SPECIFIC CHARGE e/m_0 *of the Electron* — GP II

Key Words

Specific charge and charge quantization. **Lorentz force.**
Circular motion.

Aim of the Experiment

Exemplary experiment on the atomic structure of electricity and measurement methods by the deflection of charged particles in orthogonal electric and magnetic fields.

Literature

Gerthsen Physik (22. Auflage): 8.1.1, 8.2

Bergmann-Schaefer, Band 2 (8. Auflage): 3.4.4, 7.1.2, 11.1.7, 11.1.4 , 11.3.5

Suplementary:

Bergmann Schäfer; Experimentalphysik Band II: de Gruyter.

Geiger, Scheel: Handbuch der Physik XXII. (Historische, vertiefende Anmerkungen)

Exercise

Determining the specific charge of the electron e/m_0 by measuring the magnetic deflection of electrons in a narrow beam tube.

Physical Principles

At the beginning of the 19th. centaury results of physical experiments showed clear indications for a *corpuscular* nature of the electric charge. The experiments of *Faraday* (*Michael Faraday*; 1791-1867; Eng. physicist and chemist) and the laws of electrolysis gave a definite relationship between the amount of electricity and the mass of the carrier of electricity and allowed, together with the idea of an atomic structure of material, the conclusion of the quantized nature of charge. *Helmholtz* (*Hermann von Helmholtz*; 1821-1894; Ger. physicist and physiologist) talked in a *Faraday commemoration speech* 1881 about the "*the electric charge of the ion*" and said "*just the same certain amount, let it be positive or negative electricity moves with each single valued ion or with each valence of a multi-valued ion, and accompanies it inseparably in all movements the ions make through the liquid. If carried over to the electrical processes, this hypothesis, leads, in connection with Faraday's law, to a somewhat surprising conclusion. If we assume atoms of chemical elements, then we must conclude that also electricity, positive as well as negative, is divided into certain elementary quanta which behave as atoms of electricity*." The electrolytic measurements, however, only gave mol values for ions, so that the determination of the specific charge and the ion charge presupposed the knowledge of Loschmidt's number.

The story of the discovery of the electron began in 1860 with the investigation of gas discharges and the discovery of cathode rays by *Plücker* 1859 (*Julius Plücker*; 1801-1868; German physicist and mathematician). *Hittorf* (*Johann Wilhelm Hittorf*; 1824-1914; German physicist and chemist) discovered in 1869 the magnetic- and *Goldstein* (*Eugen Goldstein*; 1850-1930; German physicist) in 1876 the electric deflection of cathode rays, whereby, *Hittorf* already suspected negatively charge particles were the actual physical nature of the cathode rays. Around 1890 *Schuster* (*K. Schuster*; *1903), *Thomson* (*Sir Joseph John Thomson*; 1856-1940; Engl. physicist) and *Wiechert* (*Emil Wiechert*; 1861-1928; German Geophysicist) performed the first measurements of the specific charge in various deflection experiments. Even though the early results of these measurements had large errors, it was found that "*the mass of the cathode rays must be about 2000 times smaller than that of the lightest ions*." Extensive measurements of the velocity (*Thomson*) and investigations on the velocity dependence of the mass and the agreement of the specific charge of the cathode rays with those of electrons released by photoelectric or thermal effects gave certainty to the existence of electrons. *Thomson* is named as the classical discoverer.

At the beginning of the 20th. centaury the value of the electron charge itself was measured (*Millikan* 1910-1925; see *MILLIKAN Experiment* in the laboratory script) and, especially due to the determination of the specific charge from the *Zeeman Effect* (*Pieter Zeeman*; 1865-1943; Dutch physicist) and from the *Rydberg Constant* (*Johannes Rydberg*; 1854-1919; Swed. physicist; see experiment *OPTICAL SPECTROSCOPY* in this laboratory script), the identity between the free electrons and the bound electrons as building blocks of atoms was proven.

The recommended values for the electron (CODATA 1999) are
(*Physikalische Blätter, March 2000*):

(1a) e/m_0 = 1.758820174 $\cdot 10^{11}$ C/kg
relative error: $1.2 \cdot 10^{-7}$

(1b) e = 1.602176462(63) $\cdot 10^{-19}$ C
relative error: $3.9 \cdot 10^{-8}$

(1c) m_0 = 9.10938188(72) $\cdot 10^{-31}$ kg
relative error: $7.9 \cdot 10^{-8}$

If one interprets the total rest mass of the electron according to *Einstein's equation ($E = mc^2$)* as the electrostatic energy content of spherical charge distribution of radius r, then we have:

$$m_0\, c^2 = \frac{e^2}{8\,\pi\,\varepsilon_0\, r} \qquad (2)$$

($E = CU^2/2$ with $C = 4\pi\varepsilon_0 r$). Solving for r and inserting the respective values gives $r = 1.4 \cdot 10^{-15}$ m. This value at least reflects correctly the order of magnitude of the electron radius as supported by deflection experiments, even when from the stand point of quantum physics it is basically unfounded to describe or explain the electron with classical concepts. In the literature it is usual to take the double value of (2) as the **classical electron radius**:

(3) $$r_0 = \frac{e^2}{4\pi \varepsilon_0 m_0 c^2} = 2.8179378(70)\cdot 10^{-15} m$$

Measurement Method

The ***Lorentz force*** acts on charged particles moving in a magnetic field ***B*** (*Hendrik Antoon Lorentz*; 1853-1928; Dutch physicist):

(4) $$\boldsymbol{F}_L = q\left(\boldsymbol{v} \times \boldsymbol{B}\right)$$

where ***v*** is the velocity of the charged particle. The *Lorentz force* always acts perpendicular to ***v***; and thus alters the direction of the velocity but not its magnitude. Particles with constant velocity in a homogenous and constant magnetic experience a constant change of direction, i.e. they trace out a circular path. The equation of motion for an electron (point particle) with $q = -e$ is:

(5) $$-e\left(\boldsymbol{v} \times \boldsymbol{B}\right) = m\dot{\boldsymbol{v}}$$

In a constant magnetic field in the *z*-direction with ***B*** = (0/0/B) the components of the equation of motion are as follows:

(6) $$\begin{bmatrix} e\,\boldsymbol{B}\,\boldsymbol{v}_x = m\dot{\boldsymbol{v}}_y \\ -e\,\boldsymbol{B}\,\boldsymbol{v}_y = m\dot{\boldsymbol{v}}_x \\ 0 = \dot{\boldsymbol{v}}_z \end{bmatrix}$$

A solution to the system of differential equations (5) is

(7) $$\begin{bmatrix} \boldsymbol{v}_x = -\boldsymbol{v}\sin\omega t \\ \boldsymbol{v}_y = -\boldsymbol{v}\cos\omega t \\ \boldsymbol{v}_z = 0 \end{bmatrix}$$

and leads to the condition ($m = m_o$):

(8) $$\omega = \frac{e}{m}\boldsymbol{B}$$

The *precession frequency* (8) of a charged particle in a magnetic field is known as the *Larmor frequency* (*Sir Joseph Larmor*, 1857-1942; Brit. physicist) and is of significance in atomic- and nuclear physics. With $\omega = \boldsymbol{v}/\boldsymbol{r}$, we have for the specific charge:

(9) $$\frac{e}{m} = \frac{\boldsymbol{v}}{\boldsymbol{r}\,\boldsymbol{B}} \quad \text{or} \quad \left(\frac{e}{m}\right)^2 = \frac{\boldsymbol{v}^2}{\boldsymbol{r}^2\,\boldsymbol{B}^2}$$

where *r* is the radius of the circular orbit.

If one accelerates electrons through an electric voltage *U*, the velocity of the electrons can be derived from the energy:

(10) $$e\boldsymbol{U} = \frac{1}{2} m\,v^2 \quad \text{or} \quad \boldsymbol{v}^2 = 2\boldsymbol{U}\frac{e}{m}$$

and inserting in (9) gives:

(11) $$\frac{e}{m} = \frac{2\boldsymbol{U}}{\boldsymbol{r}^2\,\boldsymbol{B}^2}$$

Narrow Beam Tube

The production, acceleration and investigation of the motion of electrons takes place in the ***narrow beam tube***. Free electrons are generated at the cathode of an electrode system and accelerated by a voltage (70-300 V) to the anode. A focusing electrode (Grid, *Wehnelt cylinder*; *Arthur Rudolph Wehnelt*; 1871-1944; dt. physicist) is placed between cathode and anode. The electron beam escapes into the electric field-free room through a hole in the anode.

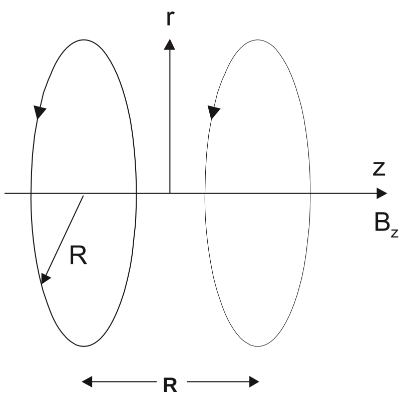
A homogenous magnetic field applied perpendicular to the direction of the beam forces the electrons into a circular orbit. The spherically shaped narrow beam tube is filled with hydrogen (about 1 Pa) so that the hydrogen molecules become optically excited by collisions with the electrons. The resulting emission of light makes the orbit of the electrons visible.

The reason why the electrons do not fan out due to mutual repulsion when entering the field-free space is connected with the focusing action of the space charge when electrons collide with the hydrogen molecules (narrow beam).

Helmholtz Coils

Helmholtz coils are used to generate homogenous and isotropic magnetic fields of medium strength. They consists of a pair of coils placed parallel and coaxial opposite each other. The magnetic field in the middle of a pair of windings in this arrangement is given by:

(12) $$\boldsymbol{B}_0 = \mu_0 \frac{8}{5\sqrt{5}}\frac{\boldsymbol{I}}{\boldsymbol{R}}$$



Points with the coordinates *z* and *r* (see figure) have the following axial and radial field components:

(13)

$$B_z = B_0 \left\{ 1 - \frac{18}{125} \left[3\frac{r^4}{R^4} - 24\frac{r^2}{R^2}\frac{z^2}{R^2} + 8\frac{z^4}{R^4} \right] + ... \right\}$$

(14)

$$B_r = B_0 \left\{ \frac{72}{125}\frac{r}{R}\frac{z}{R} \left[4\frac{z^2}{R^2} - 3\frac{r^2}{R^2} \right] + ... \right\}$$

Apparatus and Equipment

Narrow beam tube; Operating unit with adjustable anode voltage and auxiliary voltages. **Helmholtz coils**; Power supply unit for field current. Multimeters for accelerating voltages and field current.

Experiment and Evaluation

Attention: When preparing the tube for operation, take note of the information in the laboratory bench script!

Short wire pins are arranged in to tube to set the diameter of the electron orbit. The diameter specified by these pins is listed in the laboratory bench script.

A power supply unit is provided for the current supply of the ***Helmholtz coils***. When connecting the current circuit, ensure that both coils have their own connections and that the coils are connected in series. Coil data (radius, winding number) are also given in the laboratory bench script. The measurement of the accelerating voltage *U* and the magnetic current *I* is by means of two digital multimeters.

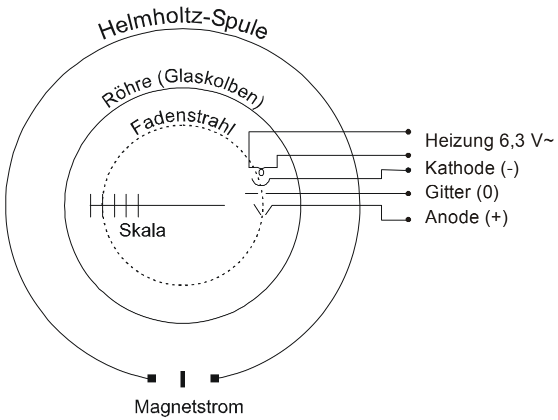
The measurements are to be carried out for the three largest orbital diameters. For evaluation (from (11) with (12)) the voltage *U* is plotted against the square of the current I^2. The specific charge is determined from the gradients of the straight-line curves.

A single final result is to be given (weighted mean value) from the three measurements.

Supplementary Questions

The experiment is carried out with electrons from a glow emission. What can be assumed about the initial energy of the electrons? How does this influence the measurement result?

How large is the linear velocity of the escaping electrons? Must a relativistic correction be taken into account?



The adjacent figure shows the basic circuit of the narrow beam tube. A special operating unit supplies the tube with voltage and current. The electrical connections for the tube are labeled on the operating unit and the sockets of the tube.

A focusing electrode (grid) is connected between the cathode and anode. The effective accelerating voltage, however, is the total voltage between cathode and anode.