FABRY-PEROT ETALON | GP II

Key Words

Diffraction and Interference; Multiple-ray interference. Interference spectrometer; *Fabry-Perot* Interferometer or *Fabry-Perot r*esonator; Optical Resonators

Aim of the Experiment

Experimental introduction to the *Fabry-Perot* interferometer as an important component in high resolution spectral equipment and in laser technology.

Literature

Standard literature (see list of standard text books); furthermore:

Bergmann·Schaefer; Lehrbuch der Experimentalphysik Band III (Optik); de Gruyter.

Exercises

1. Assembling and adjusting the apparatus.
2. Determining the plate separation of a *Fabry-Perot* etalon with the red 643.9-nm line of cadmium and calculating the (approximate) interference order.
3. Relative determination of the wavelength of the green and dark blue lines of the cadmium spectrum.
4. Estimating the line width of the interference maxima for the red line and comparison with the expected instrument line width.

Physical Principles

Multiple Ray Interference

Diffraction objects with periodic structures, as e.g., a diffraction grating, produce *multiple ray* interference with very narrow interference maxima, enabling high resolution in technical and spectroscopic applications.

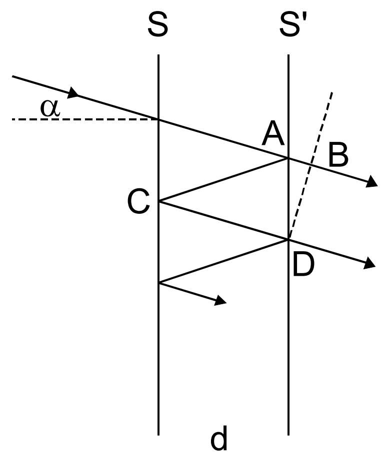
In diffraction and interference at a single or double slit, the intensity maximum appears at a point of emission where all elementary waves concerned are in-phase. A deviation in direction (detuning) from this point causes differences in the optical path length and hence portions of destructive interference, leading to a decrease in intensity compared to the maximum and the distinctiveness of the corresponding diffraction pattern (see also experiment *DIFFRACTION AND INTERFERENCE*).

If the diffraction structure repeats itself periodically, the optical path difference increases and multiplies according to the separation and number of structures and effects a much faster decrease in the intensity seen from the maximum. Multiple ray interferences are characterized by narrow maxima separated by wide dark zones (or vice-versa in multiple ray interference with reflected light.

Fabry-Perot Etalon

A *Fabry-Perot etalon* is an optical resonator formed by two plane parallel, partially reflecting surfaces and an enclosed optical medium. An incident plane wave is split into multiple, coherent partial waves due to "zigzag reflections." These interfere with one another and lead to multiple interferences in reflected or transmitted light. The ray path through an etalon is sketched in the diagram below. (Diffraction at the boundary surfaces was not taken into account since it only causes a parallel displacement of the rays).

The diagram shows the wave front of two rays emerging from the splitting of the incident ray. The path difference δ results from simple geometric considerations:



(1)

$$\delta = \overline{A}\,\overline{C} + \overline{C}\,\overline{D} - \overline{A}\,\overline{B} = 2\,d\cos\alpha$$

where d is the distance between the boundary surfaces and α is the angle of incidence of the radiation.

Accordingly, the path difference is smaller the larger the angle of incidence α is. An additional path difference due to phase jumps caused by reflections at the boundary surfaces can be neglected, since, for transmission, it amounts to a whole multiple of the wavelength. The condition for constructive interference in transmitted light is then:

(2)

$$\delta = 2\,d\cos\alpha = z\,\lambda \qquad \text{with} \qquad z = 1,2,3,...$$

whereby interference maxima appear when the angle of incidence α or the wavelength λ satisfy condition (2). The path difference in units of wavelength is called the *phase value* ϕ ($\phi = \delta/\lambda$), the whole number values z of ϕ are called the *interference order* of the maximum.

The high quality of such *optical resonators* and hence the high resolution (over a small *spectral range*) is based on multiple-ray interference and the high interference order at correspondingly large resonator dimensions. With a plate separation of 5 mm and wave length of 500 nm, the interference order is z = 20 000. At such a large optical path length even very small wavelength difference sum to give destructive interference.

Free Spectral Range

The uninterrupted region of the wavelength to be investigated using spectral equipment is called the *free spectral range* or *Dispersion range*. With the Etalon one can distinguish between two neighboring maxima by their difference in order (Δz = 1) at the same wavelength, or by small differences in wavelength $\Delta\lambda$ at the same order of interference. The interference condition (2) for a definite order in both cases is:

(3) $$(z+1)\lambda = z(\lambda + \Delta\lambda)$$

Thus the free spectral range of the Etalon is:

(4) $$\Delta\lambda = \frac{\lambda}{z} \quad \text{or} \quad \frac{\Delta\lambda}{\lambda} = \frac{1}{z}$$

Because of this comparatively small dispersion range at large z, the Fabry-Perot-Etalon is preferably suited for the investigation of virtually monochromatic light of for precise measurements of narrow wavelength ranges after spectral decomposition.

Fabry-Perot Spectrometer

When used as a spectrometer, divergent light is passed through the Etalon and the parallel rays belonging to a certain order of interference are collected by a convex lens and focused on an image plane. Because of the rotational symmetry of the optical arrangement one observes in the focal plane of the lens concentric rings of equal inclination (*Haidinger Rings*; *Wilhelm Ritter von Haidinger*, 1795-1871; Austrian Geologist and Mineralogist). If one sets fort he angle of inclination α and cos α approximately:

(5) $$\alpha = \frac{r}{f} \quad \text{and} \quad \cos\alpha = 1 - \frac{1}{2}\alpha^2$$

where f is the focal length of the lens, we get for the interference condition (2):

(6) $$z = \frac{2d}{\lambda}\left[1 - \frac{r^2}{2f^2}\right] \quad \text{or} \quad \frac{2d}{\lambda} \approx z\left[1 + \frac{r^2}{2f^2}\right]$$

With known f, equation (6) contains the quantities λ, d and z. If one measures at least two radii of the ring system, one obtains two equations of type (6) and can thus eliminate the order z. If one specifies the innermost observable ring with the index 0 (in general the ring center does not represent an interference maximum) and the following rings with the index i (i=1, 2, 3,...), then from (6) we have

(7) $$d = i\,\frac{\lambda f^2}{r_i^2 - r_0^2}$$

Relative wavelength measurements are also possible without knowledge of d or z. Let r and r' be the order to a wavelength λ and $\lambda' = \lambda + \Delta\lambda$, then from (6) we get approximately:

(8) $$\Delta\lambda \approx \frac{\lambda}{2f^2}\left(r - r'^2\right)$$

The accuracy (apart from the error in the focal length f) is determined by the measurement accuracy of the radii and the above mentioned high resolution of the Etalon does not come to bear. This is due to the fact that in (6) the order z was eliminated although it represents an essential element in the relationship (the term in the bracket: $r^2/2f^2 \ll 1$).

The Etalon must be calibrated for the exact and absolute determination of wavelength, i.e., the plate separation must be as accurately as possible, and the order exactly determined. This requires spectral analysis of several very well known wavelengths and setting the measured radii of the rings of a definite order with initial values for d and z in (6)to give a system of equations for the different wavelengths. For evaluation, the initial values for d and z are varied until the system of equations shows the largest agreement (minimum for the sum of mean square errors, i.e. the variance).

Resolution of the Fabry-Perot Etalon

The exact intensity distribution in the vicinity of an interference maximum as a function of the phase ϕ is dependent on the transmission T and reflectivity R of the plate and is described by the *Airy formula* (*Sir George Bidell Airy*; 1801-1892; Engl. mathematician and astronomer):

(9) $$\frac{I}{I_0} = \left[\frac{T}{1-R}\right]^2 \frac{1}{1 + \frac{4R}{(1-R)^2}\sin^2 \phi\pi}$$

Hence, from this relation, one can determine the full-width-at-half-maximum $2\Delta z$ of the interference maxima, giving for small angles approximately:

(10) $$2\,\Delta z = \frac{1-R}{\pi\sqrt{R}}$$

With a plate distance of 5 mm and a reflectivity of about 92 % one gets $2\Delta z$ = 0.026, hence, a width of approx. 3 % of the ring separation. The wavelength difference corresponding to the difference in order (free spectral range) is given by (4). Inserting the full width $2\Delta z$ as the criterion for the resolution gives

(11)

$$\Delta\lambda = \frac{\lambda}{z}\,0.026 = 0.0007\,\text{nm} \quad \text{or} \quad \frac{\Delta\lambda}{\lambda} = 2\cdot 10^{-6}$$

Line Width of Optical Transitions

Seen classically, a radiating atom is a damped harmonic oscillator continuously loosing energy. The associated frequency spectrum has the form of a bell-shaped curve whose width is determined by the damping constant (*decay constant*). From the aspect of quantum mechanics, the decay constant corresponds to the *mean lifetime* of the system and the finite line width is explained from *Heisenberg's Uncertainty Principle* (*Werner Heisenberg*; *1901; German Physicist). The decay constant of optical transitions is around 10^{-8} s, giving a *natural line width* of approximately 10^{-5} nm.

Under real conditions, the lines become broader due to a number of influences. In an ensemble of atoms, the mean lifetime is shortened by collisions of atoms with one another or with impurity atoms (*pressure broadening*). Atomic thermal motion and the *Doppler Effect* (*Christian Doppler*, 1803-1853; Austrian physicist and mathematician), results in a line shift when the atoms radiate, leading to a broadening of the lines due to the statistical distribution of the velocities (*Doppler broadening*). Moreover, a number of further complex interactions may occur under various conditions resulting in line broadening.

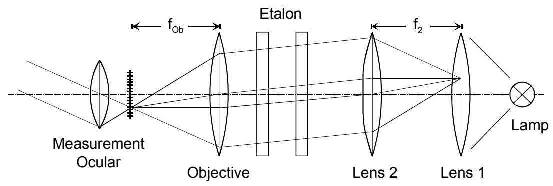
Apparatus and Equipment

Optical bank; Cd spectral lamp with power supply unit; optical lens, collimator, iris diaphragm, Fabry-Perot etalon, objective lens, ocular. Color filters (red, blue, green.)

Experiment and Evaluation

Exercise1 (Setting-up and Adjustment)

The ray path is presented in the following diagram.



The optical lens (lens 1) corresponds to the entrance slit of conventional optical ray paths and fulfils two functions. Firstly, it presents a uniformly illuminated *secondary plane light source* to generate, with the help of the collimator (lens 2), parallel light beams of different inclination. Secondly, it forms an image of the lamp approximately in the Etalon to achieve a high intensity of the interference patterns. Furthermore, an iris is available to focus the rays in the vicinity of the axis to improve the image.

The objective lens forms an image of the interference patterns in the ocular (ocular micrometer), where one can directly measure the diameter of the interference rings.

Adjusting the ray path is relatively uncritical. Important, is a careful focusing of the diffraction patterns which is subjectively difficult due to their blurred intensity distribution and a correct height adjustment to center the measuring ocular on the ring system.

Color filters are available to select the desired lines of the Cd-spectrum.

Exercise 2 (Plate Separation) and 3 (Wavelength of the Green and Blue Lines)

With the respective color filters in place, one measures the positions of the left- and right hand border of the rings for decreasing interference order. Observation of the interference patterns of the blue and green lines is physiologically problematic because of the imperfect pre-selection of the color filters and needs some time getting used to.

Evaluation is made according to equations and (7) and (8) by plotting the square of the radii against the number of rings. From the gradient of the expected lines one can determine the plate separation for the „known" red line and hence determine the wavelength of the green and blue lines.

Exercise 4 (Line Width)

The width of the red line can only be estimated due to the non-linear response of the eye. The measurement is evaluated according to (8). Compare and discuss the result with that given by equation (10). The reflectivity R of the Etalon shall be taken as 80 %.

Which addition experimental factor influences the observed line width?