition region?

```
Here are some suggested values for a proper sampling of the two
samples:

Sn in the range    (650,570) mbar    -    sampling: 4 mbar

In in the range    (440,420) mbar    -    sampling: 4 mbar

                   (420,416) mbar    -    sampling: 2 mbar

                   (416,400) mbar    -    sampling: 1 mbar

                   (400,390) mbar    -    sampling: 2 mbar
```

**b)** Determine for both samples the $B_c(T)$ value at four different temperatures (suggested values: 3 K, 2.5 K, 2 K, 1.7 K). Please check that the critical magnetic field depends quadratically upon T. Determine, by a linear regression of $B_c(T)$ versus $(T/T_c)^2$, the value of $B_{c0} = B_c(T=0\ K)$.

**c)** Please give an estimation of the values of the **binding energy** and the relative distance between two electrons forming a Cooper pair; how many other Cooper pairs can be approximately found between the two electrons forming a Cooper pair?

**d)** Please calculate the theoretical values of thermodynamical critical fields $B_{c0}$ for tin and indium and compare them with the experimental ones [1]. In order to perform such calculations you will need the data:

**Tin:**    **Molecular Weight**=118.7 g/mol;    $\rho$=7.3 g/cm$^3$;    $\gamma$=1.78*10$^{-3}$ J/mol K$^2$;

**Indium:**    **Molecular Weight**=114.8 g/mol;    $\rho$=7.3 g/cm$^3$;    $\gamma$=1.69*10$^{-3}$ J/mol K$^2$;

## 5 Reference

[1] Buckel, *Superconductivity*, VCH Verlag.

[2] M. Tinkham, *Introduction to superconductivity*, MacGraw-Hill

[3] N.W. Ashcroft, N.D: Mermin, *Festkörperphysik*, Oldenbourg Verlag.

[4] J. Bardeen, L.N. Cooper, I.R. Schrieffer, Phys. Rev. **108**, 1175 (1957).